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THE VACUUM CLEANER AS A CASE STUDY FOR TEACHING CONCEPTUAL DESIGN

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ABSTRACT

This paper is intended as an academic example for teaching Axiomatic Design in a trimestral course to engineering students or practitioners connecting with the theory for the first time. The proposed example is an application of Axiomatic Design to the selection of the best filtering system for vacuum-cleaning. Two different physical solutions are considered for collecting and retaining the solid particles: first solution is based on a filter media with a given porous size, and second one is based on a separation due to the larger density of the particles. Physical laws for both cases are given and design matrices are derived from them. Finally, the axioms are used to guide the decision making process and conclusions are given.

Keywords: Axiomatic Design, qualitative analysis, quantitative analysis, education, design matrix.

1 INTRODUCTION / STATE OF THE ART

When teaching Axiomatic Design to an audience that faces the theory for the first time, one of the principal objectives of the educator is to make his students "feel" the axioms and comprehend their implications.

A main aspect that makes Axiomatic Design such a significant theory is its capacity to make explicit the relations existing between the functional and the physical domains, pointing the ones that govern the optimal designs [Suh, 1990].

It is particularly interesting to focus on how the Independence Axiom, based on a qualitative statement: "maintain the independence of the functional requirements" [Suh, 1990], triggers a quantitative formulation based on the design matrices. According to the authors' experience, both qualitative and quantitative definitions of the Independence Axiom are often well understood by the audience, who at the beginning finds the main difficulties in the formal definition of the design problem, and later on, in the understanding of the implications that Axiom 1 has in their design routines.

On the other hand, the Information Axiom is based on a quantitative formulation: "minimize information content" [Suh, 1990]. Consequently, its entire understanding requires exploring its qualitative implications in the design process. Important efforts have been made in this sense as presented by Suh [2001] or Benavides [2012]. Full comprehension of the implications derived from a qualitative application of Axiom 2 constitutes a real challenge for the educator and for all the engineers willing to acquire the ability of using Axiomatic Design in their own design processes.

As exposed by Park [2011], "design education is more like a philosophy". As a consequence, in the framework of engineering, philosophical concepts guiding creative process have to be balanced with the accuracy of engineering laws. Nakao and Nakagawa [2011] present how the correct definition of the design problem helps the achievement of innovative products with a huge impact in the market.

In this sense, it is important to note that the analysis of a particular solution exclusively from a qualitative point of view may result in the loss of a good problem formulation. On the other hand, if only a quantitative approach is proposed, practitioners and students may get lost in the problem definition, resulting in the increased difficulty for selecting an adequate set of functional requirements (FR). Bathurst [2004] presents some of the common problems found by engineers when facing Axiomatic Design for the first time.

In order to communicate the qualitative and quantitative implications of the design axioms, it is significant to select adequate intuitive examples that could help students and practitioners to entirely understand and interiorize the theory.

The main purpose of this paper is to suggest the structure of a lecture which, based on the resolution of a pedagogical example, would help students to comprehend Axiomatic Design principles as postulated by Suh [1990; 2001]. Although this work focuses mainly on the learning of the Independence Axiom and its implications, it gives some interesting conclusions derived from the Information Axiom.

To achieve this objective, this paper focuses on the qualitative and quantitative analysis of the vacuum cleaner filtering system as a case study. First of all, a summary of the lecture's structure is presented. Next, the design problem of the vacuum cleaner is solved; first qualitatively, and later on, quantitatively. In both, the lecture's structure is conceived in order to illustrate the Axiomatic Design principles [Suh 1990; 2001] within the concrete example.

2 PROPOSED LECTURE'S STRUCTURE

The lecture's structure is based on the methodological steps described by Suh [1990; 2001]. As a first step in the education of Axiomatic Design principles, it is suitable to analyze an existing solution from a qualitative perspective. Thanks to it, the students have the opportunity to come into contact with basic design problem definition, and particularly, with two main implications of the Independence Axiom: direct dependence (caused by the formulation of needs which represent equivalent concepts) and indirect dependence (caused by the synthesis of a physical solution that couples the set of FRs).

Once students have contacted qualitatively with the implications of the design axioms, the quantitative formulation of the design problem can be suitably exposed.

The proposed structure for the lecture is presented in the next subsections.

