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An Integer Programming Formulation For The Concept Selection Problem With An *Axiomatic* Perspective (Part II): Fuzzy Formulation

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ABSTRACT

This paper is an extension to the formulation presented in Part I and deals with design situations where there is no sufficient information to warrant the use of Part I deterministic optimization. The formulation assumes the existence of a design alternatives pool with enough expertise to score ranking against the selected criteria. Part II builds upon the rationale adopted in Part I[1] for the selection criteria and the integer programming. Therefore, a function of complexity, value, cost and customer satisfaction will be used as an objective function criterion. This rationale is rooted in the concepts of QFD, axiomatic design and value engineering. The formulation is expanded to include technical and assembly feasibility as constraints. The formulation uses some concepts of fuzzy set theory to quantify complexity, a formulation ingredient.

Keywords: Axiomatic Design, Fuzzy Set Theory, Complexity, Integer Programming, QFD, Concept Selection

1. INTRODUCTION

The concept selection problem is to select the best design entity that not only satisfies the customer requirements but also outperforms the other alternative solutions based on a set of selection criteria. The selection problem involves the following three major steps: (1) identification of the selection criteria, (2) the ranking (scoring) of different design entities against the selection criteria, and (3) the identification of the 'best' (optimum) entity. The best conceptual entity is the one that scores favorably in the ranking against a criterion. However, the problem become more complex when multiple criterion are involved. The selection problem may become judgmental so it will be very prone to bias as ranking will be driven to favor some pre-selected conceptual entity. The bias problem can be eliminated by the systematic employment of the disciplined selection process. The process creditability and robustness are greatly enhanced when coupled with the state-of-the-art design theories.

In this paper, a formulation of the selection problem as an integer programming problem with, mainly, two selection criteria: customer satisfaction and design complexity is presented. The choice of design complexity as a selection criterion is stemming from the information axiom (Suh 1990) of the axiomatic design (AD) approach. In addition, the proposed formulation is built around concepts borrowed from Quality Function Deployment (QFD).

The use of probability distributions (Part I) indicate the case of the incremental design classification, i.e. experienced design situations with the needed information available to calculate complexity. Incremental design is a design that is within a *slight* variation of the current design. In many design situations, especially those classified as creative design solutions, we do not have this luxury of information and complexity can not be calculated. The type of information in the creative situation is qualitative and fuzzy in the form of engineering judgment. The existence of fuzzy information can be utilized to infer probability distribution from fuzzy distributions using the concepts of *possibility distribution, possibility-probability principle, and maximum entropy principle*. The possibility distribution is a key for our formulation that utilizes the fuzzy modeling in the cases of inexperienced design situation.

This paper is developed as follows: Section 2 contains the needed fuzzy set theory background, Section 3 is the core section of Part II and is devoted for the fuzzy formulation of the selection problem. Section 4 is the conclusion section.

2. THE FUZZY MODELING

Linguistic inexactness (imprecision) is the most common feature of many real life situations. Dutta (1985) classifies imprecision according to its source: measurement, stochastic, ambiguous definitions, incomplete knowledge, etc. In decision making, for example, the usefulness of mathematical algorithms is in having clearly defined objective criteria and constraints. They are only as good as the information they are given. Information has to be

[1] The rationale is discussed in Part I and not repeated here. The reader is encouraged to refer to Part I for clarification.

crisp (precise) in order to yield precise decisions (Zimmermann 1985).

