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TOLERANCE SYNTHESIS USING AXIOMATIC DESIGN

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ABSTRACT

This paper introduces a new approach based on Axiomatic Design to simplify the process of Tolerance Synthesis. The main advantage of this approach is that all of the information needed for the Tolerance Synthesis is easily included in the classical AD framework. The information that must be stored in the design matrices is mainly related to the production cost vs. tolerance curves and the tolerance chain needed for the synthesis phase. Tolerance Synthesis is currently one of the most proficient ways to reduce the cost of machined parts but its diffusion is limited by two main issues. First, the information needed for the synthesis is complex and difficult to manage. Second, there is a cultural limit because often a concurrent approach is not fully used, especially by small and medium enterprises (SMEs), and often the tolerances are decided by the designers while the manufacturing process (and so the cost) is chosen by the process engineer. Both of these issues could be solved by the use of Axiomatic Design resulting in a greater use of such approaches, especially for SMEs. The approach developed here introduces a cross level matrix to effectively represent the tolerance chain of the product and the idea to store the cost-tolerance function as terms of the DPs-PVs design matrix. The model developed has been applied to an industrial case study.

Keywords: Axiomatic Design, Tolerance Synthesis, manufacturing cost.

1 INTRODUCTION

Starting from the criticism of the classical approach to tolerance allocation, where the product designer chooses the tolerances following his experiences and common practices, the two step methodology of Tolerance Synthesis [1] has been developed. The general idea is to divide the tolerances in two groups: the individual tolerances, that are characteristic of a single product feature and could be associated with a single manufacturing process (i.e. the tolerance on the surface planarity is given by the milling machine and the process parameters used; the tolerance of a diameter is due to the turning process and so on) and the functional tolerances that have a strong relation with the functionality of the product (i.e. the interference of a shaft-hole assembly influences the force needed to disassemble itself) and are due to many, usually different, manufacturing processes. The relation that links the functional tolerances to the individual tolerances is called the Tolerance Chain. For tridimensional products the evaluation of the tolerance chain is not often an easy task. Many authors have proposed different approaches to obtain such data both using analytical or experimental approaches [Xu and Ji, 2002; Anselmetti *et al.*, 2003].

In the classical approach, the allocation of individual tolerances is carried out during the product design phase, but modifications can occur during the process design and manufacturing phases. The main problem is that design engineers, allocating individual tolerances, are often unaware of manufacturing processes and their capabilities. The usual allocation process is based on the preliminary definition of the functional tolerance, the one needed to satisfy the functional requirement of the product, and the subsequent evaluation of individual tolerance values that could satisfy the functional need. This method gives good results only with skilled designers. It is time consuming, costly and lacks systematization. Its main weakness lies in the fact that when the designers give the tolerances on the individual dimensions they usually have an incomplete knowledge of the manufacturing processes capabilities and tend to give stricter tolerances than necessary.

On the other hand, the Tolerance Synthesis approach aims to include the choice of the tolerance in the early stage of the design, considering their values not only for the optimization of a single characteristic, cost or performance, but taking into account both simultaneously. In order to meet this objective it is necessary to involve the competencies of both the product and the process designers in the design process.

The Tolerance Synthesis approach starts from the definition of the functional tolerances and, using the tolerance chain, determines the individual tolerances using an optimization algorithm that aims to meet the functional tolerance goal and simultaneously reduce the manufacturing cost. Such an approach is able to reduce the manufacturing cost but does not stress the choices made by the product designer regarding the optimal value of the functional tolerances. The main disadvantage of such an approach is the complexity of the operations that must be performed and the management of all the data needed.

The general idea of this paper is to use the AD framework to also store the data needed for Tolerance Synthesis in order to let this approach become more usable and widespread for manufacturing companies (like SMEs) with less structured design processes.

2 TOLERANCE SYNTHESIS APPROACH

The need for a more systematic approach to this problem was expressed several years ago. In addition to the work pioneered by Bjorke [Bjorke, 1999], many other authors [Wu *et al.*, 2009; Chase and Greenwood, 1988; Manarvi and Juster, 2004] have given important contributions to the solution of the Tolerance Synthesis issue.

A review of the recent literature suggests that existing techniques for tolerance allocation can be grouped into three categories: traditional methods, methods focusing on manufacturing, and methods focusing on quality.

