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Using Axiomatic Design and Entropy to Measure Complexity in Mass Customization

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Abstract

Nowadays, complexity issues in mass customized manufacturing are considered as topical problem. Especially, product variety induced complexity is frequently discussed in recent research works. Our focus in this paper is based on an exploration of product variety induced complexity based on axiomatic design and entropy theories. For this purpose we propose to adopt previously developed complexity measures based on so called degree of disorder. Our approach consists of transformation of graphical representation describing relation between input components of assembly node and numbers of related product configurations into design matrix of coupled design. Subsequently, we apply the measures to enumerate product variety induced complexity. Finally, we analyze mutual relations between numbers of possible product configurations and obtained values of the complexity measures.

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1. Introduction

Mass customization (MC) strategy is considered to be one of the important contemporary market mentalities, comprising a set of methods and tools, by which manufacturing and service companies can increase their competitiveness and industries can ensure their long-term profitability. The focus of MC strategy is on variety and customization of company product portfolio through flexibility and responsiveness. This can be achieved only with product families and standardized modules ready to be assembled based on customer specifications and requirements. Such an organization must be flexible and adaptive. Manufacturers of customized products are confident that if they find the right balance between product variety offer and customer requirements, it will allow them to maintain or even increase market share. Recent research studies revealed that the increasing number of product alternatives is directly related to an increase of the number of modules integrated into these varieties of products. This fact negatively affects a complexity of the product and generates turbulences in the manufacturing systems, leading to higher direct production costs. Taking this into account, the success of MC strategy depends, apart from other factors, on

the balance between product variety induced complexity and usability of mass customized manufacturing system. To solve this problem it is expected to develop suitable methods for the measurement of product variety induced complexity. The main scope of this paper is to propose and describe a model for complexity measurement and investigate the possibilities of using axiomatic design and entropy for quantifying product variety induced complexity. In this context, a methodological framework including definition of minimal preconditions of Mass Customized Assembly (MCA) and suitable complexity measures (Configuration complexity and Axiomatic Design (AD) based complexity) for system designs will be developed in this paper.

2. Related work

Mass customized manufacturing is concerned not only with production processes but influences overall set of enterprise processes. Complexity of these processes in terms of MC strategy increases due to higher dynamics of relations among system components. One of the complexity sources lies in the product configuration structure and system design. There are several approaches to explore the impact of a

product variety on manufacturing complexity, see for example [1-3]. According to the very first complexity theory adopted from information theory by Shannon [4] the most significant source of complexity is entropy. Information entropy is defined as amount of information within a system to describe the uncertainty of system states. When fewer possible states exist less information is needed to describe the system, and therefore there is less uncertainty entropy in the system. People tend to understand complexity in terms physical things, but it must be viewed from the functional perspective or domain [5]. In other words – what we want to achieve as functional requirements (FRs) and how to achieve it through design parameters (DPs). AD theory defines complexity as a measure of unpredictability of the ability to satisfy a given FR [5]. This complexity is present in all systems as a continuum. Any system or product has therefore a minimum set of independent requirements characterizing its functional need.

Applications of AD in various areas of manufacturing, decision-making appeared in literature. A hierarchical knowledge base system has been built with use of independence axiom by Chen et al. [6]. Chuang and Jiang [7] used fuzzy logic as the membership function of DP. Yi and Park [8] developed an application to enumerate and evaluate the design process according to the independence axiom of AD theory. Moreover, a component-oriented approach based on the AD has been proposed and a V-Model was extended to address component-level issues [9]. Finally, Lindkvist and Soderberk [10] applied AD and robust design to compare different assembly concept alternatives. Authors [11-13] proposed complexity/vulnerability indicators intended to measure coupling complexity in design matrices applicable during the early stages of the product design. This paper applies information coupling complexity into variety induced complexity and its management. In this context, according to Matt [14], a significant research has been carried out in the design of assembly systems for high product variety. On the other hand a number of systems have been designed without having a theoretical framework for complexity [5]. Systems where customer requirements are unpredictable are becoming more complicated even with fundamental applications of complexity management in these companies. Complexity is therefore an important problem since out-of-control complexity can bring undesired design consequences and therefore, unsatisfied customer requirements [15]. Manufacturing complexity from design perspective aimed at product alternatives is also a widely discussed topic. Piller [16], for example, proved that the extent of offered variety determines the complexity, which in turn affects the time and additional cost invested in product configurations.

