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NEW LANGUAGE AND MATHEMATICAL BACKGROUND SUPPORTING GEOMETRICAL PRODUCT SPECIFICATIONS

Paolo Chiabert
paolo.chiabert@polito.it
Politecnico di Torino
Corso Duca degli Abruzzi 24
10129 Torino
Italy

ABSTRACT

Evolution of International Standards related to the product development process requires a harmonisation of technological aspects involved in the design, manufacturing and inspection phases. This paper provides a review of the state of the art in the Geometrical Product Specifications project sponsored by International Standard Organisation and outlines some important fall-outs in technological fields. Based on the most relevant results acquired by GPS project, this paper demonstrates that new solutions can be introduced to manage the technological features in mechanical objects. One of these solutions, based on a generalised probabilistic characterisation of measured surfaces, provides a unifying approach to product geometrical description along the phases of design, manufacturing and inspection. It seems able to solve some of the most annoying problems encountered in the GPS approach such as surface partitioning, surface association and uncertainty expression.

Keywords: Standardization, Geometrical Product Specification, Geometric Dimensioning and Tolerancing, Surface/defect identification, Statistical method.

1 INTRODUCTION

In 1996 the International Standard Organization set up the Technical Committee 213 [1] charging it with the GPS project: "'Standardisation in the field of geometrical product specifications (GPS) i.e. macro- and micro-geometry specifications covering dimensional and geometrical tolerancing, surface properties and the related verification principles, measuring equipment and calibration requirements including the uncertainty of dimensional and geometrical measurements...".
The reorganisation of the whole set of standards in the GPS field arose from existing gaps and contradictions, and from a lack of generalized mathematical approach for the specification and verification of geometrical tolerances [2]. Such ambitious program required a clear definition of subjects and methodologies, codified in the GPS - Masterplan" [3]. In the past ten years, the Working Groups of ISO/TC213 re-built existing standards, deleted duplicated standards, created new standards on uncovered subjects, and provided noteworthy practical results. Moreover, the adoption of a general mathematical approach opened new perspectives and introduced new formulations of GPS fundamental issues, unifying the large amount of GPS concepts. The paper deals with this new formulation, exploring the known consequences in the field of specification, manufacturing and verification, and forecasting possible developments based on the achieved knowledge.

2 GEOMETRICAL PRODUCT SPECIFICATIONS

Geometrical Product Specifications involve several standards dealing with fundamental specification rules, global principles and definitions, specific geometrical characteristics. Standardization activities are based on the "chains of standards" philosophy.
Each geometrical characteristic (a chain) is exploited in a series of standards covering all steps in product's development: design, manufacturing, metrology, etc. Each chain of standards is composed of six chain links, grouping standards according to defined tasks. All standards must respect rules of unambiguity, totality and complementarity. GPS matrix model, depicted in Figure 1, comprehends four groups of standards.

- Fundamental, containing rules and procedures for dimensioning and tolerancing.
- Global, influencing several or all chains in the GPS standardisation;
- General, establishing drawing indications, definitions and verification principles for geometrical characteristics.
- Complementary, containing rules for specialised features or elements, depending on manufacturing process.

The six chain links are related to the following tasks.

- Codification, for drawing indication and coding of geometrical characteristics.
- Theoretical definition and values, relating numerical values and code-symbols;
- Definition for actual feature, expanding the meaning of theoretical exact feature to non-ideal real world geometry.

- Assessment of the deviation, detailing requirements for assessment of deviation between workpiece and drawn object.
- Measurement equipment requirements, describing measuring equipment and its influence on the uncertainty of the measurement process.
- Calibration requirements, detailing calibration standards and procedures.