Traditional tolerance synthesis methods are implemented separately in the design and the process planning stages. Some typical examples of these methods are reported between the 1970 and 1980 by Michael and Siddal [Michael and Siddal,1982], Speckhart [Speckhart, 1972] and Sutherland and Roth [Sutherland and Roth, 1975]. All of these proposed models allocate the tolerances in the design stage, avoiding consideration of the manufacturing processes associated with the tolerance during this selection. These approaches are focused only on the evaluation of the tolerance chain and on the forecast spread of the manufactured dimensional features. All of these researches are the basis for the following implementation of the more actual approaches such the ones based on the automated process selection [Roblens and Roy, 2004] or the simultaneous optimization of both quality and manufacturing cost [Campatelli and Del Taglia, 2004].

The common assumptions of all these works is that the functional tolerance and target value must be given by the product designer and cannot be discussed, while the search for tolerance chains and the allocation of tolerances on individual dimensions is a process that can be automated and optimised looking for a solution that minimizes the total manufacturing cost.

The approaches have developed some common steps that must be fulfilled:

1. Identify, for each functional dimension, the dimension chain that determines it and the associated tolerance chain.
2. Determine the cost-tolerance curves for manufacturing every part/feature involved in the tolerance chain. This curve should give, for every value of tolerance, the minimum manufacturing cost achievable with an optimized process plan. For small variations in the desired tolerance, it could be derived by the modification of the process parameters. For larger variations a change in the manufacturing process used is needed. The trend of these curves is usually decreasing and their mathematical model is discussed later in this paper..
3. Find the tolerance for each dimension of the chain so that the functional tolerance is respected and the total manufacturing cost (sum of single parts manufacturing costs) is minimized. This could be obtained using one non-linear optimization algorithm.

For the definition of the tolerance chain, starting from the dimensional chain, two models are the most used: the Worst Case (WC) (1) and the Root Square Statistical (RSS) (2), whose mathematical model is:

$$T_F = \sum_i^n \left| \frac{\partial f}{\partial x_i} \right| T_i \qquad (1)$$

$$T_F = \sqrt{\sum_i^n \left(\frac{\partial f}{\partial x_i} \right)^2 T_i^2} \qquad (2)$$

Where T_F is the functional tolerance, T_i are the individual manufactured tolerances, *f* is the function obtained, usually by geometric reasoning, that links the functional dimension to the individual dimension and x_i are the dimensions of the chain. It could be noted that all the coefficient of T_i in the tolerance chain are additive also if the derivates of the *f* function are negative. The choice between the Worst Case or Statistical approach depends essentially on the degree of uncertainty that characterize the tolerance data. The Worst Case is a cautious approach that can be useful if the manufacturing process used is not fully reliable for the required tolerance value. In the other case the Statistical approach is more a realistic one and can be used proficiently when all the part tolerances can be obtained reliably.

For the cost-tolerance curves the models are many and depend on the field of application. Among those most used is the one proposed by Chase [Chase, 1989] that links the cost to the tolerance using a reciprocal power function, whose coefficients are A, B and k (3).

$$\text{Cost} = A + B/t^k \qquad (3)$$

The trend of this function is presented in figure 1.

Once the cost-tolerances curves are defined the optimization is carried out selecting the value of the individual tolerances that respects the constraints (4) or (5) (depending on the choice of a WC or RSS tolerance chain computation method) and has a goal function (6) that minimizes the total cost.

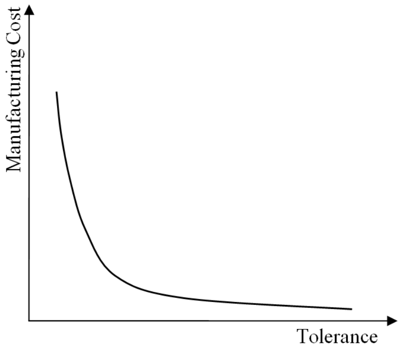


Figure 1. Trend of cost-tolerance curve.

$$\sum_i^n \left| \frac{\partial f}{\partial x_i} \right| T_i \le T_{F\ OB} \qquad (4)$$

$$\sqrt{\sum_{i}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 T_i^2} \leq T_{F\ OB} \qquad (5)$$

$$\min(C_F) = \min\left(\sum_{i}^{n} C_i\right) \qquad (6)$$

Where C_F and C_i are respectively the costs of the functional tolerance and the i-feature of the tolerance chain and $T_{F\ OB}$ is the target value of the functional tolerance. To this optimization technological constraints (7) could also be added that state that the possible tolerance value must be greater than the minimum feasible tolerance for the chosen technology.

$$\forall i \quad T_i \geq T_{i\,\min} \qquad (7)$$

Where the $T_{i\ \min}$ is the minimum tolerance that i-process could attain due to technological reasons.