Certainty formulations require structure with precise parameters. However, most real life situations are characterized linguistically with degrees of imprecision. Precision implies no ambiguity by assuming that variables, parameters, and structure represent deterministic situations as we did in Part I. The imprecision issue is further complicated in the classification of creative design. In the early phases, a design is a collection of scattered conceptual thoughts and rough drawings. The difficulty in design problem formulation often lies in establishing precise objectives, constraints and functional requirements which are uncertain , do not fall between what we consider as definite and precise. Even when the design matures to a physical entity via the mapping process, it may still need further tuning and optimization, partly, due to the uncertainty in characterizing noise factors effect during the concept design stage. It is almost the case, that, we can not make *deterministic* assertion with respect to certain phenomena because we can not measure, do not know, can not calculate all factors involved (Stark and Woods, 1986). We attribute variance between products passing the same processes to randomness by discounting the system to its average behavior. To do that, we use probability theory to handle randomness. As such, design models can't be described as unequivocal. No comprehensive design models can be written even for incremental designs situations. Unfortunately, existing knowledge is normally centered around the crisp incremental (adaptive) classifications. Under these circumstances, many suggest that, a design problem complexity can be lessened using conclusions of empirical knowledge. The result of these activities are the dominating formal models. In customer-oriented design, customers have wants and needs that are hard to interpret. They are expressed, linguistically, using terms which have no precise definition. A statement is not always right or wrong, as people are not always classified as smart or stupid, and a linear programming (LP) problem is not always feasible or infeasible. Yet, to classify an LP problem as for most classical decision making, one description or the other must be chosen. This is in accordance with the *law of excluded middle* (Klir 1988). This dichotomous property is the basis of *classical set theory*. By the same analogy, systems (solution entities) can not always classified as vulnerable or non-vulnerable, robust or not-robust. In this context, robustness may be viewed as a continuous measure of some *possibility distribution*..

An example that may be used to facilitate the fuzzy concepts is as follows. Assume that there are 4 design proposal (solution entities), say the crisp set $S = \{S_1, S_2, S_3, S_4\}$. We would like to select a solution entity at random from S. The probability distribution in this case is: $p(\{S_1\}) = p(\{S_2\}) = p(\{S_3\}) = p(\{S_4\}) = \frac{1}{4}$. If we were asked to select randomly a *successful* creative design, we can't use the probability distribution above because of the fuzziness in the word 'Successful'. The answer is in defining 'design solution' ,say Y, as a variable that takes in values in the set S, according to a probability distribution constructed around "Y is successful".

A fuzzy set accepts objects with certain degree, the so called membership function (Zadeh 1965). The fuzzy set $\tilde{A}$ is represented as : $\tilde{A} = \{(FR, \mu_A(FR)) / FR \in FRs\}$ with $m_A(FR)$ understood to represent a mapping of membership of FR, m_A: $FRs \rightarrow [0,1]$: $FR \rightarrow m_A(FR)$. It is understood that in the crisp case, $\forall FR \in A, \mu_A(FR) = 1$ and zero otherwise. Every mapping of this nature with some conceptual realization (in alignment with intuitive semantics of imprecise description of FR) is a fuzzy set. For example, FRs can be the universe of fuzzy functional requirements, such as stylish, cheap, convenient, etc.

2.1 Possibility-Probability Consistency Principle

The fuzzy information about the elements of a finite set can be represented by a possibility distribution. Possibility theory was first introduced by Zadeh (1978), as an interpretation of a fuzzy set. The concept was further developed by both Dubois and Prade (1988). Possibility is concerned with linguistic uncertainty that is assumed to be *possibilistic* rather than probabilistic. For example, the proposition "$X1$ is $\tilde{A}$" is a possibility proposition where $X1$ is a variable taking the values $x1$ and $\tilde{A}$ is a fuzzy set with $m_A(x1)$. Possibility distribution is considered some how a modeling to fuzzy restriction. Zadeh (1975), (1978) proposed the following definitions:

Definition 1

Let $\tilde{A}$ be a fuzzy set in the universe X with membership $m_A(x)$ interpreted as the compatibility of $x \in X$ with concept label $\tilde{A}$. Let $X1$ be a variable with values in X and $\tilde{A}$ acting as a fuzzy restriction, $R(X1)$, associated with X. Then the proposition "$X1$ *is* $\tilde{A}$" , which translates into $R(X1) = \tilde{A}$ associates a possibility distribution , π_x , in which $X1$ is postulated to be equal to $R(X1)$. The possibility distribution is $\pi_x = m_A(x)$.

The relation between probability and possibility has been the focus of Zadeh (1978), Dubois and Prade (1982). The possibility-probability consistency principle is the foundation of such a relationship. Based on this principle, Lueng (1980) suggested deriving the probability (p_x) of success based on fuzzy information (π_x) using the consistency principle as an evidence in the framework of *maximum entropy principle*. The important advantage of this formulation lies in the transforming the fuzzy information into a deterministic measure for creative design situations.