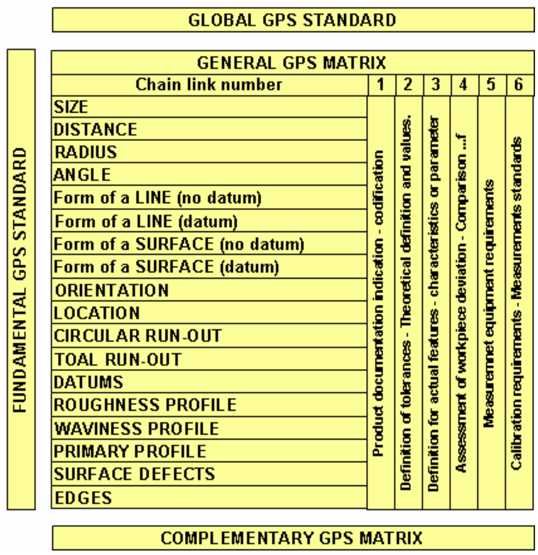

Figure 1 – ISO/TR14638 GPS Matrix

3 GEOMETRICAL DESCRIPTION OF PRODUCT MODELS

3.1 WORKPIECE, SURFACES AND FEATURES

The starting point of the GPS approach is a non ambiguous definition of workpieces along the whole production process. Although there is a unique physical realisation of workpiece, it is possible to extract several representations characterised by different information related to the context where the representation will be used. It is possible to define three contexts:

- The world of nominal definition, where the designer defines an ideal representation of the future workpiece.
- The world of specification, where the designer imagines several representations of the future workpiece.
- The world of verification, where several representations of a given workpiece are identified by the application of measuring procedures.

The boundary is the only accessible element on a real object [4,5] and it is taken in account by functional design as well as metrological verification.

Based on workpiece surface, it is possible to define several surface models, as illustrated in Figure 2:

- Nominal model, is the surface model of perfect shape defined by designers.
- Specification model, is the surface model of non-perfect shape imagined for the writing and the reading of specifications (skin model).
- Verification model, is the surface model constituted of points identified by sampling the workpiece with measuring instruments.

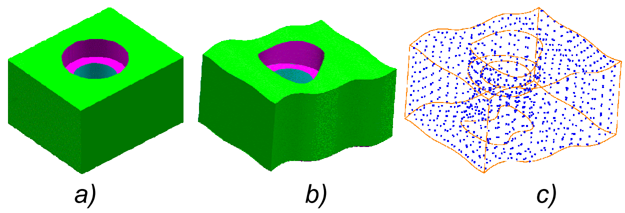

Figure 2 – Nominal model (a), Skin model (b) and Measured model (c) of a simple workpiece.

On the surface models it is possible to recognise nominal, specification and verification features, which are points, lines or surfaces having relevance with regard to some objective in the production process.

3.2 CLASSIFICATION OF SURFACES IN THE EUCLIDEAN SPACE

A GPS outstanding concept is the new classification of three-dimensional surfaces based on their invariance properties [6]. The classification of surfaces in E^3 has been derived both from the kinematic analysis of mechanisms [7] and from geometrical specification of parts [8]. The mathematical basis relies on the definition of connected Lie subgroups of $T(3) \times SO(3)$ -the group of rigid motions - where $T(3)$ [$SO(3)$] denotes the group of translations [rotations] in $\mathfrak{R}^3$, respectively. The classification of any set of points in $\mathfrak{R}^3$, requires the application of connected Lie subgroups to the candidate set: the transformed set is then compared to the original one in order to detect the presence of invariance. Actually only seven subgroups do leave invariant some proper subset of $\mathfrak{R}^3$, thus obtaining the list of classes shown in Table 1.