Only a limited number of papers addressed complexity issues in terms of mass customization so far. Therefore, our ambition in this paper is to present an approach to quantify product variety induced complexity based on combinatorial rules and to compare it with complexity based on existing Boltzmann's statistical concept in the design matrix. For this purpose we adopt previously developed complexity measures based on the so called Degree of disorder. In order to compare this approach to complexity with numbers of possible product configurations, we firstly need to describe the way to

calculate combinatorial product complexity.

3. Combinatorial product configuration complexity

3.1. Basic preconditions and notation

Based on our previous works [17, 18], a methodological framework for the generation of all possible product configurations (PCs) will be briefly outlined. The framework consists of entry components as basic elements for the calculation of PCs. Exactly three types of initial assembly components can be identified within a Mass customized assembly (MCA). A component, in this context can be understood as a part, module, and group of products, property or other characteristic of the final product. Initial assembly components are divided into the three categories, namely stable components, voluntary components and compulsory optional components. Any MCA structure consists of a number of assembly stations – nodes. These can be identified within a multi-level network t_r , where $r = 0, 1, 2, 3, \dots, m$, while t_0 identifies a final level of structure. Each assembly node understood as an individual assembly operation results with a single product, but in a long run the same node can produce a number of different product configurations depending on the number and composition of an entry components. Final products in the downstream assembly node are considered as the stable components for s subsequent operations. The number of product configurations brings uncertainty and thus complexity in these systems.

In order to formalize the three types of initial assembly components the following notation can be used:

Stable components and their number ' i ' is an obligatory initial assembly element as they are uniquely determined. An unlimited number of stable components may be further assembled with a number ' j ' of voluntary components. The selection of these components is voluntary (it is possible that $j=0$). A number ' k ' compulsory optional components is limited in selection. They are optional, but with minimum and maximum requirements ' l ' on a selection by customer, where $l \leq k < k$.

Additional individual building elements of a MCA are assembly branches in MCA assembly structure. They are important for identification and distribution of possible product configurations within an assembly process. Assembly branches can be identified on the highest layer of the model decomposition.

The three types of initial components have been combined using combinatorial rules in order to present a comprehensive model of options in relation to customer within any MCA, as presented in Fig. 1 in the form of bipartite graph. Based on this model, company decision-makers and managers are able to decide on the optimal product variety within an existing production structure or design.

3.2. Methodological Framework Scenario #1(FS1) for base and voluntary optional components

The basic precondition of any MCA is the composition of initial assembly components on entry to assembly node-

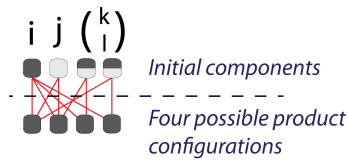


Fig. 1 Bipartite representation of MCA resulting with 4 PCs.

operation. Each such a node must consist of at least one stable component and one voluntary component. A group of entry components with one stable component is denoted as CL_i . Two product configurations arising in assembly of a single stable and voluntary optional component are considered as an available configuration for customer to choose from, while the choice depends on the number of voluntary components. If such a component is not selected, standard assembly is performed on the product without voluntary component. The group of entry components with one optional component is denoted as SCL_i . Such an assembly ($CL_i SCL_i$) is in terms of FS1 considered as a MCA since the basic precondition for such manufacturing strategy has been fulfilled.

As it was already presented, FS1 consists of only stable and of voluntary components, as can be seen in fragment of FS1 on Fig. 2, starting with single stable component, and growing number of voluntary optional components. This way, numbers of available PCs and product variations (PVs) have been determined for each initial component setup, since each PC can be assigned by a number of PVs.