2.1 QUALITATIVE ANALYSIS

For analyzing an existing solution from a qualitative perspective, we propose the following steps [Based on Suh, 1990]:

1. Qualitative formulation of the design problem
   a. Challenge definition
   b. Selection of the minimum number of independent FRs in a neutral solution environment
   c. Establishment of constraints
2. Description of the physical solution through its main DPs
3. Writing of the design matrix
4. Analysis with the use of the Independence axiom
5. Introduction to the Information Axiom in terms of probability of success
6. Propose uncoupled solutions and outline new challenges

2.2 QUANTITATIVE ANALYSIS

For analyzing an existing solution from a quantitative perspective, we propose the following steps [Based on Suh, 1990]:

1. Quantitative formulation of the design problem
   a. Challenge definition
   b. Selection of the minimum number of independent FRs in a neutral solution environment
   c. Establishment of constraints
   d. Definition of FRs
2. Description of the existing solution through its main DPs
   a. Writing of the design equations (physical laws)
   b. Identification of DPs
   c. Establishment of new constraints derived from the DPs
3. Writing of the design matrix
4. Analysis with the use of the Independence Axiom
5. Introduction to the Information Axiom in terms of probability of success
6. Selection of new DPs to achieve the optimal design and outline new challenges. The selection of the new DPs may imply the selection of a new physical solution.

3 THE VACUUM CLEANER AS A CASE STUDY

According to Suh [1990], the design problem definition is performed when the challenge is expressed and the lists of FRs and constraints are established. Because the FRs have to be stated in a neutral solution environment, the problem formulation has to be valid when analyzing two different solutions to the same design problem.

For that reason, since the methodological steps 1a, 1b and 1c are common for both the quantitative and the qualitative approaches; we will collect them in the following block.

3.1 FORMULATION OF THE DESIGN PROBLEM

3.1.1 CHALLENGE DEFINITION

Analyze two different technologies (porous filter and centrifugal separation) for filtering dust particles when vacuum cleaning. Identify their main dependences and select the best solution according to Axiomatic Design.

3.1.2 SELECTION OF THE MINIMUM NUMBER OF INDEPENDENT FRS IN A SOLUTION NEUTRAL ENVIRONMENT

The minimum list of independent FRs for the first level of hierarchy can be settled as follows (because the main objective of this paper is focused on the FRs, the set of constraints will not be established):

FR1: Clean-up dust particles
FR2: Retain dust particles
FR3: Operate for a long time

At this point, students must realize that the needs stated in FR1, FR2 and FR3 are functional requirements because they represent, in a solution neutral environment, independent concepts. The concept of direct independence is explained as a necessary condition for establishing a correct set of FRs.

3.2 QUALITATIVE ANALYSIS

3.2.1 DESCRIPTION OF THE POROUS FILTER SOLUTION THROUGH ITS MAIN DPS:

The main DPs that satisfy in the porous filter solution the aforementioned list of FRs can be settled as follows:

DP1: Vacuum
DP2: Filter pores size
DP3: Filter area

3.2.2 WRITING OF THE DESIGN MATRIX: ANALYSIS WITH THE USE OF INDEPENDENCE AXIOM

With the use of the Independence Axiom [Suh, 1990], the design matrix relating the established sets of FRs and DPs can be written:

$$\begin{pmatrix} \text{Clean-up dust particles} \\ \text{Retain dust particles} \\ \text{Operate a long time} \end{pmatrix} = \begin{pmatrix} X & X & X \\ X & X & X \\ X & X & X \end{pmatrix} \begin{pmatrix} \text{Vacuum} \\ \text{Filter pore size} \\ \text{Filter area} \end{pmatrix} \qquad (1)$$

3.2.3 ANALYSIS WITH THE USE OF THE INDEPENDENCE AXIOM

The design matrix (DM) makes explicit how the filtering system couples the functional requirements (clean-up dust particles and retain dust particles). Indeed, the more particles that are retained, the more filter pores clog, and consequently, the power for vacuuming and cleaning-up particles decreases. For a particular time of use, the conceived solution generates a dependency between functional requirements that, prior to the obtaining of the physical solution, were independent.

3.2.4 INTRODUCTION TO THE INFORMATION AXIOM IN TERMS OF PROBABILITY OF SUCCESS

As stated by Suh [Suh, 2001] in a coupled design, the variability of the DPs can generate a decrease in the probability of success of satisfying the FRs (and therefore of satisfying client needs). In this example this aspect is visible when the filter has to be removed and changed because the vacuum power is not enough to clean-up dust particles.

