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A Statistical Solution to Mitigate Functional Requirements Coupling Generated from Process (Manufacturing) Variables Integration-Part I

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Abstract

Utilizing the Axiomatic Design (AD) principles to develop a perfect product, design of a manufacturing system with minimal complexity is required. For the purpose of reducing the manufacturing system complexity, theoretically, it is preferred to integrate multiple Process Variables (PVs) of the product into a single process unit. However, due to significant presence of some active noise factors, this integration practice may result in failing to maintain the independence among some of Functional Requirements (FRs) of the product. This event is the result of statistical causal relationships unintentionally developed among a subset of the integrated PVs. In such a condition, the AD's Independence Axiom cannot be successfully satisfied and reaching a system with minimal complexity is inconceivable, even though an uncoupled or decoupled system design is apparently presented. To mitigate this kind of FRs coupling generated from the PVs integration, this study proposes partial & semi-partial correlation analysis as a statistical solution to identify the most appropriate integration choices where integrating a subset of the PVs is inevitable. Furthermore, based on the Taguchi's loss function, a quantitative criterion is established to fairly compare any two non-ideal manufacturing system designs and choose the one with relatively lower loss. The proposed approach explained in this study is verified based on hypothetical data.

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Keywords: Independence Axiom; Noise Factors; Process (Manufacturing) Variables Integration; Partial & Semi-partial Correlation Analysis

1. Introduction

In order to achieve a perfect product, which is capable of satisfying every requirement demanded by customers in an effective way, the product engineers always have to make a series of complex decisions at different stages of the product development life-cycle [1-2]. Considering all of these stages, the failure in making good decisions at the engineering design stage may cause significant and fundamental problems in development of a successful product [3-6]. In fact, any poor design decision should be regarded as a serious obstacle for reaching a successful product because it cannot be often addressed easily by simple practices such as fine-tuning and/or design iterations [4, 7-8]. Commonly, we refer to poor decisions on design of a product as "conceptual weaknesses" at the design stage. They should be reduced even if they cannot be fully eliminated [5,8]. For this particular purpose, we do require a disciplined engineering and manufacturing process design that can tie a multitude of product (system) design tools together. By definition, the "engineering and manufacturing process design" is, in fact, a set of processes and activities required to transform the customers' needs and objectives into a series of design solutions [5].

In order to address and satisfy the identified requirements of the product, there is a wide variety of engineering design approaches to develop a capable product [9-10]. However, among all of these existing approaches, the strength of the Axiomatic Design (AD) theory to design an effective product, which is potentially capable of satisfying the customers' requirements, is emphasized [7, 9,11-12].

According to Suh [7, 13-14], to design any product or engineering system, the Customers' Attributes (CAs) must be specified into a minimum set of independent requirements usually defined by engineering terms in the functional domain

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of the product. These independent requirements are to completely characterize the functional needs of the product and known as Functional Requirements (FRs) of the product. In addition, in order to fulfil the FRs, we must define or select physical solutions which are referred to as Design Parameters (DPs) in the physical domain of the product [5, 7, 15]. Finally, to manufacture the product characterized in terms of DPs, we also have to develop a process which is specified by a set of Process Variables (PVs) in process domain of the product [5, 16]. Moreover, in a consistent way, the system design process must be simplified by breaking the higher level elements down into a set of sub-elements included at the lower levels of abstraction. The decomposition process is accomplished according to zigzag procedure. In fact, on the basis of zigzagging decomposition, the hierarchy in domain is defined by zigzagging back and forth between at least two adjacent domains [17] (Fig. 1).

From the AD's principles, in order to achieve a successful product, fulfilling every identified FR is absolutely essential. Because of this, it is important that all major sources that may significantly cause some variations in the product's FRs are identified. This is, in fact, a good way to find potential factors which can significantly inhibit improvement of the probability of success in satisfying the established FRs [5, 12]. Among all of potential causes which can make considerable variations in the product's FRs, the effect of incapable manufacturing system is critical [18- 19]. In fact, relying on the AD theory, since the product manufacturing system is the engineering system intended to support the product's PVs, any technical problems for PVs may result in some difficulties in fulfilling both DPs and FRs of the product. Therefore, from this view, developing the manufacturing system based on an improper design is considered a significant factor in failing to support the PVs and satisfying the product's DPs successfully. This means that a sound design for the product manufacturing system can considerably pave the way for fulfilling the FRs of the product effectively and efficiently.