Furthermore, our previous research works [17, 18] proved that from practical and application standpoint, available PCs are much more relevant than related number of PVs. One way to prove the relevance is to compare the number of PVs against PCs for the same initial component setup. Obtained values of PVs are unrealistically larger than PCS (except for CL_1 , where both, value of PCs and PVs are equal).

3.3. Methodological Framework Scenario #2 (FS2) for stable, voluntary and compulsory optional components

| Component Class CL_j | Component Sub-class | i | j | Σ Conf. | Σ Var. |
|------------------------|---------------------|---|----------|----------------|---------------|
| CL_1 | scl_1^0 | 1 | 1 | 1 | 1 |
| | scl_2^0 | 1 | 2 | 3 | 3 |
| | scl_3^0 | 1 | 3 | 7 | 7 |
| | scl_4^0 | 1 | 4 | 15 | 15 |
| | scl_1^1 | 1 | ∞ | — | — |

Fig. 2 Fragment of Framework Scenario #1 for product class CL_1 .

In order to develop the realistic MCA framework and to employ all necessary elements of MCA, FS2 has been proposed while taking all theoretical assumptions and preconditions into account. The FS2 identifies the same numbers of product configurations (without product variations due to their low relevance) within a system of stable, voluntary optional and new – compulsory optional components. Existing rules for the generation of all possible product configurations have therefore been extended with the rules regarding the conditions for selection from compulsory optional components, which can be expressed using combinatorial number $\binom{k}{l}$ such that (unlike permutations) the order of selection does not matter.

Focus remains on the growing product variety (number of product configurations) and/or variety induced complexity.

The above presented FS2 in Fig. 3 shows that it is possible to combine practically any class and sub-class with unlimited numbers of j and k components. But for the set of CO components with number k , there are three different rules of the selection when identifying product configurations. They are:

- Individual selectivity rule - we may define exact number of

| Component Class | Component Sub-class | j | k | Condition for l/k | Σ Conf |
|-----------------|---------------------|---|-----|-------------------|---------------|
| $CL_{1-\infty}$ | scl_1^0 | 1 | 0 | 0 | 2 |
| | scl_2^1 | 1 | 2 | 1 of 2 | 4 |
| | scl_3^1 | 1 | 3 | 1 of 3 | 6 |
| | scl_3^2 | 1 | 3 | 2 of 3 | 6 |
| | scl_4^1 | 1 | 4 | 1 of 4 | 8 |
| | scl_4^2 | 1 | 4 | 2 of 4 | 12 |
| | scl_4^3 | 1 | 4 | 3 of 4 | 8 |
| | scl_5^1 | 1 | 5 | 1 of 5 | 10 |
| | scl_5^2 | 1 | 5 | 2 of 5 | 20 |
| | scl_5^3 | 1 | 5 | 3 of 5 | 20 |
| | scl_5^4 | 1 | 5 | 4 of 5 | 10 |
| | scl_1^k | 1 | k-1 | l of k | — |

Fig. 3 Fragment of Framework Scenario #2 for product classes $CL_{1-\infty}$.

'l' of components to be chosen from all 'k' of CO components, or

- Maximum selectivity rule - we may define the maximum number 'l' of CO components to combine within an assembly choice of all 'k' of CO components (note that 'l' is max. k-1), or finally
- Minimum selectivity rule – we may choose/combine at least 'l' CO of components from available 'k' of CO components (note that 'l' is min. 1).

The above presented FS2 is different from our previously published works as it takes individual selectivity rules into account and identifies available product configurations accordingly.

4. Application of complexity measures of design solutions by the use of AD theory

In order to apply the complexity measure of design solution for quantification of product variety induced complexity, we firstly need to describe the above mentioned models of product configuration calculations by axiomatic design matrices.

