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AXIOMATIC DESIGN OF CHAOTIC COMPONENTS OF SURFACE TEXTURES

Christopher A. Brown
brown@wpi.edu
Worcester Polytechnic Institute
Worcester, MA 01609-2280

ABSTRACT

The axiomatic design of surface textures is considered from the perspective of scales of interaction and the scales of the texture on the surface, which can be represented in an area-scale plot. Area-scale analysis by virtual tiling is briefly explained. It is shown how two functions one requiring a smooth surface and one requiring a rough surface can be functionally integrated but physically decoupled by scale on the same surface.

Keywords: surface texture design, fractals, chaotic geometry, area-scale analysis

1 INTRODUCTION

The objective of this paper is to explain how axiomatic design can be applied to the design of surface textures with chaotic components. The advantage of axiomatic design (Suh 1990) in this context is that it provides a clear framework for the elements of design and the rules, or axioms, for relating them.

Most real surfaces, and maybe all surfaces of engineering interest, have chaotic components in their textures at some sufficiently fine scale of observation, frequently within the scales of interest. The scales of interest are determined by the scales of the interactions with the texture that control the texture-related performance phenomena. This assumes that there is a scale that can be related to interactions with a surface, and that this scale determines what size features on the surface will influence the interaction. The design considerations here are limited to surfaces that have chaotic components at some scales that are of practical interest. The texture at these scales still influences the performance, or functional requirements of the surface and is influenced by the process parameters.

The geometry of surfaces can be conveniently divided into two elements, form and texture. Texture is often decomposed into three components: roughness, waviness and lay (ASME B46 1995). In this paper, form will be considered as any component of the geometry of a part that was specified in the design and can easily be described with conventional, or Euclidean, geometry. Euclidean geometry means that this component can be described with smooth, or at least piecewise smooth curves. Every other aspect of the geometry of the surface of a part will be considered texture, whether designed or created as a consequence of the manufacturing process or use. And, in this paper, texture will not be further decomposed.

The independence axiom (Suh 1990) could be said to optimize the relations between the elements of the functional domain and the physical domain, as well as the relations between the elements of the physical domain and the process domain. And compliance with the independence axiom avoids coupling and multiple adjustment iterations. The elements of the physical domain are central to considerations of concurrent engineering, which encompasses all three domains. Appropriate geometric description of the chaotic components of textures is necessary for describing the elements of the physical domain, which is necessary for the application of the independence axiom and central to concurrent engineering of surface textures.

There is relatively little experience describing chaotic geometry. Conventional descriptions of surface textures are largely recognized as inadequate for quantitative analysis of measured surface textures (e.g., Cailler et al. 1989, Bailey 1977). There are at least three reasons why the conventional approaches are inadequate. First, due to historical technical limitations on measurement instruments, the measurements of the surface textures have not been taken at a fine enough scale to include the scales of interaction for the function or process phenomena of interest. Second, the conventional analysis methods are not sensitive to the scale of surface features in any convenient way. And, the conventional analyses generally fail to capture the essence of the texture that is linked to the performance or process. Therefore it is necessary to explore new possibilities for measuring and analyzing surface textures.

The information axiom (Suh 1990) maximizes the probability of success. It can be applied to quality control of surface textures in two ways by minimizing the probability of, one, accepting a part that will not perform within the tolerance for the functional requirements, and, by minimizing the probability of rejecting a part that will. For this it is necessary to maximize the probability that the measurement will contain the essence of the texture that is responsible for the phenomena of interest. Then maximizing the probability that the characterization of the measurement will still contain the essence of the texture responsible for the phenomena of interest.

2 DESIGN DOMAINS IN SURFACE METROLOGY

The three design spaces are applied to texture design in Fig. 1. The functional domain contains the functional requirements (FRs) that will be satisfied by some aspect of the surface texture. A priori it may not be known which FRs will be satisfied by the surface texture, and historically many designers have attempted to satisfy FRs with anything but surface texture, because it is relatively poorly understood. The elements of the physical domain, design parameters (DPs), in consideration in this paper compose the surface texture. The elements of the process domain, the process variables (PVs), are those aspects of the surface creation process, generally some component of the manufacturing process which create the surface texture.

