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A STRUCTURAL OPTIMIZATION METHODOLOGY USING THE INDEPENDENCE AXIOM

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ABSTRACT

The Design Axioms provide a general framework for design methodologies. The axiomatic design framework has been successfully applied to various design tasks. However, the axiomatic design is rarely utilized in the detailed design process of structures when the optimization technology is carried out. The relationship between the axiomatic design and optimization is investigated and the Logical Decomposition method is developed for a systematic structural optimization. The entire optimization process is modified to satisfy the Independence Axiom. In the decomposition process, design variables are grouped according to sensitivities. The sensitivities are evaluated by the Analysis of Variance (ANOVA) to avoid considering only local values. The developed method is verified by examples such as the twenty-five member transmission tower and the two-bay-six-story frame.

Keywords : design axioms, optimization, logical decomposition method, sensitivity, analysis of variance(ANOVA)

1 INTRODUCTION

Design is defined as an interplay between "what we want to achieve" and "how we want to achieve" in an engineering process [Suh, 1990]. Traditionally, engineering design has been carried out by the experience or intuition of expert engineers. Recently, two design axioms have been created to help designers develop their designs in an objective and scientific way. They are (1) Independence Axiom and (2) Information Axiom. These axioms can be applied to all design processes in a general way. Many successful case studies have been developed to prove the validity of the axioms [Suh, 1990; Suh, to be published; Albano and Suh, 1992]. A design process conducted by the axioms is called axiomatic design or axiomatic process. In the axiomatic approach, a conceptual design can be carried out systematically and practical aspects can be included easily compared to other design methods such as design optimization. A special feature of the axiomatic design is that the design parameters (DPs, design variables) are determined independently for the corresponding functional requirements. Therefore, multiple functional requirements can be satisfied independently.

During the last two decades, engineering optimization has been developed tremendously [Haug and Arora; 1979, Arora, 1989; Haftka and Zafer, 1992]. In optimization, a given function is minimized (or maximized) while constraints are satisfied. The optimization technology is very well exploited for the automation of a design process. As the finite element method is established and applied, optimization is developed for the structural design. These days, engineering optimization is regarded as structural optimization. Although structural optimization gives an excellent design solution, it is difficult to consider the practical aspects because all the processes in the optimization must be defined mathematically with functions. Basically, optimization has one objective function (a single functional requirement) with multiple design variables (design parameters). Therefore, all the design variables are coupled in some sense and determined by an all—n-one approach. If we have multiple objective functions (functional requirements in the axiomatic approach), they should be modified into a single function. Thus, we may have different optimum according to the modification.

The design process can be divided into the conceptual design and the detailed design. As mentioned earlier, the axioms can be applied excellently in the conceptual design. There is a case study for detailed structural design with the axiomatic approach [Albano and Suh, 1992]. However, the decision making process for the detailed design is relatively simple in that research. Because optimization is superior in the detailed design of a complex system, this research was initiated to include optimization in the axiomatic design process for the detailed design.

A brief description of the developed scheme is as follows [Lee, 1998]: suppose we have multiple functional requirements (design objectives) and design parameters (design variables). A design matrix is defined for the axiomatic design according to the sensitivities. Analysis of variance (ANOVA) is utilized for the sensitivity information to cover some sectional trends [Taguchi, 1987]. Design variables can be grouped via the sensitivity information and the number of the groups is the same as that of the functional requirements. Good design matrices are uncoupled and decoupled ones. The relation between the mathematical optimum condition and the design matrices is investigated mathematically. A design scheme is defined from the investigation. Various standard examples are solved for the verification of the developed method.

2 THE RELATION BETWEEN AXIOMATIC DESIGN AND OPTIMIZATION

Design axioms have been created and applied to various engineering problems by Prof. N.P. Suh [Suh, 1990; Suh, to be published]. They provide a general framework for engineering activities. Because many references are available for the theory and application [Suh, 1990; Suh, to be published][Lee, 1998; Taguchi, 1987; Suh, 1995a; Suh, 1995b; Suh, 1984; Do, 1997], the detailed explanation is omitted here. Also, the structural optimization theories and applications are available in references [Albano and Suh, 1992; Huag and Arora, 1979; Arora, 1989; Haftka and Zafer, 1992].