4.1. Transforming nodes of MCA into axiomatic design matrices

According to Axiomatic design definition [5], design process is present in four main domains: customer, physical, process and functional. After several iterations, design process transforms customer needs into functional requirements (FR) and constraints, which are later transformed into design parameters (DP). Complexity in context of AD is defined, as the measure of uncertainty in achieving FRs. Within a design hierarchy, the dependencies between the FRs and DPs can be represented by the following equation:

$$FR = [A] DP, \tag{1}$$

where each element of the matrix [A] can be expressed as $A=FR/DP$, and Equation 1 can be understood as choosing the right set of DPs to satisfy given FRs. Therefore each A element of the matrix indicates dependency of FR_m on DP_n , while m indicates number of FRs and n is for number of DP_s within the same matrix. If the value of any element A refers to '0', then FR does not depend on the DP, and vice versa for 'X'. In the context of AD matrices, three types of design exist: uncoupled, decoupled and coupled.

T1: If $m>n$, then the design is either coupled or not all FRs cannot be satisfied.

T2: If $m<n$, then the design is redundant with excess DPs. Such situation does not appear in our MC design matrices, but in both cases, the design matrix [A] is not squared.

T3: If $m=n$, then the matrix is diagonal and the design has exact solution, since each FR is satisfied with exactly one DP. This situation rarely happens in real process or product design and Axiom 1 is not satisfied.

Since most of the designs are originally coupled and there is currently no relevant method for computing information content of such designs, we propose an approach to calculate complexity as information content in accordance with Axiom

2. In this approach we transform each possible individual non-modular assembly node into a coupled design matrix as shown in Fig. 4. Example of design matrices for product classes $CL_2 SCL^0_1$, $CL_2 SCL^0_3$, $CL_2 SCL^2_2$ can be seen in Fig. 4 (exponents '0' and '2' with SCL represent numbers of available 'k' of CO components.)

In Fig. 4a, we start with $i=2$ and $j=1$ components. FRs under our design view are represented by the number of all possible product configurations at the given node. It is because product options practically reflect customer requirements in a selection procedure. DPs are represented by pre-determined initial components of the assembly node. Then, transformation of the concept for the generation of all possible product configurations into axiomatic design matrix is feasible. Analogically, we can transform any model of assembly node with arbitrary combination of initial components into axiomatic design matrix (see examples in Fig. 4b for $i=2$ and $j=3$ components and Fig. 4c for $i=2$, $j=2$ and $k=2$ components). Obtained design matrices represent coupled designs, for which is characteristic that matrix contains mostly non-zero elements and thus the FRs cannot be satisfied independently.

4.2. Application the concepts of entropy and disorder to quantify product variety induced complexity

Many authors, e.g. [19, 20], have treated the relations between entropy and disorder or chaos. According to Martin et al. [21], the concepts of entropy and disorder are inherently linked, but disorder is only a metaphor for entropy, not the definition. Guenov [12] introduced three complexity indicators for architectural design, which are relatively simple and easy to apply, and are sufficiently accurate for the early stages of systems design.

If we selected N_l points (out of total N) to be combined

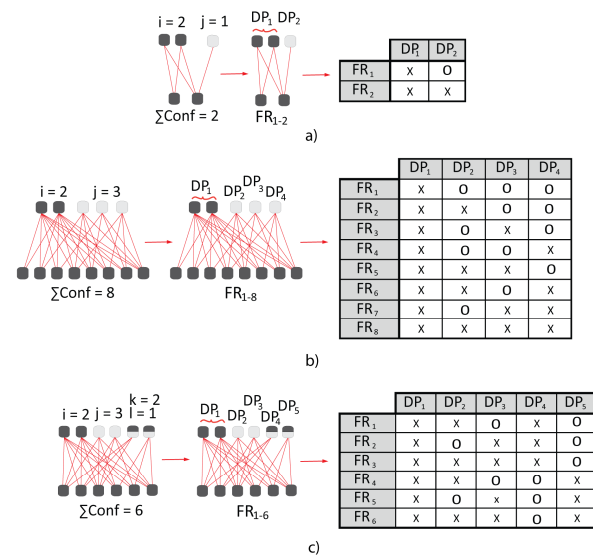


Fig. 4 Transformation of assembly node coupled design matrices with (a) $i=2, j=1$, (b) $i=2, j=3$, (c) $i=2, j=2, k=2$ and $l=1$.