With respect to presenting an efficient design for an engineering system, often, relying on the Information Axiom of the AD, it is argued that "a simple design is the best design" [20, 5-6, 12, 21]. From this, one may deduce that a good design for the product's manufacturing system makes one PV satisfy multiple DPs. This means that a coupled design looks better. However, from the AD's point of view, this is the case where multiple PVs are established to successfully satisfy the corresponding DPs of the same number. Nonetheless, these PVs may also be tightly integrated on a single process entity as well. In this study, we call such a design practice as "process integration" and it may be recommended.

Concerning the process integration practice, in order to verify the PVs integration practice usefulness in achieving a simpler system design, here we suitably employ the Second (Information) Axiom of the AD theory. For this purpose, consider an uncoupled process (manufacturing) design in which the PVs are not integrated and each PV particularly uses an independent "process entity" to satisfy its associated DP which has already been established in the physical domain. In addition, let consider $P_{DP_i}^{L}$ (i=1, 2, 3, …, n) be the statistical probability of satisfying the i^{th} DP at a given L^{th} level of the system's levels of abstraction. Hence, the overall probability of success for the manufacturing (process) design, $P_{Manufacturing}$, can be expressed as the Eq. (1) [4, 7];

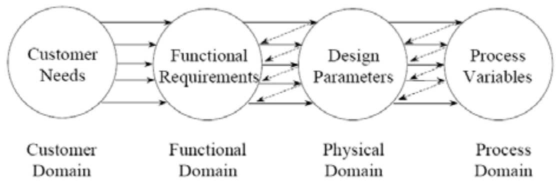

Fig.1. Mapping between Adjacent Domains in Axiomatic Design (AD)

$$P_{Manufacturing}^{L} = P_{\{n\}}^{L} = P\big((dr_1^l \le DP_1 \le dr_1^u), \dots, (dr_n^l \le DP_1 \le dr_n^u)\big) \quad (1)$$

Where; dr_i^l and dr_i^u represent the "upper design range" and "lower design range" of the DP_i, respectively.

In addition, with no loss of generality, assume that the DPs are also statistically independent of each other. Hence, the Eq. (1) can be rewritten as the Eq. (2); [4, 7]

$$P_{\{n\}}^{L} = \prod_{i=1}^{n} P((dr_i^l \le DP_i \le dr_i^u)) \quad (2)$$

In this type of process (manufacturing) design, since PV_i independently employs a particular Process Entity (PE_i) to satisfy its associated DP_i (i=1, 2, 3, …, n), we have to use the concept of "conditional probability" to accurately measure probability of satisfying the DP_i . For this purpose, as the Eq. (3) expresses, we need to study the probability of satisfying the DP_i with respect to success/failure of the PE_i;

$$P\big((dr_i^l \le DP_i \le dr_i^u)\big) = P((dr_i^l \le DP_i \le dr_i^u)|PE_i)).P(PE_i) + P((dr_i^l \le DP_i \le dr_i^u)|PE_i^{C})).P(PE_i^{C}) \quad (3)$$

Where; PE_i and PE_i^{C} represent the "success" and "failure" of the PE_i to support the PV_i for fulfilling the DP_i, respectively.

Moreover, without loss of generality, assume that the probability of success of each PE in supporting the PV_i is the same. That is, P (PE_i) =P (for i=1, …, n). Hence, the Eq. (2) can be rewritten as the Eq. (4);

$$P_{\{n\}}^{L} = P^n.\prod_{i=1}^{n} P\big((dr_i^l \le DP_i \le dr_i^u)|PE_i\big) \quad (4)$$

Therefore, the "information content" and "complexity" of such a system design can be obtained by the Eq. (5);

$$\begin{aligned} C_{Manufacturing} &= I_{Manufacturing} \\ &= -\log_2 P_{\{n\}}^{L} \\ &= -\log_2\{P^n.\prod_{i=1}^{n} P\big((dr_i^l \le DP_i \le dr_i^u)|PE_i)\big)\} \\ &= (n.\log_2 P) + \sum_{i=1}^{n} \log_2 P\{\big(dr_i^l \le DP_i \le dr_i^u\big)|PE_i\} \end{aligned} \quad (5)$$