3.2.5 PROPOSING UNCOUPLED SOLUTIONS AND OUTLINE NEW CHALLENGES

The coupling identified leads to the formulation of a new challenge: "how to retain dust particles without losing vacuum power and maximizing the time of use".

There are different solutions in the market that solve this dependency. One of them is the one patented by Dyson: the centrifugal vacuum cleaner based on cyclone technology. In this solution, the FR "retain dust particles" is satisfied by a separation of the dust particles with the use of the centrifugal force. This solution responds to the following new design matrix.

$$\begin{pmatrix} \text{Clean-up dust particles} \\ \text{Retain dust particles} \\ \text{Operate a long time} \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{pmatrix} \begin{pmatrix} \text{Vacuum} \\ \text{Cyclone} \\ \text{Container capacity} \end{pmatrix} \quad (2)$$

In this case, the DM shows a decoupled design. Indeed, the filtering system for retaining dust particles does not affect the vacuum power, and consequently, the functionality of cleaning-up dust particles.

It's remarkable to note the huge effect that this new concept had in the market, showing up the deep impact that the reduction of the number of dependencies has into the achievement of more competitive products.

At this point it is useful to induce the students to think about the independency obtained with respect to the porous bag required for the conventional filter vacuum cleaner. Additionally, they can be proposed to think in terms of probability of success, determining which of the solutions has a higher probability of satisfying FRs.

3.3 QUANTITATIVE ANALYSIS

3.3.1 DEFINITION OF FRS

In order to achieve the quantitative analysis, the set of FRs has to be defined in terms of the appropriate physical variables:

FR1: Clean-up dust particles: u_1

FR1 represents the functionality of cleaning-up particles, which might be represented by the variable u_1 which represents the speed of particles that are cleaned.

FR2: Retain dust particles: d_{min}

FR2 may be stated as follows: separate all the particles that have a size bigger than d_{min}.

FR3: Maximize operational time: t_{max}

FR3 might be stated as the time in which FRs are satisfied.

3.3.2 DESCRIPTION OF THE POROUS FILTER SOLUTION THROUGH ITS MAIN DPS

Writing of the design equations (physical laws):

In order to obtain the physical laws that apply to the problem, let us consider the following system as a simplification of the vacuum cleaner we want to analyze:

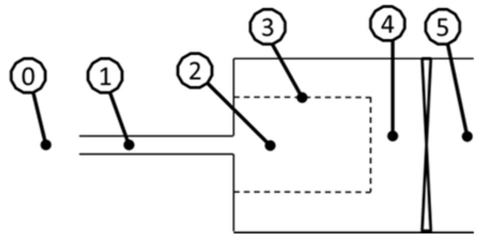

Figure 1. Solution based on porous filter.

where, 0 = room, 1 = tube, 2 = dust container before filter, 3= filter, 4= dust container after filter, and 5 = fan. The physical laws applying to them for an ideal gas are collected in Table 1.

Table 1. Main physical laws for filtering solution.

Zones	Speed	Pressure	Enthalpy	Mass flow	State equation
0	$u_0 = 0$	p_0	h_0	$\dot{m}$	$\frac{\gamma p_0}{\gamma - 1} = \rho_0 h_0$
1	u_1	p_1	$h_1 = h_0 - \frac{u_1^2}{2}$	$\dot{m} = \rho_1 A_1 u_1$	$\frac{\gamma p_1}{\gamma - 1} = \rho_1 h_1$
2	$u_2 = 0$	$p_2 = p_1$	$h_2 = h_0$	$\dot{m}$	$\frac{\gamma p_2}{\gamma - 1} = \rho_2 h_2$
3	u_3	p_3	$h_3 = h_0 - \frac{u_3^2}{2}$	$\dot{m} = \rho_3 A_3 u_3$	$\frac{\gamma p_3}{\gamma - 1} = \rho_3 h_3$

4	$u_4 = 0$	$p_4 = p_3$	$h_4 = h_0$	$\dot{m}$	$\frac{\gamma p_4}{\gamma - 1} = \rho_4 h_4$
5	$u_5 \approx 0$	$p_5 = p_0$	$h_5 = h_0 + \frac{\dot{W}}{\dot{m}}$	$\dot{m}$	$\frac{\gamma p_5}{\gamma - 1} = \rho_5 h_5$