2.2 The Maximum Entropy Formulation

There would be much controversy if the designer assigns, rather than assesses, the probability of success in the concept phase to quantify complexity. From the perspective of developments discussed in the previous sections, it would appear that the problem is simply deciding how to encode available information. However, the problem is not that simple. It is indeed difficult to answer fundamental questions about design knowledge. Often we can be fairly explicit about what we know in regard to a specific question. However, this knowledge can be incomplete and must be encoded in a possibility distribution before we can make use of inferential methods. In Part I, the author introduced the concept of entropy, with the average entropy is given $H = -\sum p_i \log p_i$. The concept of entropy, and its extended notions, is used to handle the issue of uncertainty. Jaynes (1957) proposed the principle of maximum entropy, and this principle has been employed in different disciplines, e.g., thermodynamics (Tribus 1961) and urban modeling (Wilson 1970). The maximum entropy principle addresses the assignment of prior probabilities based on prior knowledge. Jaynes (1957) showed that the least presumptuous way to assign prior probability is by maximizing the entropy function in Eq. (17) subject to the normalization constraint, Eq.(18). In this meaning, Jaynes (1957) added: "The minimally prejudiced probability distribution is that which maximizes entropy subject to constraints supplied by the given information."

Maximum entropy is most beneficial when the knowledge is characterized in average form. The formulation of maximum entropy can be characterized as follows: a *DP* is a variable that can have different possible nominal values in the concept stage, but we do not know the value. However, we know the possibilities, and we wish to find the probabilities. We would like to generate a probability distribution which agrees with the averages but are maximally non-committal with respect to anything else.

In the following formulation we will treat the variable *DP*, as a discrete variable that takes its value from the universal set of *DPs*. In the continuous *DP* form, we need to substitute sums by integrals and take the differences between discrete values. Sometimes we are only concerned with a discretized version of the continuous variable, i.e. DP_1, DP_2, ..., DP_d and a discrete formulation will fit.

As such, the problem is expressed mathematically by Leung (1980) as follows:

$$Max.H = -\sum_{i=1}^{d} p_i \log p_i \qquad (17)^2$$

Subject To:

$$\sum_{i=1}^{d} p_i = 1 \qquad (18)$$

$$\text{"}\pi \textit{ is consistent with } p\text{"} \qquad (19)$$

where p_i is the probability that *DP* will have the value DP_i, $DP_i \in \left[DP_i - \frac{\delta}{2}, DP_i + \frac{\delta}{2} \right]$ and d is the number of discrete intervals.

[2] Note that the equations numbering starts with (17) to stress the logical flow between Part I & Part II.

The distribution which maximizes Eq. (17) is considered a minimally prejudiced assignment in that it makes the distribution maximally vague or general. The term 'minimally prejudiced' implies that the distribution is so general and is maximally influenced by new data. Eq. (19) indicates that at least one of the assertions is true. In Eq. (20), π is the possibility distribution (the membership function) of the set. Zadeh (1978) suggested the following definition of the consistency principle

$$\sum_{i=1}^{d} p_i \pi_i = \alpha \qquad (20)$$

where $\alpha \in [0,1]$ and near one.

Dubois and Prade (1982) proposed their own definition of the consistency principle: a probability distribution (p) and a possibility distribution (π) are consistent if $\forall DP \subset DPs$

$$\pi(DP_i) \geq p(DP_i) \quad \forall DP_i \in DP \qquad (21)$$

Both definitions in Eq.s: (20) or (21) can replace Eq.(19).

Example

The surface finish of a transmission oil pan is a significant design parameters for sealing. A design organization is considering using a silicon elastomer as a possible replacement for the current solid plastic seal. The use of silicon elastomers has very attractive cost advantages over the current design. The design organization has no experience with silicon applications, and they would like to determine the nominal value of the surface finish of the oil pan that will maximize the probability of success. The Material Engineering Department was consulted and provided the following possibility distribution of success at four possible nominal discretized values, $\{DP_1, DP_2, DP_3, DP_4\}$, of the surface finish

$$\begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \\ DP_4 \end{Bmatrix} = \begin{Bmatrix} 0.50 \\ 0.60 \\ 0.80 \\ 1.00 \end{Bmatrix}$$

The design organization would like to know the probability of success at the following consistency levels $\alpha = 0.85, 0.90, 0.95$.

Solution

We can use the GAMS to solve the program: Eq.(17), Eq.(18), and Eq.(20) to obtain the probability distribution shown in Table 1.