Class of Symmetry C_i (i=1,...,7)	Group of Symmetry G_i	Reference Set[a] R_i	dim(G_i)
1) Spherical	$G_1=SO(3)$	Point	3
2) Cylindrical	$G_2=T(1)\times SO(1)$	Straight Line	2
3) Planar	$G_3=T(2)\times SO(1)$	Plane	3
4) Helicoidal	$G_4=T(1)\times SO(1)$ with pitch $\mu \neq 0$	Helix	1
5) Axial	$G_5=SO(1)$	(Point, Straight Line)	1
6) Prismatic	$G_6=T(1)$	(Straight Line, Plane)	1
7) Trivial	$G_7=I_3$	(Point, Straight Line, Plane)	0

Tab. 1 – Classes of invariance in the Eucliden space E^3

In detail, given any $S \subset \mathfrak{R}^3$, let: $Aut(S)$ be the group of automorphisms of S, i.e. $Aut(S)=\{ g \in T(3)\times SO(3): gS=S\}$; $Aut_0(S)$ be the connected component of $Aut(S)$ that contains the identity

rigid motion I_3. Set S is assigned to class C_i if and only if $Aut_0(S)=G_i$. In [6] it is shown that, if S is either closed or has a closed limit set, then $Aut(S)$ is a Lie group and therefore $Aut_0(S)$ is a connected Lie group. Since all such groups are listed in Table 1, set S turns out to be classifiable.
The above condition on S is actually so broad that it can be assumed to hold true for all practical purposes thus allowing a full classification of those proper subsets of $\Re^3$ that are relevant in engineering applications.

Such result allows a dramatic simplification in the analysis of geometrical features. According to Table 1, seven semi-parametric models can describe all elementary surfaces building up any complex mechanical model.
Moreover, each semi-parametric model has an intrinsic euclidean reference system localising and orienteering the surface in the space. The MRGE (Minimum Reference Geometric Element), is hence composed of a complete or incomplete set of three elements (point, line and plane) derived from the semi-parametric model associated to the real surface.
A further step in the classification derived from the consideration that mechanical parts are characterised by relationships between different surfaces.
This introduced a powerful classification method based TTRS (Topologically and Technologically Related Surfaces) [8].
A TTRS is defined as the minimal semantic unity where it is possible to define a relative positioning of a couple of surfaces. In a broader interpretation, a TTRS can be composed of two surfaces, or two TTRSs, or a surface and a TTRS, belonging to the same solid. The reorganisation of all the possible relationships between two TTRSs (single surfaces, or TTRSs) produced the 28 situations represented in Table 2.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	C_5 C_1						
C_2	C_7 C_5	C_7 C_6 C_2					
C_3	C_5	C_7 C_5 C_6	C_6 C_3				
C_4	C_7	C_7 C_4	C_7	C_7 C_4			
C_5	C_7 C_5	C_7 C_5	C_7 C_5	C_7	C_7 C_5		
C_6	C_7	C_7 C_6	C_7 C_6	C_7	C_7	C_7 C_6	
C_7	C_7	C_7	C_7	C_7	C_7	C_7	C_7

Table 2 – TTRSs classification.

Composing these elementary situations it is possible to describe any kind of mechanical object thus providing a universal geometrical modelling tool.
TTRSs approach has a natural application to the resolution of three dimensional metrological problems.

4 OPERATOR, OPERATION AND DUALITY PRINCIPLE

TTRSs theory introduces a structured procedure in the design of complex shapes by identifying the relationships which have to be controlled in order to assure the products quality.
Such design controls, universally known as geometrical tolerances, have to be applied to the real surfaces of the manufactured object. However, the association between designed and real surfaces is not explicitly defined, thus preventing any practical application: there is no connection between the specification operator, detailing the design functional requirements, and the verification operator, checking the compliance of inspected parts with those requirements.
The specification process translates the design intent into requirements for specific GPS characteristics, by means of a specification operator composed of specification operations.
The verification process verifies the workpiece characteristics according to the actual specification operator by means of actual verification operations.
There is a substantial parallelism between verification and specification activities: the former is an inspection procedure performed on the physical part, the latter could be thought as a conceptual inspection procedure carried out on the skin model [9]. The correspondence between design and inspection activities has been translated by ISO/TC213 in the duality principle, i.e. a bi-univocal relationship between specification and verification operations illustrated in Figure 3.