Assuming an isentropic evolution between 0-1, 2-3, and 4-5, i.e., $p_1 / p_0 = (\rho_1 / \rho_0)^\gamma$, $p_3 / p_2 = (\rho_3 / \rho_2)^\gamma$, $p_5 / p_4 = (\rho_5 / \rho_4)^\gamma$; and assuming that the variations of density are small, we can retain the first terms of Taylor expansion in order to solve the system of equations in terms of the FR selected. The following transfer equations are deduced (see appendix for details):

Design equation for *FR1- u₁*

$$u_1 = \sqrt[3]{\frac{2\dot{W} / (\rho_0 A_1)}{1 + \left(\frac{A_1}{A_3}\right)^2}} \tag{3}$$

Design equation for *FR2 – d_d*

$$d_d \geq d_{pores} \tag{4}$$

Design equation for *FR3 – t_max*

Considering the limit as the moment when the whole filter is clogged:

$$t_{\max} = \frac{N}{n \frac{\dot{m}}{\rho_0}} = \frac{N}{n \sqrt[3]{\frac{2\dot{W}A_1^2}{1 + \left(\frac{A_1}{A_3}\right)^2}}} = \frac{N}{n \sqrt[3]{\frac{2\dot{W}A_1^2}{1 + \left(\frac{A_1}{N \frac{\pi d_{pores}^2}{4}}\right)^2}}}_{\max} \tag{5}$$

Definition of DPs

The DPs that derive from design equations are A_1, N, d_{pores} and $\dot{W}$.

3.3.3 Writing of the Design Matrix for Porous Filter Solution

According to the design equation (3), none of the terms of the first row of the DM are zero, consequently:

$$\frac{\partial u_1}{\partial \dot{W}} \neq 0; \frac{\partial u_1}{\partial A_1} \neq 0; \frac{\partial u_1}{\partial d_{pores}}; \neq 0; \frac{\partial u_1}{\partial N} \neq 0 \tag{6}$$

According to the design equation (4), the terms of the second row of the DM are:

$$\begin{aligned} &\frac{\partial d_d}{\partial \dot{W}} = \frac{\partial d_d}{\partial A_1} = \frac{\partial d_d}{\partial N} = 0 \\ &\frac{\partial d_d}{\partial d_{pores}} \neq 0 \end{aligned} \tag{7}$$

Finally, analyzing the design equation (5) for FR3,

$$\frac{\partial t}{\partial \dot{W}} \neq 0; \frac{\partial t}{\partial A_1} \neq 0; \frac{\partial t}{\partial d_{pores}} \neq 0; \frac{\partial t}{\partial N} \neq 0 \tag{8}$$

This results in the following design matrix:

$$\begin{pmatrix} \Delta u_1 \\ \Delta d_d \\ \Delta t \end{pmatrix} = \begin{pmatrix} \frac{\partial u_1}{\partial \dot{W}} & \frac{\partial u_1}{\partial A_1} & \frac{\partial u_1}{\partial d_{pores}} & \frac{\partial u_1}{\partial N} \\ 0 & 0 & \frac{\partial d_{dustpart}}{\partial d_{pores}} & 0 \\ \frac{\partial t}{\partial \dot{W}} & \frac{\partial t}{\partial A_1} & \frac{\partial t}{\partial d_{pores}} & \frac{\partial t}{\partial N} \end{pmatrix} \begin{pmatrix} \Delta \dot{W} \\ \Delta A_1 \\ \Delta d_{pores} \\ \Delta N \end{pmatrix} \tag{9}$$

3.3.4 Analysis with the Use of the Independence Axiom

As it can be observed through the design matrix, the solution based on a filter for retaining dust particles couples the FRs. Indeed, due to the fact that the number of the filter pores diminishes with time, and that the mass flow has to be conserved throughout sections 1, 2, 3, the effective area of the filter $A_3 = N \frac{\pi d_{pores}^2}{4}$ diminishes.

As a consequence, the vacuum power (represented by u_1) decreases during the operational time. This coupling is particularly critical because as it can be observed, even if the other control parameters vary in order to compensate this coupling, the diameter of the filter pores cannot be as big as desired, because it would imply the not achievement of FR2: $d_{pores} \leq d_{\min}$.