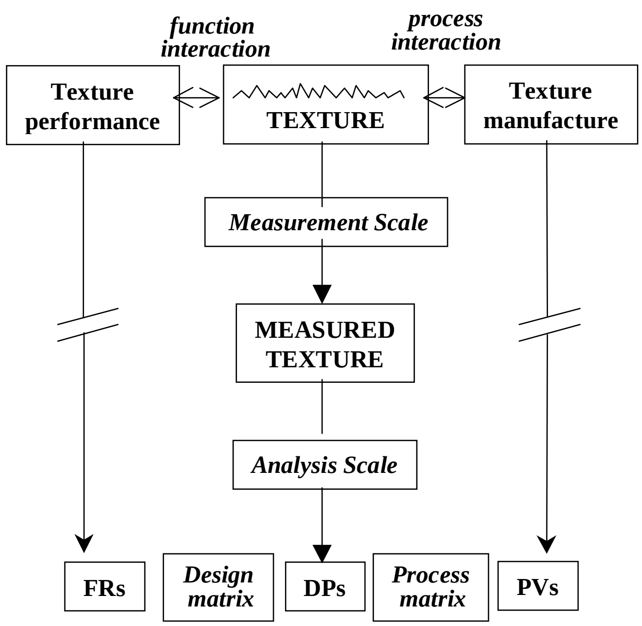
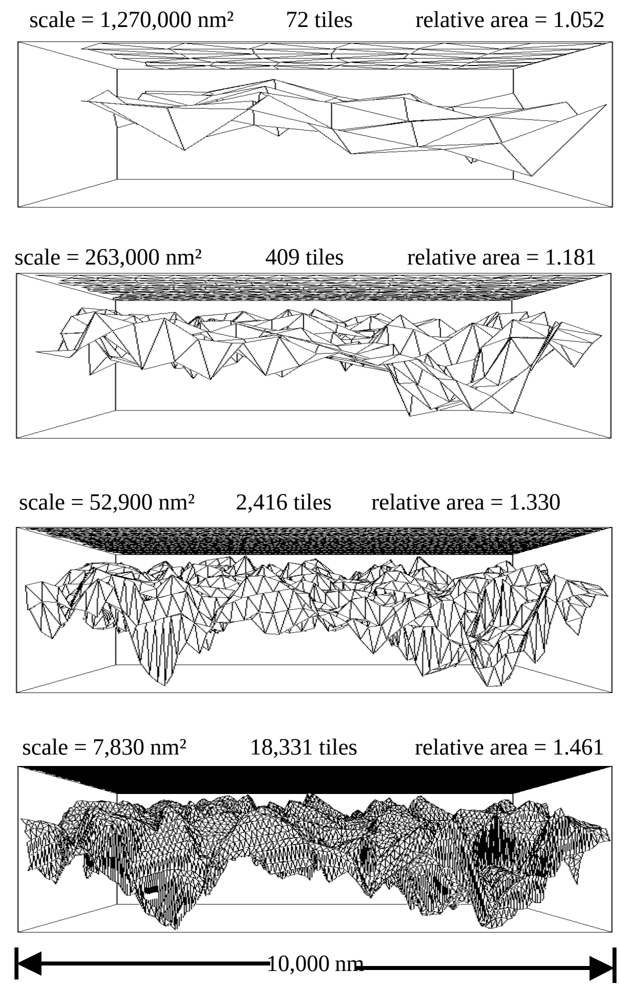


Figure 1. The three design domains for describing the texture in the physical domain, showing the importance of including the scales of interaction in the texture measurement and analysis.

The challenge is to describe the surface texture in such a way that it can be related with confidence to the FRs and the DPs. At some sufficiently fine scale, frequently well within the scale of interest, the geometry under consideration is chaotic, and not piecewise continuous or smooth. Generally the texture geometry will be nowhere differentiable. This kind of geometry defies ordinary geometric descriptions. Appropriate description of the physical elements of the surface is essential for the application of axiomatic design.

Figure 2. Four examples of virtual tiling by the patchwork



method on a measured surface. The scales of analysis, number of tiles and relative areas are given with the tilings. The diamond coating on a silicon substrate was measured by scanning tunneling microscopy at UNCC.