Table 1: The probability distribution obtained from the fuzzy data p

α	*Dist.*	*DP1*	*DP2*	*DP3*	*DP4*
	π	0.500	0.600	0.800	1.000
0.85	p	0.092	0.130	0.260	0.518

0.90	*p*	0.047	0.079	0.226	0.649
0.95	*p*	0.013	0.030	0.156	0.800

The sum of the probability of success of the combined discretized ranges $\sum_{i=3}^{4} DP_i$, $DP_i \in \left[DP_i - \frac{\delta}{2}, DP_i + \frac{\delta}{2} \right]$ is greater than 80% even at low consistency levels. The combined range represents the design range, *dr*. If the system range is the whole range, i.e., $\sum_{i=1}^{4} DP_i$, then the area P_{sr} is unity. Thus the common range area, P_{cr}, is the same as the design range. By using the definition of complexity adopted in axiomatic design (Eq.(3), Part I), the following probability of success (Table 2) and complexity levels can be determined

The probability of success for a given functional requirement (e.g. the sealing function) can be calculated when the respective *DP* is selected (e.g. a compression-based gasket vs. a chemical elastomer). Since an *FR* is a function of random

Table 2: The complexity levels of the probabilities

α	P_{cr}	P_{sr}	p	H (nats)
0.85	0.778	1.000	0.778	0.251
0.90	0.875	1.000	0.875	0.134
0.95	0.956	1.000	0.956	0.045

variables, the probability of success, as well as complexity levels can be found. Hence, H_{ik} can be calculated and substituted into the formulation presented in Part I as follows:

1. Determine the discrete set values for all the *DPs* and *PVs* at instance '*k*' identified by the physical and process mappings.
2. Determine the membership function of the fuzzy set, "successful", around these set values
3. Solve the discrete mathematical formulation [Eq.s: (17)-(18), (20) or (21)] to obtain the probabilities of success
4. Substitute the probabilities in Eq. (3) to obtain H_{ik}
5. Repeat Steps: 1 through 4 for every instance, a *DP*, of the functional structure, i.e. all feasible physical solution entities
6. Substitute H_{ik} in the integer programming formulation that was presented in Part I, Eqs.:(6) -(10) or Eqs: (11) through (16).
7. Solve to select the best solution entity.

In addition to the probability-consistency principle, there have been many attempts to combine probabilistic and fuzzy measures in a discrete framework. Zadeh(1968) first introduced the entropy of a fuzzy set with respect to a discrete probabilistic as the weighted Shannon entropy. Other frameworks to combine probabilistic and fuzzy measures was suggested by Xie and Bedrosian (1984), Pal and Pal (1992). We found that these measures are hard to justify in our context.

3. THE CONCEPT SELECTION PROBLEM: A FUZZY FORMULATION

The above development to obtain an estimate for design complexity from fuzzy data simplifies the effort to find a solution to a fuzzy version of the integer programming formulation of the selection in Part I. In this case information content is quantified using deterministic quantities rather than fuzzy quantities. Another approach is to fuzzify the integer programming itself as a totality. The fuzzification of the integer program can be carried out by fuzzyfing any combination of the variables *PI, W,* or *AW*. In the derivation below, the variables *W* and *AW* were used as fuzzy concepts. Extension of the development to include *PI* does not contribute to the formulation clarity and it was dropped. Nevertheless, the reader can follow the derivation here to include *PI* as a fuzzy concept when desired.

The modeling of *W* and *AW* as fuzzy numbers allow more realistic and robust representation of the imprecision and the linguistic inexactness experienced in the selection process. In this case, the 'function weight' rating can be viewed as a fuzzy linguistic variable (denoted as $\tilde{W}$) for *QFD's correlation factor* and the 'attribute weight' rating can be viewed as a fuzzy linguistic variable for *QFD's importance factor*. Both factors take linguistic values in a set of rating with elements modeled as fuzzy numbers. A fuzzy number is a convex normalized piecewise continuous fuzzy set on the real line. For computational efficiency, Dubois and Prade (1979) suggested a fuzzy number representation that depends on the identification of two reference functions: *L* for left and *R* for right and the spreads α and β, respectively. A fuzzy number $\tilde{A}$ in *LR* representation can be written as $\tilde{A} = (t, \alpha, \beta)_{LR}$ and is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{t-x}{\alpha}\right) & x \le t \\ R\left(\dfrac{x-t}{\beta}\right) & x \ge t \end{cases} \tag{22}$$

$x \in R$, the set of real numbers. For example, the 'attribute weight' variable (*AW*) can be fuzzified when it assumes labels in the set {low, medium, high}. Each value in this label set can be modeled as a fuzzy number that is described by the parameteric form in Eq. (22). Note that $AW_j = AV_j \bullet IR_j \bullet SP_j$ can only be fuzzified by fuzzifying at least one of its arguments.