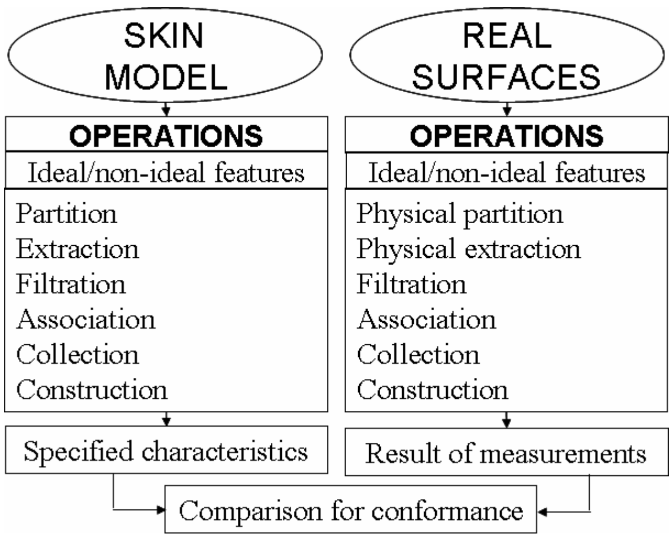

Figure 3 – GPS duality principle between specification and verification activities

According to the duality principle, a complete description of operations in the specification field induced a complete definition of dual operations in the verification field. In order to explore the potentiality of GPS operators, the following list contains operations of specification for non-ideal models:

- Partition: identification of bounded features from non-ideal or ideal features.

- Extraction: identification of specific points from non-ideal feature.
- Filtration: reduction of information from a non-ideal feature.
- Association: fitting ideal features to non-ideal features according to a criterion.
- Collection: considering more than one features together.
- Construction: building ideal features from other ideal features with constraints.

Actually, it is possible to obtain a coherent framework for the geometrical control of mechanical parts just providing a description of specification operations while delegating to the duality principle the definition of verification operations. Obviously, moving from the nominal models towards the real models, operators become less precise or, in tighter words, the uncertainty in the control system increases.

5 EXTENDED UNCERTAINTY

ISO focused TC213 task on the areas of specification and verification, recognizing a substantial connection between design and verification phases. However, the missing involvement of manufacturing area does not authorize to ignore the impact of this task on technological aspects.
GPS standards define operating procedures in design and verification phases and strongly influence the manufacturing phase, which is enclosed between them. On the other side, technological skills need to assure the desired level of uncertainty in the manufacturing process. In this context, ISO/TC213 defined a generalized uncertainty principle, illustrated in Figure 3, explaining the difference between the real product and the nominal object.

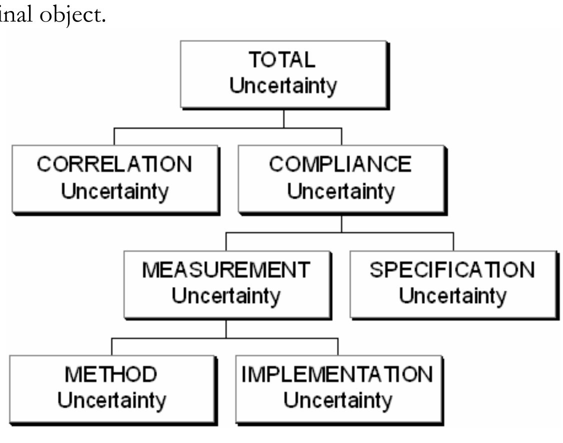

Table 3 – GPS uncertainty model (ISO/TS17450-2).

According to the practice described in the GUM [10], two levels of uncertainty have been identified.

- Geometrical uncertainty, composed of correlation and specification uncertainty.
- Measurement uncertainty, derived from method and implementation uncertainty.

Correlation uncertainty implies an incorrect or uncompleted relationship between intended functionality and geometrical control specifications. It is related to the physics phenomena governing the product functionality and to the approximation of simulation's results involved by applied models.
Specification uncertainty arises from incorrect or incomplete standards applied to geometrical product's definition as well as from continuous development of GPS language. Two kinds of specification uncertainty are available: the former related to operators and application of specifications and the latter just related to specifications themselves.
The verification process involves a dispersion of the measured values with respect to the true value of measured points. ISO/TC213 proposed two different types of measurement uncertainty.