3 PROPOSED APPROACH

The general idea is to integrate the classical AD approach with some additional information that would let the tolerance synthesis problem become a lot simpler. This approach is intended, obviously, only for physical products with special regards to high precision mechanical assemblies. A similar approach has been used by Goncalves-Coelho and Mourao [Goncalves-Coelho and Mourao, 2007] to find a better production process considering the overall process performance and the number of items to be produced. In this case the objective it is not to define the best process but to find the optimal production tolerance within a specific process, and later find the fit process parameter. The idea of using AD to represent a flow of information is proposed also by Leu *et al.* [Leu *et al.*, 2009] where AD is used as framework for the design of complex systems and includes also the flow of information needed between the sub-systems. In this case the information flow has to be cross-level to link the functional characteristics, usually the first level FRs, to lower level components.

The preliminary assumptions for this work are:

- The FRs are related to the functionality of the product and, in most of the cases for a mechanical product, can be related to a functional dimension (DP) and the satisfaction of this requirement can be evaluated starting from the tolerance of the DP value.
- Every DP (physical features) of the decomposition tree could be attributed a tolerance; for the lowest level this tolerance could be associated with a geometrical feature and so to a specific manufacturing process while the upper levels are "calculated" tolerances.
- The tolerance of an upper level DP is given by the tolerances of the lower level DPs in the same decomposition branch; the relation between the upper level DP and the other components is the tolerance chain that can be computed using the WC or RSS methods.
- The PVs at the lowest level, the ones used to manufacture the geometrical features of a mechanical product, can be associated with a curve cost-tolerance function.

From these preliminary issues is possible to define which additional information must be added to the classical AD representation to implement a Tolerance Synthesis. These are detailed in the following list:

- The FRs definition must be always analytical: the design range (tolerance) must be expressed for each FR.
- The DPs must be characterized by a physical dimension that will be represented using a target value and a tolerance range.
- The PVs of the lowest level will be the real manufacturing processes responsible for the lowest level DPs and will be linked to a database of cost-tolerance functions.
- There will be the need to create a Tolerance Design Matrix (TDM) that will take an initial input from the DMs of various level. This matrix will allow the construction of the tolerance chain needed for the optimization.

In general to be used this approach needs a lower than usual degree of abstraction in the definition of the design: FRs and DPs must all be measurable and the tolerance ranges for both must be defined. These constraints are hard to respect when considering processes or immaterial products, such as software, but they are well suited for mechanical assemblies.

The innovative content of this approach is the use of the TDM to link the tolerances of the upper level FRs to the basic manufacturing operations that produce the lowest level DPs. The TDM could be assembled starting from the upper level Design Matrix. The analysis of the matrix could suggest which relation could be further expressed in a mathematical form and which are not consistent with the problem. In case of an uncoupled design with a diagonal matrix, the tolerances of, for example DP2, will be given by the tolerances of DP2xx only. In case of a coupling between the DPs to satisfy the same functions the DP2 tolerance will be probably due also to the DP1xx or DP3xx elements. An example of TDM is presented in figure 2. In this, the upper level DPs are reported as columns and the leaf DPs are shown as rows. In this case, DP1 and DP2 are uncoupled while DP3 is coupled with DP1. Starting from the Design Matrix the designer could analyze, and eventually compute, the values of the possible tolerance chain relations considering only the leaf DPs indicated by the DM relation. In figure 2, for example, the designer needs to analyze only the values in bold, while the others could be automatically considered zero. The elements of the matrix will be the sensitivity (partial derivate of the function f in (1) or (2)) with respect to the DPs. Only in the case of a 1D tolerance chain could the value be simply 1 or 0. These values could be also negative depending on the dimensional chain of the DP; however these will be responsible for a positive contribution in the tolerance chain due to the mathematical form of (1) and (2).

$$\begin{array}{c|ccc|} & DP1 & DP2 & DP3 \\ \hline FR1 & X & 0 & 0 \\ FR2 & 0 & X & 0 \\ FR3 & X & 0 & X \\ \hline \end{array} \qquad \begin{array}{c|ccc|} & DP1 & DP2 & DP3 \\ \hline DP11 & \mathbf{2.3} & 0 & \mathbf{0} \\ DP12 & \mathbf{0} & 0 & \mathbf{1.2} \\ DP13 & \mathbf{-4.5} & 0 & 0 \\ DP21 & 0 & \mathbf{0.7} & 0 \\ DP22 & 0 & \mathbf{2.4} & 0 \\ DP31 & 0 & 0 & \mathbf{3.2} \\ DP32 & 0 & 0 & \mathbf{2.2} \\ \hline \end{array}$$

Figure 2. Design matrix and Tolerance Design Matrix.