3.3.5 Introduction to the Information Axiom in Terms of Probability of Success

As commented in the qualitative analysis, the coupling generates a progressive loss of vacuum power. This decrease induces a smaller probability of success for satisfying FR1: clean-up dust particles.

3.3.6 Selection of New DPs to Uncouple Solutions and Outline New Challenges: Cyclone Based Vacuum Cleaner

Axiomatic Design identifies how far designs are from the optimal solution. Therefore, it answers why solutions become separated from the best design, making explicit their critical points [Suh, 1990].

In this particular case, Axiomatic Design shows how the physical solution based on a filter generates a coupled design. The tendency indicated by DPs is that in order to eliminate the functional coupling, the porous filter has to be removed, requiring a new DP that would uncouple the solution. The next subsection analyses how a different physical solution decouples the system.

3.3.7 Description of the Cyclone Based Solution Through Its Main DPs

In order to obtain the main DPs that describe the cyclone based solution, let us consider the following system:

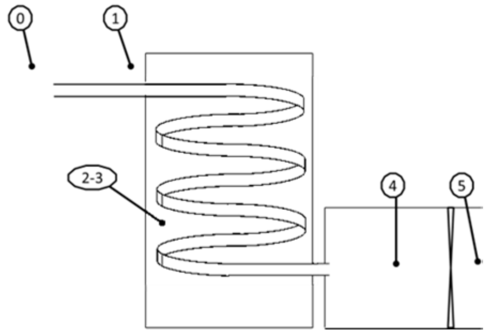

Figure 2. Solution based on centrifugal force.

Writing of the design equations (physical laws)

Physical laws are equivalent to the ones exposed previously, considering that in this case, zone 2 and 3 are equivalent.

Table 2. Main physical laws for centrifugal solution.

Zones	Speed	Pressure	Enthalpy	Mass flow	State equation
0	$u_0 = 0$	p_0	h_0	$\dot{m}$	$\frac{\gamma p_0}{\gamma - 1} = \rho_0 h_0$
1	u_1	p_1	$h_1 = h_0 - \frac{u_1^2}{2}$	$\dot{m} = \rho_1 A_1 u_1$	$\frac{\gamma p_1}{\gamma - 1} = \rho_1 h_1$
2-3	$u_{2-3} = 0$	p_{2-3}	h_{2-3}	0	-
4	$u_4 = 0$	$p_4 = p_1$	$h_4 = h_0$	$\dot{m}$	$\frac{\gamma p_4}{\gamma - 1} = \rho_4 h_4$
5	$u_5 \approx 0$	$p_5 = p_0$	$h_5 = h_0 + \frac{\dot{W}}{\dot{m}}$	$\dot{m}$	$\frac{\gamma p_5}{\gamma - 1} = \rho_5 h_5$

Applying the laws previously exposed and solving the system in terms of the FR selected, and considering that in cyclone case $A_3 >> A_1$ we obtain:

Design equation for *FR1- u₁*

$$u_1 = \frac{\dot{m}}{\rho_0 A_1} = \sqrt[3]{\frac{2\dot{W}}{\rho_0 A_1}} \quad (10)$$

Design equation for *FR2 – d_d*

The differential equation that describes the radial displacement of a dust particle inside the cyclone is:

$$\frac{4}{3}\pi\left(\frac{d_d}{2}\right)^3 \rho_d \ddot{x} = \frac{4}{3}\pi\left(\frac{d_d}{2}\right)^3 \rho_d \frac{u_1^2}{R} - \frac{1}{2}\rho_0 \dot{x}^2 c_d \left(\frac{\pi d_d^2}{4}\right) \quad (11)$$

For large size particles or for low radial speeds the following inequality holds:

$$\frac{\frac{1}{2}\rho_0 \dot{x}^2 c_d \frac{\pi d_d^2}{4}}{\frac{4}{3}\pi\left(\frac{d_d}{2}\right)^3 \rho_d \frac{u_1^2}{R}} << 1 \quad (12)$$