On the other hand, regarding a manufacturing process) design in which the PVs (PV_1, PV_2, …, and PV_n) are integrated on a single Process Entity (PE). Similarly, here we use the "conditional probability" concept to measure probability of satisfying the DPs. Therefore, "the overall probability of success" for such a system design can be expressed as Eq. (6);

$$P^L_{Manufacturing} = P^L_{\{n\}}$$
$$= P((dr^l_1 \le DP_1 \le dr^u_1), \dots, (dr^l_n \le DP_1 \le dr^u_n))$$
$$= P((dr^l_1 \le DP_1 \le dr^u_1), \dots, (dr^l_n \le DP_1 \le dr^u_n)|PE).P(PE) +$$
$$P((dr^l_1 \le DP_1 \le dr^u_1), \dots, (dr^l_n \le DP_1 \le dr^u_n)|PE^C).P(PE^C) \quad (6)$$

Assume that the probability of success PE in supporting the PVs to satisfy the corresponding DPs is also P. That is, P (PE) =P. Hence, the Eq. (6) can be rewritten as the Eq. (7);

$$P^L_{\{n\}} = P.\prod_{i=1}^{n} P((dr^l_i \le DP_i \le dr^u_i)|PE) \quad (7)$$

Therefore, the "information content" and "complexity" of such a system design can be obtained by the Eq. (8);

$$C_{Manufacturing} = I_{Manufacturing}$$
$$= -\log_2 P^L_{\{n\}}$$
$$= -\log_2\{P.\prod_{i=1}^{n} P((dr^l_i \le DP_i \le dr^u_i)|PE))\}$$
$$= (\log_2 P) + \sum_{i=1}^{n} \log_2 P((dr^l_i \le DP_i \le dr^u_i)|PE)\} \quad (8)$$

Therefore, comparing Eq. (5) and Eq. (8), it is clearly concluded that integration of PVs on a single process entity may result in relatively lower information content. However, this conclusion is true where there is no "noise factors" in the system. This is the important point the present study is to address.

With respect to the practice of PVs Integration on a single manufacturing unit, sometimes, due to the presence of some active noise factors, such as time, location, worker's skills, common limited resources, etc., which may be frequently experienced in manufacturing environments, the integration of PVs cannot maintain the inherent independence of the PVs. Such a condition particularly occurs where causal relationships between some of PVs are developed. In fact, often, following integrating PVs in one single process unit, some strong statistical causal relationships between some of PVs have been developed unintentionally. That is, in such a condition, the behaviors of some of PVs depend causally on behaviors of the others. For example, consider a statistical causal dependency between functions of two different CNC machines (as two DPs of a given product) integrated on a common multi-skill worker to manufacture the product of interest. Concerning this case, it is clear that the worker's skills to work with the machines are, in fact, the corresponding PVs required to fulfill the specified DPs. Here the number of settings the machines may require to properly operate can be regarded as one the possible active noise factors. In fact, if the number of required settings for two machines significantly increases, the worker may not appropriately divide his/her available time between two machines and, as a result, the machines will not be served properly. In such a situation, the functions of the machines may therefore depend causally on each other even though the established PVs are apparently mapped into the DPs via an uncoupled process design. As another example for clarifying the considerable role of noise factors in developing undesirable causal relationships between some of integrated PVs, consider a manufacturing process in which a common robot must load/unload two different machines. Clearly, the functions of these two machines integrated on this common industrial robot are, in fact, regarded as two DPs established in physical domain of product intended to be manufactured. In this example, the distance between these two machines is considered to be a serious noise factor. In fact, in this case, the location of the facilities may make some difficulties for the robot in effectively employing the machines. In other words, in this case, the inappropriate locations of the machines will lead to developing a statistical causal relationship between the functions PVs) of the employed robot.