2.1 AXIOMATIC DESIGN AND OPTIMIZATION WITH MULTIPLE FUNCTIONAL REQUIREMENTS

In the axiomatic design, uncoupled or decoupled design is a good design. The design matrix is a diagonal matrix for the uncoupled design and a triangular matrix for the decoupled design. The functional forms of the relations with two functional requirements are as follows:

$$f_1 = f_1(x_1), \quad f_2 = f_2(x_2) \; : uncoupled\ design \qquad (1)$$
$$f_1 = f_1(x_1), \quad f_2 = f_2(x_1, x_2) : decoupled\ design \qquad (2)$$

where f_1 and f_2 are functional requirements and x_1 and x_2 are corresponding design parameters. The design parameters can be determined separately in the uncoupled design and x_1 and x_2 should be determined sequentially in the decoupled design. If the optimization process follows this sequence for the decoupled design, mathematical optimum may not be obtained. In the optimization, the gradients of f_1 and f_2 must be zeros at optimum. In the uncoupled design in Eq. (1), the optimum values for f_1 and f_2 can be obtained independently and the process is the same as that of the axiomatic design. In the first step of the decoupled design, x_1 is determined from $f_1(x_1)=0$ and x_1 is fixed in the next step as x_1^*. In the second step, x_2 is determined from $f_2(x_1^*,x_2)=0$. The solution from the above process may not be the optimum evaluated when x_1 is not fixed in the second step. Therefore, the decoupled design may not be good one when the design solution is calculated by a mathematical optimization. Above statements are valid for the constrained problem if the functions are replaced by Lagrangians.

2.2 THE RELATION WITH A MULTI-OBJECTIVE FUNCTION IN OPTIMIZATION

In optimization, multiple objective functions are modified to a single function which is called a multi-objective function. A typical representation of the multi-objective is as follows:

$$f = w_i f_i \qquad (3)$$

When we have two objective functions in optimization, the representation for the axiomatic design are the same as Eqs. (1)-(2). In an uncoupled case, it is obvious that the solution of $(w_1f_1+w_2f_2)=0$ is the same as that from $f_1=0$ and $f_2=0$. Therefore, the solutions from the single objective function and the axiomatic process are the same. However, it is proved in reference [Haftka and Zafer, 1992] that the solutions are discerned in the decoupled case. It is noted that the decoupled case defined in Eq. (2) should not be solved in the axiomatic approach.

2.3 NEARLY UNCONPLED DESIGN

Suppose there is a coupled design as follows:

$$\begin{array}{ll} f_1 = f_1(x_1, x_2),\ f_2 = f_2(x_1, x_2) : coupled\ design & \\ Influence\ of\ x_1\ on\ f_1 \quad Influence\ of\ x_2\ on\ f_1 & (4) \\ Influence\ of\ x_2\ on\ f_2 \quad Influence\ of\ x_1\ on\ f_2 & \end{array}$$

According to the amount of the influence, the problem can be a nearly uncoupled problem. Therefore, the axiomatic process can be applied. The small influence may not be ignored in some problems. For theses cases, a design flow is suggested in Fig. 1 by an iterative manner.

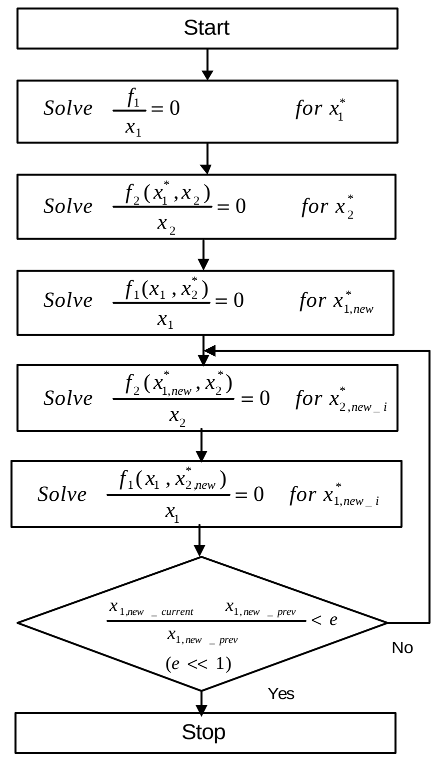


Figure 1. The flow of finding a solution in the nearly uncoupled design

3 AN AXIOMATIC DESIGN METHODOLOGY FOR STRUCTURAL OPTIMIZATION

Generally, the number of design variables is very large compared to the number of objective functions in structural optimization. However, the number of design parameters must be the same as that of the functional requirements in axiomatic design. When the number of design parameters is large, the parameters can be grouped to have similar characteristics. That is, important parameters to a specific functional requirement can be grouped into one set of parameters. Therefore, the number of the groups can be the same as that of the design objectives. Most of the structural optimization problems are coupled by all the design variables. It is almost impossible to make a perfect uncoupled problem. However, different objective functions may have different sensitivity amount for different design variables. Therefore, the design variables can be grouped according the sensitivity information and the grouping process can make a nearly uncoupled design. The pertinence is backed up by the tolerance in the following theorem:

Theorem 8 Independence and Tolerance
A design is an uncoupled design when the designer specified tolerance is greater than

$$\sum_{\substack{j \neq i \\ j=1}}^{n} \left| \left(\partial FR_i / \partial DP_j \right) \Delta DP_j \right| \quad ; i = 1, \ldots, n \qquad (5)$$

In which case the off-diagonal elements of the design matrix can be neglected from design consideration.

The design variables can be decomposed into groups by sensitivity analysis. The partial derivative in Eq. (5) is the sensitivity amount. The mathematical derivative may have local information only. Therefore, it is suggested to use analysis of variance (ANOVA) for the sensitivity information. The sums of squares in the ANOVA include the sensitivity in certain ranges. As mentioned earlier, when the off-diagonal elements in the design matrix are not totally negligible, the iterative process in Fig. 1 can be utilized.

Generally, the design variables are grouped according to the locations of parts in structural design. We call this decomposition the physical decomposition. The decomposition in this research is named as "logical decomposition" as opposed to the physical decomposition. The logical decomposition by the grouping of the design variables finds an appropriate design window. The flow of the logical decomposition is illustrated in Fig. 2.

4 EXAMPLES

Two standard problems in structural optimization are solved to show the validity of this research.

4.1 TWENTY-FIVE TRANSMISSION TOWER

The twenty-five member transmission tower is illustrated in Fig. 3. Objective functions are f_1=mass and f_2=the displacement in x direction at node 2. External loads are F_x=24kN and F_z=18kN at nodes 1 and 2, and F_x=30kN and F_y=40kN at nodes 3 and 6. Design variables are A1 (areas of members 1 to 9), A2 (areas of members 10 to 13), A3 (areas of members 14 to 25), Z3,Z4,Z5 and Z6 (coordinates in z direction at nodes 3,4,5 and 6 respectively). At the interested range, three levels for each design variables are defined. The orthogonal array L_{18} is utilized to calculate the square sums in ANOVA table [Taguchi; 1987]. The relative square sums are illustrated in Figs. 4 and 5.