Under this condition Eq. (11) yields to:

$$m_d \ddot{x} = m_d \frac{u_1^2}{R} \quad (13)$$

This equation can be integrated to obtain:

$$\dot{x} = \frac{u_1^2}{R} t \quad (14)$$

$$x = \frac{1}{2}\frac{u_1^2}{R} t^2 \quad (15)$$

The time spent by the particle inside the cyclone is:

$$t = \frac{2\pi R N_c}{u_1} \quad (16)$$

Taking into account Eqs. (12, 13, 14, 15 and 16), we can write the condition for neglecting the aerodynamic forces:

$$d_d >> 3\pi^2 N_c^2 c_d \frac{\rho_0}{\rho_d} R \quad (17)$$

A particle will escape from the cyclone towards the container if $x \geq d_1$, where d_1 represents the diameter of the tube (note that $A_1 = \pi d_1^2 / 4$). Thus a particle will reach the container if the following inequality is satisfied:

$$N_c \geq \frac{1}{\pi}\sqrt{\frac{d_1}{2R}} \quad (18)$$

It is convenient to remark that this condition is easily satisfied, and hence, the FR is satisfied by all the particles that have a large size as stated by Eq. (17). For particles with a much lower diameter than that, the aerodynamic force will become dominant and the radial velocity will become constant as stated by:

$$\dot{x} = \sqrt{\frac{4}{3c_d}\frac{\rho_d}{\rho_o} d_d \frac{u_1^2}{R}} \quad (19)$$

$$x = \sqrt{\frac{4}{3c_d}\frac{\rho_d}{\rho_o} d_d \frac{u_1^2}{R}}\, t \quad (20)$$

$$d_d = \frac{3c_d}{16\pi^2}\frac{\rho_0}{\rho_d}\frac{d_1^2}{R N_c^2} \quad (21)$$

Design equation for *FR3* – t_{max}

Considering the limit as the moment where the whole dust container is full:

$$t_{\max} = \frac{\dfrac{V_{23}}{4\pi\left(\dfrac{d_d}{2}\right)^3}}{\dfrac{3}{n\dfrac{\dot{m}}{\rho_0}}} = \frac{3V_{23}}{2\pi d_d^3 n\sqrt[3]{2\dot{W}A_1^2}} \qquad (22)$$

Definition of DPs

The DPs that derive from design equations Eq. (10, 21 and 22) are $d_1, A_2, N_c, R \; and \; \dot{W}$

3.3.8 Writing of the Design Matrix for Cyclone Based Solution

According to the design equations, the resultant design matrix can be written as follows:

$$\frac{\partial u_1}{\partial \dot{W}} \neq 0; \frac{\partial u_1}{\partial d_1} \neq 0; \frac{\partial u_1}{\partial R} = \frac{\partial u_1}{\partial N_c} = \frac{\partial u_1}{\partial A_2} = 0 \qquad (23)$$

$$\frac{\partial d_d}{\partial \dot{W}} = \frac{\partial d_d}{\partial d_1} = \frac{\partial d_d}{\partial A_2} = 0; \frac{\partial d_d}{\partial R} \neq 0; \frac{\partial d_d}{\partial N_c} \neq 0 \qquad (24)$$

$$\frac{\partial t}{\partial \dot{W}} \neq 0; \frac{\partial t}{\partial d_1} \neq 0; \frac{\partial t}{\partial A_2} \neq 0; \frac{\partial t}{\partial R} = \frac{\partial t}{\partial N_c} = 0 \qquad (25)$$

This results in the following design matrix:

$$\begin{pmatrix} \Delta u_1 \\ \Delta d_d \\ \Delta t \end{pmatrix} = \begin{pmatrix} \dfrac{\partial u_1}{\partial \dot{W}} & \dfrac{\partial u_1}{\partial d_1} & 0 & 0 & 0 \\ 0 & \dfrac{\partial d_d}{\partial d_1} & \dfrac{\partial d_d}{\partial R} & \dfrac{\partial d_d}{\partial N_c} & 0 \\ \dfrac{\partial t}{\partial \dot{W}} & \dfrac{\partial t}{\partial d_1} & 0 & 0 & \dfrac{\partial t}{\partial A_2} \end{pmatrix} \begin{pmatrix} \Delta \dot{W} \\ \Delta d_1 \\ \Delta R \\ \Delta N_c \\ \Delta A_2 \end{pmatrix} \qquad (26)$$

3.3.9 Analysis with the Use of Independence Axiom

As it can be observed, the solution obtained eliminates the main functional dependence that was present in the porous filter solution. As it is derived from the design matrix, in the cyclone based solution vacuuming dust particles does not dependent on the system used to filter them.