ignoring permutations (such as AB/BA), then the combinatorial description is as follows:

$$C_N^{N_1} = \frac{N!}{(N-N_1)!N_1!} \tag{2}$$

Then, the first measure omega ‘Ω’ coming out of Equation (2) is often called a Degree of disorder, and can be expressed as follows:

$$\Omega = C_N^{N_1} * C_{N-N_1}^{N_2} * C_{N-N_1-N_2}^{N_3} * \dots * C_{N-N_1-\dots-N_{K-1}}^{N_K} = \frac{N!}{N_1!N_2!\dots N_K!} \tag{3}$$

Where ‘N’ is number of interactions within a measured design matrix, and N_1, N_2, \dots, N_K are number of interactions per each DP of the same matrix. In order to obtain less exponential and more tangible values of Ω, the second indicator was proposed, namely $Ln\Omega$.

For the second indicator, let f_j be the density in Q_j , i.e. the number of molecules per unit of μ-volume:

$$f_j = N_j/v^\mu, \tag{4}$$

Boltzmann defines a function for a statistical entropy description as follows:

$$H = \sum_{j=1}^K [f_j \ln f_j] v^\mu, \tag{5}$$

Then, a Systems Design Complexity (SDC), as denoted by authors, is expressed as follows:

$$SDC = \sum N_j \ln N_j, \tag{6}$$

where the volume is equal to unity and ‘ N_j ’ is interpreted as a number of interactions per single DP of the measured designed matrix. Our initial research question was: “How these indicators could be effectively used to quantify product variety induced complexity?” In order to verify their usability, it is useful to describe them briefly. Calculated values of the Ω, $Ln\Omega$ and SDC for all product classes and sub-classes have been determined for both, FS1 and FS2 (see Tab. 1 and 2).

DPs are represented by the number of input assembly components (columns N), while all stable components are always represented by a single DP.

As can be seen from the Table 1 with complexity indicators, values of CC for two classes grow incrementally,

Tab. 1 Fragment values of FS1 complexity indicators for all classes CL.

| Component Class | Component Sub-class | | ΣConf | N | DPs | Ω | ln[Ω] | SDC |
|-----------------|---------------------|----|-------|------|-----|---------|-------|--------|
| | S | VO | | | | | | |
| CL1 | 1 | 1 | 1 | 2 | 2 | 2 | 0,69 | 1,39 |
| | 1 | 2 | 3 | 7 | 3 | 210 | 5,3 | 6,1 |
| | 1 | 3 | 7 | 19 | 4 | 1,7E+09 | 21,3 | 30,3 |
| | 1 | 4 | 15 | 47 | 5 | 7,5E+28 | 66,5 | 107,2 |
| | 1 | 5 | 31 | 111 | 6 | 5,3E+79 | 183,6 | 328,3 |
| | 1 | 6 | 63 | 255 | 7 | | | 926,4 |
| | 1 | 7 | 127 | 573 | 8 | | | 2478,4 |
| | 1 | 8 | 255 | 1266 | 9 | | | 6334,7 |
| CL2+ | 2 | 1 | 2 | 3 | 2 | 3 | 1,1 | 4,7 |
| | 2 | 2 | 4 | 8 | 3 | 420 | 6,0 | 8,3 |
| | 2 | 3 | 8 | 20 | 4 | 4,4E+09 | 22,2 | 33,3 |
| | 2 | 4 | 16 | 48 | 5 | 2,2E+29 | 67,6 | 110,9 |
| | 2 | 5 | 32 | 112 | 6 | 1,9E+80 | 184,8 | 332,7 |
| | 2 | 6 | 64 | 256 | 7 | | | 931,6 |
| | 2 | 7 | 128 | 574 | 8 | | | 2484,2 |
| | 2 | 8 | 256 | 1267 | 9 | | | 6341,3 |

Tab. 2 Fragment values of FS2 complexity indicators for classes CL_{1-∞}.