The designer in this case is supported by the AD decomposition tree to create a list of possible tolerances that create the tolerance chain. This first support to the Tolerance Synthesis process is enhanced by the organization of all of the needed information for this process inside the AD framework. Once the Tolerance chain is built starting from the DMs information, it is possible to use the cost-tolerance information to apply an optimization algorithm that will give as output the best value for the DPs tolerances that will fulfil the functional requirements and simultaneously will minimize the manufacturing costs.

In figure 3 is reported a scheme of AD with the added link needed for tolerance synthesis.

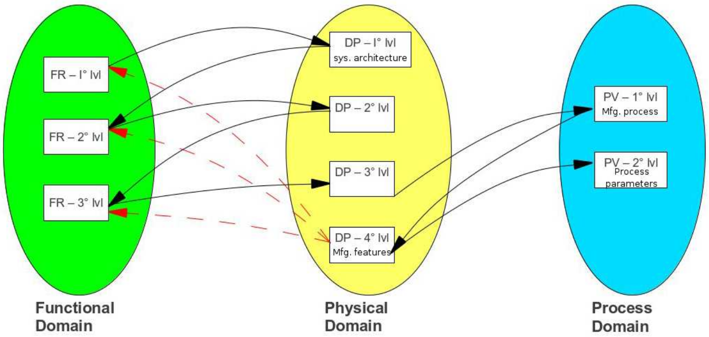
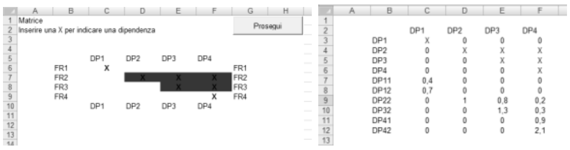


Figure 3. Scheme of Tolerance AD, tolerance link in dashed line.

In order to have a smooth transition to this approach a simple software program based on some Microsoft Excel sheets with embedded VBA code has been developed. Its functions are the following:

- There is an automatic test of the introduced DM in order to rearrange it and provide the indication of its coupling degree. In case of a coupled matrix the user must agree clicking on a pop-up menu to proceed.
- The blank TDM is created starting from the indication of the upper level DPs. This matrix must the filled with the sensitivity.
- It allows user to choose the manufacturing process cost-tolerance curves to be associated to the PVs from a database.
- The tolerance synthesis is carried out thanks to the Microsoft Excel "Solver" algorithm automatically.

A screenshot of the software is reported in figure 4.



Matrice
Inserire una X per indicare una dipendenza

	DP1	DP2	DP3	DP4	
FR1	X				FR1
FR2		X	X	X	FR2
FR3			X	X	FR3
FR4				X	FR4
	DP1	DP2	DP3	DP4	

	DP1	DP2	DP3	DP4
DP1	X	0	0	0
DP2	0	X	X	X
DP3	0	0	X	X
DP4	0	0	0	X
DP11	0,4	0	0	0
DP12	0,7	0	0	0
DP22	0	1	0,8	0,2
DP32	0	0	1,3	0,3
DP41	0	0	0	0,9
DP42	0	0	0	2,1

Figure 4. Screenshot of the developed software.

4 CASE STUDY: FORTINI CLUTCH

This approach has been tested on a famous case study, the Fortini clutch [15] that has been used for many evaluations of the efficiency of the tolerance allocation approach. The product is a one way clutch assembly that is composed of a hub, four rollers and a cage, as presented in figure 5.

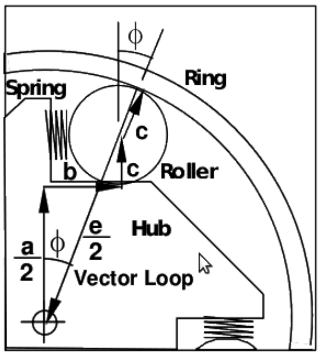


Figure 5. Scheme of Fortini clutch assembly.

The FRs and DPs decomposition of this assembly is presented in figure 6.

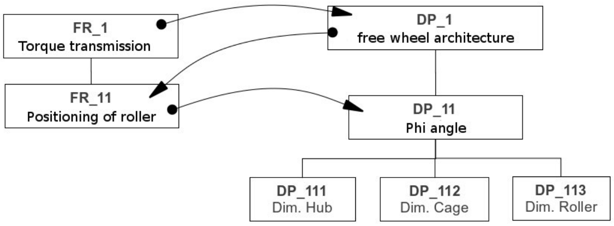


Figure 6. FRs/DPs of clutch.

The lowest level DPs have been connected with the related PVs to which also have been associated with the cost-tolerance curve characteristics of the specific manufacturing process and stored in a database, as presented in figure 7.