In the *LR* representation, if $\tilde{B} = (s, \gamma, \tau)_{LR}$ then $\tilde{A} + \tilde{B} = (t+s, \alpha+\gamma, \beta+\tau)_{LR}$ and $\tilde{A} - \tilde{B} = (t-s, \alpha+\tau, \beta+\gamma)_{LR}$. For extended product, we have the following rules: $\tilde{A}.\tilde{B} \approx (ts, s\alpha + t\gamma, s\beta + t\tau)_{LR}$ when $\tilde{A}$ and $\tilde{B}$ are both positive numbers; $\tilde{A}.\tilde{B} \approx (ts, -s\beta - t\tau, s\alpha - t\gamma)_{LR}$ when $\tilde{A}$ and $\tilde{B}$ are both negative numbers, and $\tilde{A}.\tilde{B} \approx (ts, s\alpha - t\tau, s\beta - t\gamma)_{LR}$ when $\tilde{A}$ is positive and $\tilde{B}$ is negative.

Let $\tilde{W}_{ik} = (t_{ik}, \alpha_{ik}, \beta_{ik})_{LR}$ and $A\tilde{W}_j = (s_j, \gamma_j, \tau_j)_{LR}$. Let the set of customer attributes, *C*, be partitioned into four subsets. Let: $C^{\langle +,+\rangle}, C^{\langle +,+\rangle} \subset C$, be the subset, with cardinality $J^{\langle +,+\rangle}$, where both $\tilde{W}_{ik}$ and $A\tilde{W}_j$ are positive fuzzy numbers; $C^{\langle -,-\rangle}$, $C^{\langle -,-\rangle} \subset C$, be the subset, with cardinality $J^{\langle -,-\rangle}$, where both $\tilde{W}_{ik}$ and $A\tilde{W}_j$ are negative fuzzy numbers; $C^{\langle +,-\rangle}, C^{\langle +,-\rangle} \subset C$, be the subset, with cardinality $J^{\langle +.-\rangle}$, where $\tilde{W}_{ik}$ is a positive fuzzy number and $A\tilde{W}_j$ is a negative fuzzy number; $C^{\langle -,+\rangle}, C^{\langle -,+\rangle} \subset C$ be the subset, with cardinality $J^{\langle -,+\rangle}$, where $\tilde{W}_{ik}$ is a negative fuzzy number and $A\tilde{W}_j$ is a positive fuzzy number. Then $C = C^{\langle +,+\rangle} \cup C^{\langle -,-\rangle} \cup C^{\langle +,-\rangle} \cup C^{\langle -,+\rangle}$ and $J = J^{\langle +,+\rangle} + J^{\langle -,-\rangle} + J^{\langle +,-\rangle} + J^{\langle -,+\rangle}$. The fuzzy integer programming formulation can be written as

$$Max. \quad \tilde{O} = \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\tilde{W}_{ik}\right)_{LR} . \left(A\tilde{W}_j\right)_{LR} . \left(\frac{Y_{ik}}{\sum_{i=1}^{m} H_{ik} Y_{ik}}, 0, 0\right)_{LR} \tag{23}$$

Subject To:

$$\sum_{k=1}^{K} Y_{ik} T_{ik} = 1 \qquad \forall i,\ i = 1,\ 2,\ \ldots,\ m,\quad DP_k \in \boldsymbol{F} \tag{24}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{J}\left(\tilde{W}_{ik}\right)_{LR} . \left(A\tilde{W}_j\right)_{LR} . \left(Y_{ik}, 0, 0\right)_{LR} > \left(\sum_{i=1}^{m_d}\sum_{j=1}^{J}\left(\tilde{W}_{ik}\right)_{LR} . \left(A\tilde{W}_j\right)_{LR} . \left(Y_{ik}, 0, 0\right)_{LR}\right)_{datum} \tag{25}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} H_{ik} Y_{ik} < \left(\sum_{i=1}^{m_d} H_i\right)_{datum} \tag{26}$$

$$\sum_{i=1}^{m-1}\sum_{u=1+i}^{m} Y_{ik} Y_{ul} Z_{kl} \le m-1,\ i \ne u;\ i = 1,2,\ldots,m-1; \quad u = 2,3,\ldots,m \tag{27}$$

$$Y_{ik} = 0 \text{ or } 1 \tag{28}$$

By applying the *LR* mathematics, the sum of the *LR* fuzzy number representation in each subset can be calculated as follows