As a consequence, in the aforementioned solution the vacuum power does not decrease during the operational time. In this case, the limit is imposed by the volume of the dust container, and not by the system used to separate particles. In this sense, the quantitative analysis confirms the dependencies identified in the qualitative study.

As it can be noted, in the quantitative analysis presented (for both filter and cyclone based solutions) the number of DPs available is bigger than the number of FRs. Particularly, each FR depends on more than one and only one DP, conducting to redundant designs in terms of the number of DPs, and generating coupled or decoupled designs in terms of independency.

This situation is to be expected when the physical laws that allow designers to achieve the quantitative study of designs are settled. In general, the number of DPs that derive from the laws of physics is much bigger than the minimal set of independent FRs. For that reason, Axiomatic Design constitutes a valuable tool to minimize the impact that a bigger number of DPs generates in the definition of new designs. By minimizing the dependencies between FRs and DPs and by selecting the appropriate DP that maximizes the probability of success, Axiomatic Design theory keeps the inherent complexity of the physical problem minimal [Lu and Suh, 2009].

3.3.10 Proposing Uncoupled Solutions and Outline New Challenges

Although the main functional coupling is solved with the described solution, at this point it is convenient to induce students to think about how this solution could be improved. More specifically, students can be asked to think about how the obtaining of a non-redundant design could be achieved. For example, they can be asked for analyzing if the DPs could be combined in dimensionless variables and mainly, which of them should be fixed as constant values. Additionally, students should be invited to evaluate each derivative of the design matrix, and particularly, the relative weight of each term, concluding which terms should be frozen and what tendencies the DPs should follow in order to maximize independency and the probability of success.

4 CONCLUSION

This paper proposes the structure of a lecture whose purpose is to introduce students and practitioners the basics of Axiomatic Design through the case study of an existing product which presents different configurations. The aim is to examine whether the design is optimal or not.

The case study is solved both qualitatively and quantitatively, and it shows how the compliance or not with the design axioms introduces a rationale that certainly identifies the critical points where the synthetized solutions move away from the optimal. This identification constitutes a valuable guide for designers and decision makers, even when just a qualitative study can be conducted, in order to direct their creativity into the optimal solution, what confers a precious tool to validate designs before investing resources to develop them. In addition, it shows how the accomplishment of the Axiom 1 can lead to the accomplishment of the Axiom 2.

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APPENDIX

LIST OF VARIABLES

u_i	*Air speed in zone i*
p_i	*Air pressure in zone i*
h_i	*Air enthalpy in zone i*
m	*Mass of the air*
$\dot{m}$	*Flow mass of the air*
γ	*Heat capacity ratio of the air*
ρ_0	*Air density*
ρ_d	*Dust particles density*
d_d	*Dust particles diameter*
n	*Number of dust particles per volume unit*
d_{pores}	*Filter pores diameter*
N	*Number of filter pores*
A_3	*Porous filter area*
d_1	*Tube 1 diameter*
A_1	*Tube 1 area*
V_{23}	*Dust container capacity*
$\dot{W}$	*Fan power*
R	*Radius of curvature of cyclone*
N_c	*Number of cyclone turns*
$\ddot{x}$	*Radial acceleration inside cyclone*
$\dot{x}$	*Radial speed inside cyclone*
x	*Radial position inside cyclone*
c_d	*Drag coefficient*

PROCEDURE TO OBTAIN THE DESIGN EQUATIONS

1. Density resolution in all the zones

$$\left(\frac{\rho_1}{\rho_0}\right)^{\gamma-1} = 1-\left(\frac{\rho_0}{\rho_1}\right)^2 \frac{1}{2h_0}\left(\frac{\dot{m}}{\rho_0 A_1}\right)^2$$

$$\frac{\rho_1}{\rho_0} = 1-\varepsilon_{10} \quad \text{Incompressible regime.}$$

$$\varepsilon_{10} = \frac{1}{2(\gamma-1)h_0}\left(\frac{\dot{m}}{\rho_0 A_1}\right)^2 + 0(\varepsilon_{10}{}^2)$$

$$\rho_2 = \frac{\gamma p_2}{(\gamma-1)h_2} = \frac{\gamma p_1}{(\gamma-1)h_0} = \frac{\gamma p_0}{(\gamma-1)h_0}\left(\frac{\rho_1}{\rho_0}\right)^{\gamma} = \rho_0(1-\varepsilon_{10})^{\gamma}$$