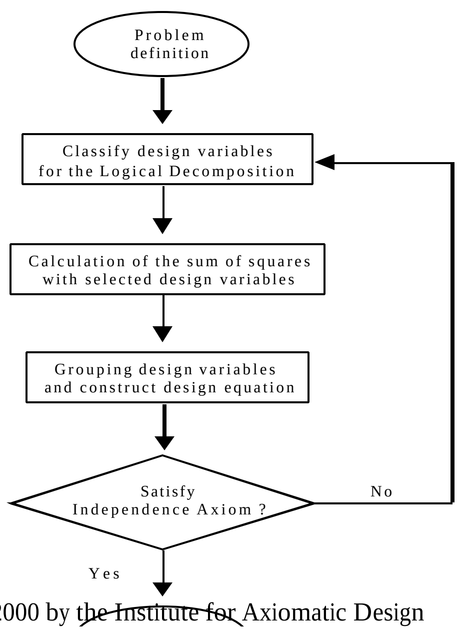


Figure 2. The flow of the logical decomposition

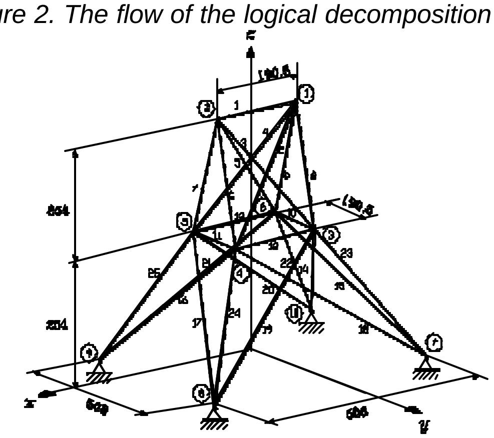


Figure 3. 25-member transmission tower

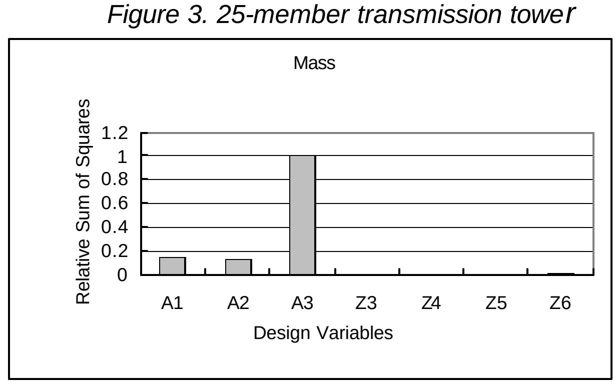


Figure 4. Relative sum of squares for the mass of 25-member transmission tower

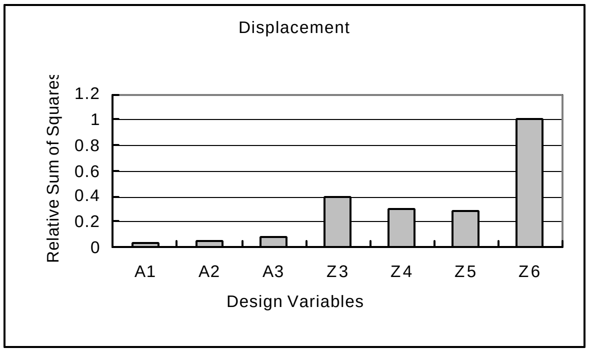


Figure 5. Relative sum of squares for the displacement of 25-member transmission tower

According to the preceding analysis, the design variables are grouped into two groups TDP1 and TDP2 as shown in Table 1.

Table 1. Group design variables for 25-member transmission tower

Group Design Variable	*Design Variable*
TDP1	*A1, A2, A3*
TDP2	*Z3, Z4, Z5, Z6*

The design equation is established as follows:

$$\begin{Bmatrix} \min \ mass \\ \min \ disp. \end{Bmatrix} = \begin{bmatrix} & O \\ O & \end{bmatrix} \begin{Bmatrix} TDP\,1 \\ TDP\,2 \end{Bmatrix} \qquad (6)$$

The zeros in Eq. 6 are not exact zeros. Therefore, it is a nearly uncoupled design. As mentioned earlier, an iterative process in Fig. 1 can be utilized. The optimization formulations are as follows:

$$\begin{aligned} &find \ TDP1 \\ &to \min mass \\ &subject\ to\ s \quad s_{all.} \end{aligned} \qquad (7)$$

$$\begin{aligned} &find \ TDP2 \\ &to \min displacement \\ &subject\ to\ s \quad s_{all.} \end{aligned} \qquad (8)$$

where s is a stress at each element and $s_{all.}$ is the allowable stress.

The structural optimization is carried out by a commercial software called GENESIS [VMA, 1998]. The optimum solution is obtained by three iterations as shown in Table 2. The optimum solution is as follows:

$$[A_1^*, A_2^*, A_3^*, Z_1^*, Z_2^*, Z_3^*, Z_4^*] = [1.9155, 0.66958, 2.1406, 47.01, 105.274, 70.436, 0.10]$$

Table 2. Results of the design for 25-member transmission tower

	Mass(kg)	*Displacement(mm)*
Initial response	*2.0620E+03*	*18.222E+00*
1st iteration	*1.6300E+03*	*17.379E+00*
2nd iteration	*1.6530E+03*	*17.377E+00*
3rd iteration	*1.6530E+03*	*17.377E+00*
Optimized response	*1.6530E+03*	*17.377E+00*

4.2 TWO-BAY, SIX-STORY FRAME

Optimization is carried out for the structure illustrated in Fig. 6. The problem is well described in references [Haug and Arora, 1979] and [Lee, 1998]. The design objectives are f_1=mass and f_2=y-directional displacement at node 2. Design variables are y coordinates of nodes 1,4,7,10,13 and 16. The nodes at the same story are constrained to have the same coordinates by the design variable linking.