For $j \in C^{\langle +,+\rangle}$ we have

$$\left(\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle +,+\rangle}} t_{ik} s_j Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle +,+\rangle}} \left(t_{ik}\gamma_j + s_j \alpha_{ik}\right) Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle +,+\rangle}} \left(t_{ik}\tau_j + s_j \beta_{ik}\right) Y_{ik}\right)_{LR} \tag{29}$$

For $j \in C^{\langle -,-\rangle}$ we have

$$\left(\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle -,-\rangle}} t_{ik} s_j Y_{ik}, -\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle -,-\rangle}} \left(t_{ik}\tau_j + s_j \beta_{ik}\right) Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle -,-\rangle}} \left(-t_{ik}\gamma_j + s_j \alpha_{ik}\right) Y_{ik}\right)_{LR} \tag{30}$$

For $j \in C^{\langle +,-\rangle}$ we have

$$\left(\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle +,-\rangle}} t_{ik} s_j Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle +,-\rangle}} \left(t_{ik}\tau_j - s_j \alpha_{ik}\right) Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle +,-\rangle}} \left(t_{ik}\gamma_j - s_j \beta_{ik}\right) Y_{ik}\right)_{LR} \tag{31}$$

For $j \in C^{\langle -,+\rangle}$ we have

$$\left(\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle -,+\rangle}} t_{ik} s_j Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle -,+\rangle}} \left(-t_{ik}\tau_j + s_j \alpha_{ik}\right) Y_{ik}, \sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{j\in J^{\langle -,+\rangle}} \left(-t_{ik}\gamma_j + s_j \beta_{ik}\right) Y_{ik}\right)_{LR} \tag{32}$$

The overall objective function $\tilde{O}$ in the *LR* format is given in Eq.(33).

$$
\tilde{O}=\left(\begin{array}{l}
\dfrac{\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{J}t_{ik}s_jY_{ik}}{\sum_{i=1}^{m}\sum_{k=1}^{K}H_{ik}Y_{ik}},\\[2ex]
\dfrac{\sum_{i=1}^{m}\sum_{k=1}^{K}Y_{ik}\left(\begin{array}{l}\sum_{j=1}^{j\in J^{\langle+,+\rangle}}\left(t_{ik}\gamma_j+s_j\alpha_{ik}\right)-\\ \sum_{j=1}^{j\in J^{\langle-,-\rangle}}\left(s_j\beta_{ik}+t_{ik}\tau_j\right)\\ \sum_{j=1}^{j\in J^{\langle+,-\rangle}}\left(s_j\alpha_{ik}-t_{ik}\tau_j\right)+\\ \sum_{j=1}^{j\in J^{\langle-,+\rangle}}\left(t_{ik}\tau_j-s_j\alpha_{ik}\right)\end{array}\right)}{\sum_{i=1}^{m}\sum_{k=1}^{K}H_{ik}Y_{ik}},\\[2ex]
\dfrac{\sum_{i=1}^{m}\sum_{k=1}^{K}Y_{ik}\left(\begin{array}{l}\sum_{j=1}^{j\in J^{\langle+,+\rangle}}\left(t_{ik}\tau_j+s_j\beta_{ik}\right)+\\ \sum_{j=1}^{j\in J^{\langle-,-\rangle}}\left(s_j\alpha_{ik}-t_{ik}\gamma_j\right)\\ \sum_{j=1}^{j\in J^{\langle+,-\rangle}}\left(s_j\beta_{ik}-t_{ik}\gamma_j\right)+\\ \sum_{j=1}^{j\in J^{\langle-,+\rangle}}\left(t_{ik}\gamma_j-s_j\beta_{ik}\right)\end{array}\right)}{\sum_{i=1}^{m}\sum_{k=1}^{K}H_{ik}Y_{ik}}
\end{array}\right)_{LR} \tag{33}
$$