$$\frac{\rho_2}{\rho_0} = 1-\gamma\varepsilon_{10} + 0(\varepsilon_{10}{}^2)$$

$$\left(\frac{\rho_3}{\rho_0}\right)^{\gamma-1}\left(\frac{\rho_0}{\rho_2}\right)^{\gamma-1} = 1-\left(\frac{\rho_0}{\rho_3}\right)^2 \frac{1}{2h_0}\left(\frac{\dot{m}}{N\rho_0 A_3}\right)^2$$

$$\frac{\rho_3}{\rho_0} = 1-\varepsilon_{30} \quad \text{Filter not clogged; incompressible regime.}$$

$$\varepsilon_{30} = \varepsilon_{10}\left[\left(\frac{A_1}{NA_3}\right)^2 + \gamma\right] + 0(\varepsilon_{10}{}^2)$$

$$\rho_4 = \frac{\gamma p_4}{(\gamma-1)h_4} = \frac{\gamma p_3}{(\gamma-1)h_0} = \frac{\gamma p_2}{(\gamma-1)h_0}\left(\frac{\rho_3}{\rho_2}\right)^{\gamma} = \frac{\gamma p_1}{(\gamma-1)h_0}\left(\frac{\rho_3}{\rho_2}\right)^{\gamma}$$

$$\rho_4 = \frac{\gamma p_0}{(\gamma-1)h_0}\left(\frac{\rho_1}{\rho_0}\right)^{\gamma}\left(\frac{\rho_3}{\rho_2}\right)^{\gamma} = \rho_0(1-\varepsilon_{10})^{\gamma}(1-\varepsilon_{30})^{\gamma}\left(\frac{\rho_0}{\rho_2}\right)^{\gamma}$$

$$\frac{\rho_4}{\rho_0} = 1-\gamma\varepsilon_{10}\left[1+\left(\frac{A_1}{NA_3}\right)^2\right] + 0(\varepsilon_{10}{}^2)$$

$$\frac{\rho_0}{\rho_5} = 1+\frac{\dot{W}}{\dot{m}h_0}$$

2. **Obtaining of u_i and $\dot{m}$ as a function of DPs**

$$\dot{m} = \frac{\dot{W}}{h_0}\frac{1}{\frac{\rho_0}{\rho_5}-1} = \frac{\dot{W}}{h_0}\frac{1}{\frac{\rho_0}{\rho_4}\frac{\rho_4}{\rho_5}-1} = \frac{\dot{W}}{h_0}\frac{1}{\frac{\rho_0}{\rho_4}\left(\frac{p_4}{p_5}\right)^{1/\gamma}-1} =$$

$$= \frac{\dot{W}}{h_0}\frac{1}{\left[1+\gamma\varepsilon_{10}\left[1+\left(\frac{A_1}{NA_3}\right)^2\right]\right]\left(\frac{p_3}{p_2}\right)^{1/\gamma}\left(\frac{p_1}{p_0}\right)^{1/\gamma}-1}$$

$$= \frac{\dot{W}}{h_0}\frac{1}{\left[1+\gamma\varepsilon_{10}\left[1+\left(\frac{A_1}{NA_3}\right)^2\right]\right]\frac{\rho_3}{\rho_0}\frac{\rho_0}{\rho_2}\frac{\rho_1}{\rho_0}-1} = \frac{\dot{W}}{h_0}\frac{1}{(\gamma-1)\varepsilon_{10}\left[1+\left(\frac{A_1}{NA_3}\right)^2\right]}$$

$$\dot{m} = \frac{2\dot{W}}{\left(\frac{\dot{m}}{\rho_0 A_1}\right)^2\left[1+\left(\frac{A_1}{NA_3}\right)^2\right]}$$

$$\dot{m} = \sqrt[3]{\frac{2\dot{W}\left(\rho_0 A_1\right)^2}{1+\left(\frac{A_1}{NA_3}\right)^2}}$$

$$u_1 = \frac{\dot{m}}{\rho_0 A_1} = \sqrt[3]{\frac{2\dot{W}/\left(\rho_0 A_1\right)}{1+\left(\frac{A_1}{NA_3}\right)^2}}$$