Frequently the practice in industry is to somehow get around the DPs of the surface texture. Sometimes the practice is to overwhelm the influence of the surface texture with some other aspect of the surface that is easier to control or measure, principally composition. Sometimes this is not possible because of constraints on the chemical composition of the surface, or sometimes because the FRs are sufficiently sensitive to the texture, or the demands on performance are great enough that the texture must be considered. In these cases often the practice is to adjust the PVs and test the FRs and ignore the description of the texture, or use a description that has a narrow domain of applicability tied to the specific machines and processes.

Fractal geometry has been proposed as a method for describing chaotic shapes (Mandelbrot 1977), and this has been applied to surface textures by a number of researchers (e.g., Tricot et al. 1988, Thomas and Thomas 1988, Russ 1994, Brown et al. 1993). The end result of many of these applications is a fractal dimension, which can be a reasonable description of the geometric complexity of the texture. However, there are two difficulties with such a simple result. Most engineering surfaces are multi-fractal with respect to scale. That is, at some sufficiently large scale they are smooth, and aptly described by Euclidean geometry, and while at fine scales they are complex, the nature of the complexity can vary with scale, and different fractal dimensions can apply over different scale ranges (Brown et al. 1993). The other difficulty with the fractal dimension is that, with the exception of capacitance of a cracked surface and its effect on the electrochemical impedance spectra (Sapoval et al. 1993), to this author's knowledge the fractal dimension does not appear in any fundamental relationship describing surface function. Scale-sensitive fractal analyses in general (Brown et al. 1998) and area-scale fractal analysis in particular (Brown 1993) addresses these two difficulties. First, it is scale sensitive, so that it identifies the scale ranges over which the calculated fractal dimensions apply. Second, area-scale analysis calculates the relative area at each scale (Fig. 2), and this can be related to surface performance in fundamental ways theoretically and experimentally (Siegmann and Brown 1999, Shipulski and Brown 1994). In the case of understanding the contribution of substrate roughness to adhesive strength, for example, the relative area at a particular scale is the parameter of interest, and the ability to calculate fractal parameters is a byproduct of the determination of relative areas over sufficiently wide scale range.

2.1 DESIGN AND PROCESS MATRICES

Figure 1 shows the importance for the design and process matrices of scale in measuring and characterizing the surface texture. In order to define the relations between the FRs and DPs and between the DPs and PVs the texture must be characterized at the scales that correspond to the interactions that control the FRs, and those scales that are influenced by the PVs. If these scales are not included in the characterization of the texture that is used for the DPs then either it will not be possible to find relations required to construct the design and process matrices, or if they are found, the domain of applicability will be limited.

3 AREA-SCALE ANALYSIS

Area-scale analysis is an extension of length-scale analysis, i.e., it is Richardson analysis applied to measured surfaces (Brown et al. 1993), overcoming the difficulties described by (Russ 1994). The measured surface is represented by heights, z on a regular grid in x and y. The tiling method, as illustrated in Fig. 2 uses repeated virtual tiling of the measured surface with triangular tiles in a patchwork fashion (Brown et al. 1994). Within each repetition the triangular tiles all have the same area in a three-dimensional space, although

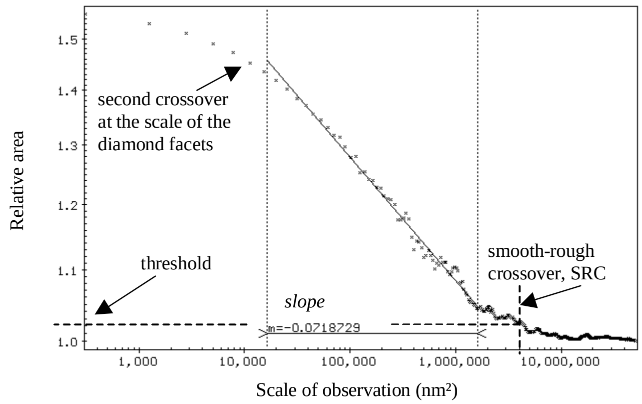


Figure 3. Area-scale plot of the same measured surface used in Fig.2. At large scales the relative areas are 1, at finer scales the relative areas increase regularly on the log-log plot until the scale of the facets on the diamond crystals are reached forming a second crossover to lower complexity at the fine scales.

the shape is allowed to vary within certain limits. The area of the triangle represents the scale of measurement, and with each repetition a different area triangle is used, so that a range of scales is analyzed.