An optimum and feasible physical configuration is the one that maximizes the mean of $\tilde{O}$ while minimizing imprecision, that is, minimizing the left and right spreads in its *LR* representation. Using this reasoning, the problem can be formulated as a deterministic non-linear {0,1} integer programming problem. That is, the problem is transferred from a fuzzy non-linear {0,1} integer program to a deterministic non-linear {0,1} integer program with constraints derived from fuzzy quantities. In the deterministic domain, the objective function can take a quotient form where the numerator is the mean of $\tilde{O}$ while the denominator is the sum (or the product) of the left and right spreads of Eq.(33). The spread sum form of the objective was adopted and is given in Eq.(35). In addition, the fuzzy constraint in [Eq.(25)] should be converted to its deterministic form. Let the datum design performance (the right hand side of Eq.(25)) be given in Eq.(34).

$$
\left(\sum_{j=1}^{J}\tilde{W}_{ik}A\tilde{W}_j\right)_{datum}=(w,\varepsilon,\omega)_{LR} \tag{34}
$$

Then by employing the subtraction rule of fuzzy numbers of *LR* representation and by using $(0,0,0)_{LR}$ as a neutral element for the addition operation we get constraint Eq.s: (36)-(38). The overall program can be assembled by appending constraints Eq.s: (24),(26),(27), and (28) and is given by Eq.s: (35)-42.

Max.

$$
\frac{\left(\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{J}t_{ik}s_jY_{ik}\right)\left(\sum_{i=1}^{m}\sum_{k=1}^{K}H_{ik}Y_{ik}\right)}{\left(\sum_{i=1}^{m}\sum_{k=1}^{K}Y_{ik}\left(\begin{array}{l}\sum_{j=1}^{j\in J^{\langle+,+\rangle}}\left(t_{ik}\gamma_j+s_j\alpha_{ik}\right)-\\ \sum_{j=1}^{j\in J^{\langle-,-\rangle}}\left(s_j\beta_{ik}+t_{ik}\tau_j\right)+\\ \sum_{j=1}^{j\in J^{\langle+,-\rangle}}\left(t_{ik}\tau_j-s_j\alpha_{ik}\right)+\\ \sum_{j=1}^{j\in J^{\langle-,+\rangle}}\left(s_j\alpha_{ik}-t_{ik}\tau_j\right)\end{array}\right)\right)+\left(\sum_{i=1}^{m}\sum_{k=1}^{K}Y_{ik}\left(\begin{array}{l}\sum_{j=1}^{j\in J^{\langle+,+\rangle}}\left(t_{ik}\tau_j+s_j\beta_{ik}\right)+\\ \sum_{j=1}^{j\in J^{\langle-,-\rangle}}\left(s_j\alpha_{ik}-t_{ik}\gamma_j\right)+\\ \sum_{j=1}^{j\in J^{\langle+,-\rangle}}\left(t_{ik}\gamma_j-s_j\beta_{ik}\right)+\\ \sum_{j=1}^{j\in J^{\langle-,+\rangle}}\left(s_j\beta_{ik}-t_{ik}\gamma_j\right)\end{array}\right)\right)} \tag{35}
$$

Subject To:

$$
\left(\frac{\sum_{i=1}^{m}\sum_{k=1}^{K}\sum_{j=1}^{J}t_{ik}s_jY_{ik}}{\sum_{i=1}^{m}\sum_{k=1}^{K}H_{ik}Y_{ik}}\right)-w>0 \tag{36}
$$

$$\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} Y_{ik} \left(\begin{array}{l} \sum_{j=1}^{j \in J^{\langle +,+\rangle}} \left(t_{ik}\gamma_j + s_j\alpha_{ik}\right) - \sum_{j=1}^{j \in J^{\langle -,-\rangle}} \left(s_j\beta_{ik} + t_{ik}\tau_j\right) \\ \sum_{j=1}^{j \in J^{\langle -,+\rangle}} \left(s_j\alpha_{ik} - t_{ik}\tau_j\right) + \sum_{j=1}^{j \in J^{\langle +,-\rangle}} \left(t_{ik}\tau_j - s_j\alpha_{ik}\right) \end{array} \right)}{\sum_{i=1}^{m}\sum_{k=1}^{K} H_{ik}Y_{ik}} + \omega > 0 \quad (37)$$