The initially proposed tolerance values, for the three features of the assembly, are reported in table 1 together with the B and k coefficients for the cost-tolerance curve of the manufacturing process. The coefficient A represents the fixed costs so has no relevance for the optimization. The total cost of this configuration is 5.42$.

Table 1. Initially proposed tolerances for the assembly.

Dimension	**Nominal Value (mm)**	**Initial tol. (mm)**	**B**	**k**
Hub width - a	55,00	0,10	0,10	-0,45
Roller radius - c	11,40	0,01	0,05	-1,13
Ring diameter - e	101,60	0,02	0,015	-0,79

The functional dimension in this case is the contact angle (ϕ) between the rollers and the cage measured between the two lines from the centre of the hub that connects the contact point and is tangent to the roller. In order to work properly it is generally assumed [Fortini, 1967] that the value of this angle must be 7° ± 1°. The angle could be expressed by the function (8) using the Vector Loop approach [Chase, 1999].

$$a + c + c\cos(\phi) - e\cos(\phi) = 0 \qquad \phi = \arccos\left(\frac{a+c}{e-c}\right) \tag{8}$$

Deriving this function using the formula (1) is possible to define the tolerance chain and evaluate the sensitivity coefficient using the finite difference (9) and fill up the TDM that in this case will be very simple due to the presence of a single functional tolerance DP11:

$$\begin{array}{cc} & DP11 \\ DP111 & \\ DP112 & \\ DP113 & \end{array}\!\!\!\begin{array}{c} \\ \left[\begin{array}{c} 2{,}65 \\ 10{,}55 \\ 2{,}63 \end{array}\right] \end{array} \tag{9}$$

This could be written in the form of a tolerance chain equation (10) and the optimization process could be carried out easily.

$$T_\phi = \frac{\partial\phi}{\partial a}T_a + \frac{\partial\phi}{\partial c}T_c + \frac{\partial\phi}{\partial e}T_e = 2{,}65\cdot T_a + 10{,}55\cdot T_c + 2{,}63\cdot T_e \tag{10}$$

The result of the optimization is responsible for a reduction in the cost of the assembly of about 21% (final cost 4.30$) and new tolerance values for the three mechanical features, as reported in Table 2. It could be noted that the tolerance of the less expensive process has been tightened while the other two have been widened.

Table 2. Tolerance/cost for the assembly's features.

Dimension	Initial tol. (mm)	Final tol. (mm)
Hub width - a	0,10	0,05
Roller radius - c	0,01	0,012
Ring diameter - e	0,02	0,076

5 CONCLUSION

The introduction of a Tolerance Design Matrix in the AD framework allows designers to easily carry out a Tolerance Synthesis analysis in order to optimize the production cost. The TDM can be created starting from the FR/DP design Matrix, saving time for the designer (especially for complex products) and reducing the probability of errors. This matrix would link the upper level DPs to the leaf DPs in order to represent a tolerance chain. For optimization the cost-tolerances curves also must be included as coefficients in the DP/PV design matrix. These values can be stored in a database, such as been created in the developed software.

The AD framework added with this additional information becomes a powerful tool to easily perform the Tolerance Synthesis, increasing the use of this complex, but cost-saving, approach to more cases.

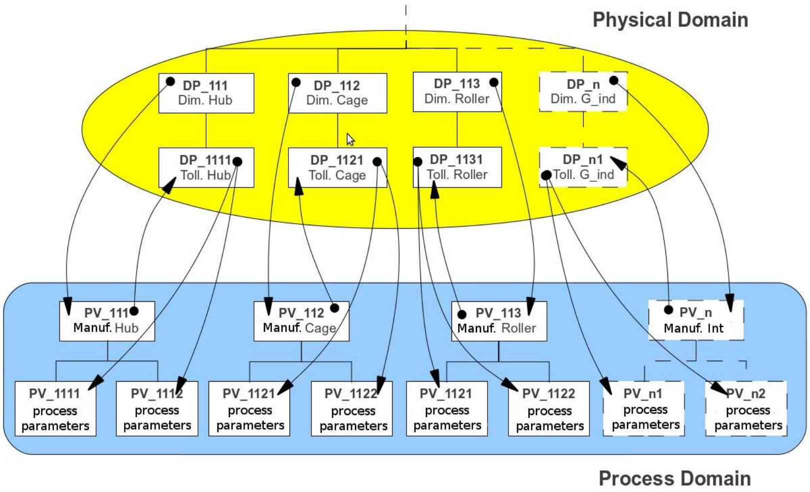


Figure 7. DPs/PVs of clutch.

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