| S | VO | CO | Condition for CO | ΣConf | N | DPs | Ω | ln[Ω] | SDC |
|-------------------|--------|--------|------------------|-------|-------------|-------------|-------------|--------|-------|
| CL _{1-∞} | 1 | 0 | 0 | 2 | 3 | 2 | 3 | 1,1 | 1,4 |
| | | 2 | 1 of 2 | 4 | 10 | 4 | 18900 | 9,8 | 9,7 |
| | | 3 | 1 of 3 | 6 | 15 | 5 | 37837800 | 17,4 | 18,2 |
| | | | 2 of 3 | 6 | 21 | 5 | 8,55513E+11 | 27,5 | 30,7 |
| | | 4 | 1 of 4 | 8 | 20 | 6 | 1,57135E+11 | 25,8 | 27,7 |
| | | | 2 of 4 | 12 | 42 | 6 | 1,51593E+28 | 64,9 | 83,6 |
| | | | 3 of 4 | 8 | 36 | 6 | 1,43045E+24 | 55,6 | 65,2 |
| | | 5 | 1 of 5 | 10 | 25 | 7 | 1,11314E+15 | 34,6 | 38,0 |
| | | | 2 of 5 | 20 | 70 | 7 | 1,27325E+52 | 120,0 | 166,1 |
| | | | 3 of 5 | 20 | 90 | 7 | 6,67365E+69 | 160,8 | 232,0 |
| | | | 4 of 5 | 10 | 50 | 7 | 6,55433E+32 | 75,6 | 114,3 |
| | | 6 | 1 of 6 | 12 | 30 | 8 | 1,20174E+19 | 43,9 | 48,9 |
| | 2 of 6 | | 30 | 105 | 8 | 1,36536E+84 | 193,7 | 280,8 | |
| | 3 of 6 | | 40 | 180 | 8 | n/a | n/a | 567,0 | |
| | 4 of 6 | | 30 | 165 | 8 | 7,5406E+140 | 324,4 | 502,1 | |
| | 5 of 6 | | 12 | 78 | 8 | 1,43801E+64 | 147,7 | 178,7 | |
| | o | l of k | - | - | - | - | - | - | - |
| | 2 | 0 | 0 | 3 | 7 | 3 | 210 | 5,3 | 6,1 |
| | | 2 | 1 of 2 | 6 | 20 | 5 | 1,62955E+11 | 25,8 | 28,4 |
| | | 3 | 1 of 3 | 9 | 30 | 6 | 6,52797E+18 | 43,3 | 51,2 |
| | | | 2 of 3 | 9 | 39 | 6 | 2,90509E+26 | 60,9 | 73,5 |
| | | 4 | 1 of 4 | 12 | 40 | 7 | 8,08467E+26 | 62,0 | 76,3 |
| | | | 2 of 4 | 18 | 78 | 7 | 4,44573E+59 | 137,3 | 190,8 |
| | | | 3 of 4 | 12 | 64 | 7 | 9,39695E+48 | 112,8 | 142,2 |
| 5 | | 1 of 5 | 15 | 50 | 8 | 1,83983E+39 | 90,4 | 90,4 | |
| | | 2 of 5 | 30 | 130 | 8 | 1,1909E+172 | 396,2 | 371,0 | |
| | | 3 of 5 | 30 | 160 | 8 | 2,7915E+136 | 314,2 | 482,0 | |
| | | 4 of 5 | 15 | 95 | 8 | 2,37897E+79 | 182,8 | 235,8 | |
| 6 | | 1 of 6 | 18 | 60 | 9 | 1,21409E+44 | 101,5 | 131,4 | |
| | 2 of 6 | 45 | 195 | 9 | n/a | n/a | 619,1 | | |
| | 3 of 6 | 60 | 320 | 9 | n/a | n/a | 1153,0 | | |
| | 4 of 6 | 45 | 285 | 9 | n/a | n/a | 987,6 | | |
| | 5 of 6 | 18 | 132 | 9 | 1,5224E+118 | 272,1 | 355,4 | | |
| o | l of k | - | - | - | - | - | - | - | |
| 3 | 0 | 0 | 7 | 19 | 4 | 1745944200 | 21,3 | 30,3 | |
| | 2 | 1 of 2 | 14 | 52 | 6 | 5,55671E+35 | 82,3 | 114,1 | |
| | 3 | 1 of 3 | 21 | 78 | 7 | 1,5753E+58 | 134,0 | 194,3 | |
| | | 2 of 3 | 21 | 99 | 7 | 3,51198E+78 | 180,9 | 260,6 | |
| | 4 | 1 of 4 | 28 | 104 | 8 | 5,71576E+81 | 188,3 | 280,9 | |
| | | 2 of 4 | 42 | 198 | 8 | n/a | n/a | 641,5 | |
| | | 3 of 4 | 28 | 160 | 8 | 2,4779E+136 | 314,1 | 482,1 | |
| | 5 | 1 of 5 | 35 | 130 | 9 | 6,7354E+109 | 252,9 | 358,7 | |
| | | 2 of 5 | 70 | 330 | 9 | n/a | n/a | 1113,3 | |
| | | 3 of 5 | 70 | 400 | 9 | n/a | n/a | 1368,0 | |
| | | 4 of 5 | 35 | 235 | 9 | n/a | n/a | 677,4 | |
| | 6 | 1 of 6 | 42 | 156 | 10 | 2,6951E+127 | 293,4 | 467,5 | |
| 2 of 6 | | 105 | 494 | 10 | n/a | n/a | 1972,3 | | |
| 3 of 6 | | 140 | 799 | 10 | n/a | n/a | 3527,9 | | |
| 4 of 6 | | 105 | 704 | 10 | n/a | n/a | 3010,0 | | |
| 5 of 6 | | 42 | 324 | 10 | n/a | n/a | 1132,4 | | |
| o | l of k | - | - | - | - | - | - | - | |