$$\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} Y_{ik} \left(\begin{array}{l} \sum_{j=1}^{j \in J^{\langle +,+\rangle}} \left(t_{ik}\tau_j + s_j\beta_{ik}\right) + \sum_{j=1}^{j \in J^{\langle -,-\rangle}} \left(s_j\alpha_{ik} - t_{ik}\gamma_j\right) \\ \sum_{j=1}^{j \in J^{\langle -,+\rangle}} \left(s_j\beta_{ik} - t_{ik}\gamma_j\right) + \sum_{j=1}^{j \in J^{\langle +,-\rangle}} \left(t_{ik}\gamma_j - s_j\beta_{ik}\right) \end{array} \right)}{\sum_{i=1}^{m}\sum_{k=1}^{K} H_{ik}Y_{ik}} + \varepsilon > 0 \quad (38)$$

$$\sum_{k=1}^{K} Y_{ik}T_{ik} = 1 \qquad \forall i,\ i = 1,\ 2,\ \ldots,\ m, \quad DP_k \in F \quad (39)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} H_{ik}Y_{ik} < \left(\sum_{i=1}^{m_d} H_i\right)_{datum} \quad (40)$$

$$\sum_{i=1}^{m-1}\sum_{u=1+i}^{m} Y_{ik}Y_{ul}Z_{kl} \le m-1,\ i \ne u;\ i = 1,2,\ldots,m-1; \quad u = 2,3,\ldots,m \quad (41)$$

$$Y_{ik} = 0 \text{ or } 1 \quad (42)$$

The decision variables are the binary variables Y_{ik} which indicates the physical component (DP_k) that delivers the functional requirement FR_i while maximizing the customer satisfaction, and minimizing uncertainty in the linguistic formulation process as well as to design complexity. Minimizing uncertainty increases the designer overall confidence in the selected *DPs* and guards the selection process from being biased toward solutions of questionable confidence. This transferred deterministic formulation of the fuzzy selection problem allows analysis to be conducted at the micro level, i.e. the attributes (parameters) of the fuzzy number. A macro level formulation can be obtained when a fuzzy number is replaced by a crisp score, e.g. its centroid or weighted average (Chen and Hwang 1992). The crisp score is a function of the left and right parts of the membership function.

Solution to the program in Eq.s: (35)-(42) can be obtained by branch-and-bound enumeration method. In our case, there are m binary variable, the Y_{ik}'s, which result in exactly $\prod_{i=1}^{m} 2^i = 2^m$ different integer vector solution. However, as m gets larger, it may be extremely difficult, computationally, to explicitly enumerate all integer solutions. However, with suitable selected constraint criteria the exhaustive enumeration can be reduced by eliminating sets of the vector solutions that does not fit the criteria or result in improved solutions. These sets are implicitly enumerated. The branch-and-bound method requires a solution as a starting requirement which can be obtained by relaxing the integrality constraints in Eq.(42) to $0 \le Y_{ik} \le 1 \ \forall i, i = 1,\ldots,m, k = 1,\ldots,K$. In this case, the integer program is converted to a non-linear continuous problem since the decision variables can assume any value in [0,1], inclusive. The objective function becomes a quotient of two linear functions and is subject to linear constraints. A program of this type is called a 'fractional program' or a 'programming problem with linear fractional functionals'. The different cases of solution treatment can be found in (Murty 1983). Solutions are obtained after converting the fractional program to a linear program with suitable transformation of variables. Once the continuous solution is obtained, the branch-and-bound enumeration method can be employed.

4. CONCLUSIONS

The concept selection problem can be solved using the integer programming formulation proposed here. The selection criteria include the complexity, customer satisfaction, and design value. Design complexity is measured by information content using Shannon entropy which in turn takes the probability of success as arguments. In many situations, these probabilities can not be quantified directly. The combined use of fuzzy concepts and maximum entropy principle enable the inference of the probability distribution from the fuzzy information. Once these probabilities are found, they can be substituted in the entropy to quantify complexity. The complexity is then is used in either: a deterministic integer programming [Eq.s: (11)-(16) of Part I] or a fuzzy formulation that can be transferred to the deterministic domain after some manipulation [Eq.s: (35)-(42) of Part II]. The result in either formulation is a selected system that has the components to maximize customer satisfaction, value, and simplicity in an assembly feasible configuration.

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