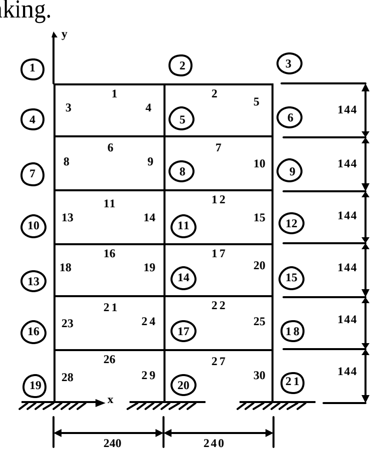
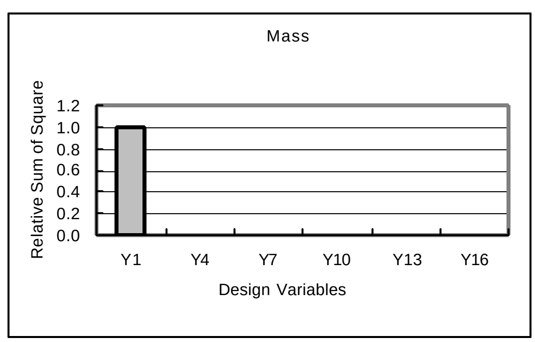


Figure 6. Two-bay, six story frame

The orthogonal array is L_{18} utilized to calculate the square sums with three levels of design variables [Taguchi, 1987].

Figure 7. Relative sum of squares for the mass of two-bay, six-story frame



As shown in the figures, the design variable Y1 has a large influence on the mass and the others are important to the displacement. Design variables can be decomposed. The group design variables followed by the decomposition are shown in Table 3 and the design equation is established in Eq. (9).

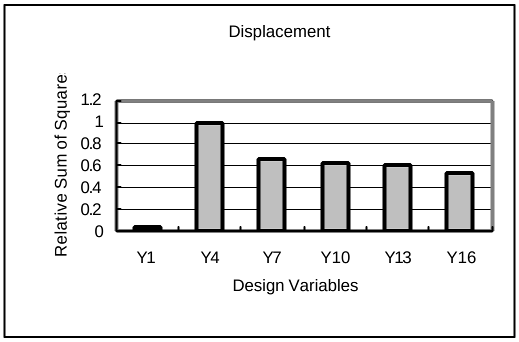


Figure 8. Relative sum of squares for the displacement of two-bay, six-story frame

Table 3. Group design variables of two-bay, six-story frame

Group Design Variable	*Design Variable*
SDP1	*Y1*
SDP2	*Y4, Y7, Y10, Y13, Y16*

$$\begin{Bmatrix} \min mass \\ \min displacement \end{Bmatrix} = \begin{bmatrix} & O \\ O & \end{bmatrix} \begin{Bmatrix} SDP1 \\ SDP2 \end{Bmatrix} \qquad (9)$$

The off-diagonal terms in Eq.(9) are not exactly zeros. Because they can be ignored, the design equation is a nearly-uncoupled one. The optimization process can be applied by an iterative manner in Fig. 1 and the result is shown in Table 4. The mass and displacement are reduced by 2.7% and 7.2%, respectively.

Table 4. Results of the design of two-bay, six-story frame

	Mass(kg)	*Displacement(mm)*
Initial response	*7.8368E+03*	*62.054E+00*
1st iteration	*7.8283E+03*	*58.508E+00*
2nd iteration	*7.8176E+03*	*58.005E+00*
3rd iteration	*7.6221E+03*	*57.607E+00*
4th iteration	*7.6221E+03*	*57.607E+00*
Optimized response	*7.6221E+03*	*57.607E+00*

5 SUMMARY

A design methodology has been developed to use the axiomatic design in the optimization process in detailed design. The following statements are summarized.

(1) When the axiomatic design is applied, it is proved mathematically that the optimization problem must be decomposed into an uncoupled problem. However, if the problem is a nearly uncoupled problem, an iterative method is suggested.
(2) A "logical decomposition" method is suggested to solve the complex optimization problem with decomposition. The logical decomposition can be obtained by sensitivity information. The square sums can be utilized efficiently to cover some sectional information.
(3) Examples have been solved to show the validity of the developed methodology. In a truss example, the mass is reduced by 19.8% and the displacement is reduced by 4.6%. The frame optimization reduces the mass and displacement by 2.7% and 7.2%, respectively.

6 ACKNOWLEDGMENTS

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