The results of the tiling are shown in Fig. 3, where the relative areas are plotted versus the scale of measurement. The relative area is the area measured by the tiling exercise, i.e., the number of triangles times the scale, or area of the triangular tile, divided by the nominal area, i.e., the projection of the region covered by the tiling projected onto the x-y plane.

At sufficiently large scales the relative areas are close to one. Any interactions with the surface at these scales will see the surface as smooth. At some finer scales the relative areas are sufficiently greater than one so that interactions at these scales will see the surface as being rough. A threshold in relative area can be selected so that the scale of the smooth-rough crossover (SRC) can be defined. The slope of the log-log plot is an indication of the geometric complexity. The more negative the slope, the greater the complexity. At scales below the SRC on many surfaces the log of the relative area increases linearly for two or more orders of magnitude with a decrease in the log of the scale. These are regions of geometric self-similarity, constant complexity and constant fractal dimension. Other crossover scales can describe the scales of changes in fractal dimension and the geometry can be described as multi-fractal in scale. The relative area at a particular scale itself can be the parameter of interest, as it describes the geometric opportunity for interactions with the surface (Siegmann and Brown 1999).

It should be noted that there are other scale-sensitive methods for characterizing surface textures, which can also be used in this way, e.g., volume scale (Brown et al. 1998, Brown et al. 2000). These other methods result in a plot showing some geometric property on one axis, usually the vertical axis and the scale at which the geometric property was determined on the other. The geometric property can be selected to be consistent with the model for interaction with the texture.

4 INFORMATION CONTENT: TEXTURE, MEASURED TEXTURE, PARAMETERS

The information contained in the texture of a surface is represented in Fig. 4. When the surface is measured, the measured surface represents at best a sub-set of the total information content. The measured surface might also contain information not in the surface, such as information about the condition of the sensor used to measure the surface and information on the noise present during the measurement. The information about the texture needed to make the relation with an FR or DP necessary for the design or process matrix may or may not be included in the measured surface, or may be only

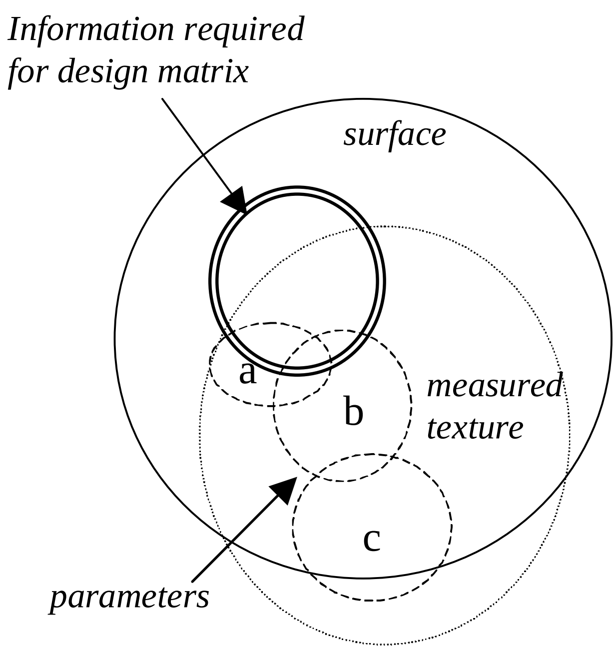


Figure 4. Information diagram, showing the information inherent in the texture. The solid circle represents the total information in the texture. The concentric circles indicate the information needed to make a correlation. The large dashed circle represents the information in the measured texture. The smaller dashed circles represent the texture characterization parameters. Parameters a and c are orthogonal

partly included, so as to limit the domain of applicability of the relation. A texture characterization parameter contains a subset of the measured surface, and since many texture characterization parameters are calculated after filtering, they may also contain information about the filter, which is not part of the measurement. Ideally, some filters are intended to remove erroneous information in the measurement, others are intended to restrict the information in the parameter so that it has a better chance of containing the information needed to make the relations.