Note: “n/a” values represent Omega and Ln(Omega) values that are not available due to high variety.

unlike values of the Ω, where these reach extremely high values (e.g. CL₁, SCL₁⁰, where Ω = 5,3.10⁷⁹). Therefore, it was impossible to obtain appropriate values of LnΩ. Only values of SDC seem to grow incrementally. Thus, we can state, that SDC indicator can be effectively used for measurements of structural complexity of MCA.

Subsequently, a graph showing trend of growing CC and SDC values from FS1 has been constructed. Selected indicators and their trend lines can be seen in Fig. 5.

For the FS2, the same complexity indicating values have been obtained for specific initial components setup and for all possible individual selection from 2 to 6 CO components. Again, values of Ω and LnΩ are limited due to extremely high factorial numbers but appropriate values of SDC are available for further analysis.

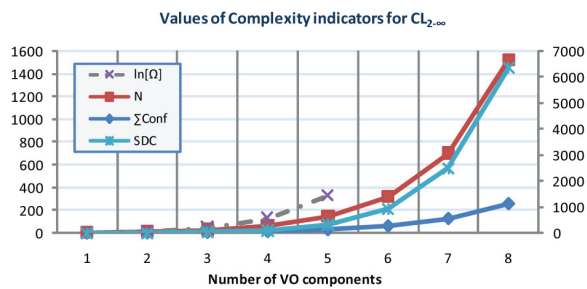


Fig. 5 Growing trend of complexity indicators in class $CL_{2-\infty}$.

6. Conclusions

In this paper, our ambition was to present a method to quantify product variety induced complexity based on combinatorial rules (CC) and to compare it with complexity based on existing Boltzmann's statistical concept in the design matrix (SDC) and Degree of disorder-based Ω , and $\ln\Omega$. Based on obtained results it is possible to state that all three newly applied complexity measures for design solutions can be theoretically employed to quantify product variety induced complexity. However, SDC suits the best for the given purpose, as it has been proved based on the presented testing of selected problems.

This paper is not intended to develop AD theory in the very basic concept, but to apply existing principles to quantify a product variety induced complexity using the approach of Guenov [21]. Subsequently, a comparison of three different approaches showed that there is visible mutual relation between the CC and SDC measures what evidently justifies a usability of SDC as the product variety induced complexity indicator. The most important finding of this study is that SDC indicator brings more realistic complexity values than CC indicator as it was described in Section 5 of this paper.

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