- Method uncertainty, related to differences existing between specification and verification operator.
- Implementation uncertainty, connected to standard deviation of measurement process.

Such generalised formulation of uncertainty requires huge research efforts to achieve the same level of confidence available on the measurement uncertainty.

6 PROBABILISTIC CHARACTERIZATION OF SURFACES

As stated above, the achievement of a complete and coherent standardization in the GPS field belongs to a (rather) remote future. At this time, the research teams supporting the TC213 have produced a deepen knowledge on GD&T and provided an innovative approach, while respecting the traditional practice codified in fifty years of industrial experience.
But there is space and need for new ideas in this exciting research area. Also in the stable field of product shape design, it is possible to develop innovative solutions. An example is the statistical approach to the description of product shape. It is based on the most relevant theoretical results provided by TC213 as well as on powerful statistical analysis tools.
The statistical tool constituting the skeleton of this approach is the Parzen method [11], which provides an estimate of a generic unknown Probability Density Function (PDF) from a set of samples drawn form the PDF itself.
Let $W=\{w_1,\ldots,w_m\}$ a set of real-valued random variables and $p(W)$ the unknown PDF of W; $\{W_i\}$ i=1,…,n a set of n samples drawn from $p(W)$. The Parzen method builds up an estimate of $p(W)$ according to the following formula:

$$\tilde{p}(W) = \frac{1}{n}\sum_{i=1}^{n}\prod_{j=1}^{m} g(w_j; w_j^i, \sigma_j^2) \qquad (1)$$

where $g(x;\mu,\sigma^2)$) is a predefined elementary PDF of the real-valued random variable x with mean μ and variance σ^2. The milestones of the proposed method are:

- the classification of three dimensional surfaces, according to the seven classes of symmetry groups, allows the modelling of seven different PDF having, by construction, the symmetric properties characteristic of each class [12];
- the hypothesis of surface invariance properties preservation in the set of measured points, true under

specific conditions [13], allows the extension of derived results from the set of points to the sampled surface;

- the statistical interpretation of the set of measured points, allows the definition of each measured point $P \in \Re^3$ as a noisy version of an unknown point Q lying on the surface, thus justifying the research of the relation between P and Q.

From a practical point of view, the proposed method ranks the seven-modelled PDFs, showing defined invariant properties, against the set of measured points and computes the log likelihood. This is a measure of the probability that the sample had been drawn form a surface having a given invariance property.

Figure 4 – Complex surface with axial symmetry

The method can be easily extended to the automatic identification of elementary features within complex objects, as illustrated in Figure 4.

7 CONCLUSION

The Axiomatic Design, since its materialization in 1990, modified the designers approach to the product development process. Cleraness, efficiency, quality are some of the advantages provided by the application of AD in engineering activities.
Nevertheless, the most important value of Axiomatic Design is the theory itself, based on a clear and univocal description of domains, rules, relationships and parameters.
The basic principles of Axiomatic Design are today present in many disciplines which are looking for a more precise, economical and reliable structure.
This is the case of Geometrical Product Specifications project which is going to define the tools necessary to create a mathematically-based framework in the definition of the shape of the product along its lifecycle.
The paper outlines a brief summary of the most recent developments in the standardisation of geometrical product specifications, originated by the coordination of several international researches.
Based on these new concepts, acknowledged in the future ISO normative, the paper illustrates an innovative stochastic approach to the recognition of three dimensional surfaces. It substantially differs from the filtering operation proposed in the TC213 because it is based on a powerful PDF estimator implementing the Parzen mathematical tools.
Experimental tests suggest a high efficiency in the identification of invariant properties in single surface as well as the recognition and decomposition of complex TTRS. The generality of the proposed approach could provide a new tool to measure the geometrical errors in manufacturing parts as a degree of violation of invariance properties.

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