The best parameter is clearly the one that contains the information necessary to make the relation. Often it is unknown what part of the information in the texture is required to form the relation with the FRs and PVs. In this case it looks from the diagram in Fig. 4, as if the best parameter or parameter set would be that which contains the most information, thereby maximizing the possibility of getting the required information. This looks like maximizing the information content, and therefore violating the information axiom, which prescribes minimizing the information (Suh 1990). To clarify this it is necessary to consider the definition of information in the context of design.

The information content of a texture characterization parameter, or set of texture characterization parameters, can be defined as the reciprocal the probability of describing the measured surface within some resolution. The number of surfaces that are described by the parameter, or set of parameters, can determine the probability of describing a certain measured surface with a particular parameter:

$$p = 1/n \tag{1}$$

$$I = \log(1/p) \;\Rightarrow\; I = \log(n) \tag{2}$$

where p is the probability of describing the measured surface and n is the number of surfaces that could be described by the parameter. This number, n, is not infinite, as might be thought, because the measured surface has a finite number of measured heights, and each height has a certain resolution, associated with the measurement and digitizing, and there is a limited to the range of heights.

To calculate the information content in a measurement or in a quality assurance system, the probabilities of accepting bad parts and rejecting good parts needs to be considered. The more precise and accurate the measurement and parameter, and the finer the resolution and sensitivity of the measurement, the lower is the probability of accepting a bad part or rejecting a good one. The probability of success, used for defining the information content in design can be defined as one minus the probabilities of inappropriate rejection or acceptance:

$$p = 1 - pr - pa \tag{3}$$

where pr is the probability of rejecting a good part and pa is the probability of accepting a bad one. In the context of surface textures, the selection of the characterization parameter, or parameter set will influence pr and pa.

When two or more parameters are selected to characterize the DPs surface texture, then they should be satisfying a similar number of FRs and be satisfied by a similar number of PVs. In this case, ideally the FRs, DPs and the PVs should be independent. One way of studying the independence of texture characterization parameters is to measure a variety of surfaces, which were manufactured throughout the range of the DPs, calculate the characterization parameters, then systematically regress each texture characterization parameter with every other one. The degree of orthogonality of the texture characterization parameters can be assessed by the regression coefficients. The regression coefficients can be conveniently arranged in a cross correlation table (Nowicki 1985, Brown et al. 1990).

5 DECOUPLING WITH SCALE IN THE AREA-SCALE SPACE

Many surfaces must fulfill two functions, one that requires a smooth surface and one that requires a rough surface. Often these two functions can be functionally decoupled by the scale of the interactions controlling the two functions, while remaining physically integrated on the same surface. A road for example should be smooth on the scale of the wheel-road interaction to provide a smooth ride, and rough on the scale of the interaction with the tire material and water layers, to provide sufficient traction. The walls of a cylinder should be smooth to provide a seal with the piston rings, but rough on the scale of the oil retention to provide lubrication.

The area-scale space (Fig. 3) can be used to design surfaces so that they can be rough for interactions at fine scales and smooth for interactions at large scales. To do this the smooth-rough crossover should be intermediate between the scales of interaction for the functions requiring rough and smooth surfaces. Once it is assured that the SRC is comfortably below the scale of the interaction for the smooth function (in order to maximize the tolerance and minimize the information content). Then it may be desirable to increase the relative area with respect to reduction of the scale in order to maximize the area for interaction, and therefore strengthen the interaction, at the fine scales.

The process to produce the surface must be similarly designed so that the surface can be produced. Two different finishing processes may be required to produce the texture at the required scales. This is the case with plateau honing of cylinder liners, which are first rough ground and then honed to produce a smooth plateau with fine scale scratches for retaining oil.

6 CONCLUDING REMARKS

To create design matrices and have effective quality assurance, the scale of measurement, and analysis used in describing the surface texture as a DP should be consistent with the scales of interaction for the FRs and PVs.

The concept of information content can be applied to texture characterization parameters, considering the probability of describing the surface of interest.

Area-scale analysis and area-scale plots can be an effective aid in designing surface textures by providing a space which can be used to physically integrate and functionally separate two or more interactions by the scale of interaction.

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