Obviously, any causal relationship between any certain pair of PVs is considered a significant cause for failing to maintain the independency of DPs and FRs, in turn. In fact, since the DPs and FRs are theoretically functions of the PVs, any strong causal relationship between any given pair of PVs will subsequently result in developing some type of causal relationships between the corresponding FRs as well. Clearly, this means violating the First (Independence) Axiom of the AD theory. In other words, a real coupling among a subset of the product's FRs is expected to be met where the PVs integration leads to some significant causal relationships. This is the main challenge the present study is going to address as a serious problem in manufacturing system design context. Hence, the contribution of the current work can be outlined as the followings;

- Clarifying effect of significant causal correlation among a certain subset of the product's PVs -- which may arise through some active noise factors in PVs integration-- as a serious obstacle for satisfying the product's FRs effectively even though an uncoupled or decoupled manufacturing system design is apparently presented.
- Establishing a quantitative criterion, based on the Taguchi's Loss Function, to fairly compare any two different design decisions on integrating the PVs in one single process unit and choose the one with relatively lower loss.
- Proposing a statistical solution to identify the most appropriate integration choices where integrating a subset of the PVs is inevitable.

2. FRs Coupling which Originates from Integration of PVs

In this section, we are going to begin with delineating the real difference of two terms Coupling and Correlation between any two design variables (e.g. FRs/DPs/PVs of a product of interest). The "coupling" between a given two design variables is certain and may be established by a poor mapping practice while the "correlation" is hypothesized to be a sign of a statistical relationship between them. That is, any two design variables can be statistically correlated while they are not coupled. However, this is the case where there does not exist a significant statistical causal relationship between these two design variables. This means that we can experience an apparent uncoupled or decoupled system design in which some of the design variables in a specific domain are actually coupled because their corresponding elements in adjacent domain have significant causal correlations. For the purpose of understanding such a condition more, for example; consider a two-DP uncoupled process design as the Eq. (9) expresses;

$$\begin{bmatrix} DP_1 \\ DP_2 \end{bmatrix} = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} \cdot \begin{bmatrix} PV_1 \\ PV_2 \end{bmatrix} \tag{9}$$

Or, in other words;

$$\begin{aligned} DP_1 &= B_{11}.PV_1 \\ DP_2 &= B_{22}.PV_2 \end{aligned} \tag{10}$$

In addition, assume that PV_2 is a function of PV_1since the PV_2 is causally correlated to PV_1. That is;

$$PV_2 = g(PV_1) + \varepsilon \quad ; \quad \varepsilon \sim N(0, \sigma^2) \tag{11}$$

Where; the ε represents random component of this statistical causal relationship between the $PV_1$and the PV_2. Moreover, it is assumed that the ε has a "Normal" probability distribution with mean zero and variance σ^2. Thus, the Eq. (10) can be rewritten as the Eq. (12);

$$\begin{aligned} DP_1 &= B_{11}.PV_1 \\ DP_2 &= B_{22}.(g(PV_1) + \varepsilon\,) \end{aligned} \tag{12}$$

Therefore, DP_1 and DP_2 are coupled to each other although the Eq. (9) does not apparently imply such a fact.

Hence, on the basis of argument above, it is clearly concluded that any (causal) correlation among the PVs will also result in generating a (causal) correlation among the DPs directly. In this regard, in order to explore the magnitude of the (causal) correlations between a given pair of the DPs in terms of degree of correlation between its corresponding PVs, let's consider the Eq. (13) as a process design equation which represents the pattern of mapping practice between physical and process domain of the product;

$$\begin{bmatrix} DP_1 \\ DP_2 \\ \cdots \\ DP_n \end{bmatrix} = \begin{bmatrix} B_{11} & 0 & \cdots & 0 \\ 0 & B_{22} & \cdots & 0 \\ \vdots & . & \ddots & \vdots \\ 0 & 0 & \cdots & B_{nn} \end{bmatrix} \cdot \begin{bmatrix} PV_1 \\ PV_2 \\ \cdots \\ PV_n \end{bmatrix} \tag{13}$$

Also, let $\Sigma_{\boldsymbol{DP}}$represent the "Variance-Covariance Matrix" of the vector $\boldsymbol{DP}$, VCM$_{\boldsymbol{DP}}$. According to the AD theory, it is clear that the concerned DPs can be mathematically expressed as the Eq. (14);

$$\boldsymbol{DP} = [\boldsymbol{B}].\boldsymbol{PV} \tag{14}$$

Where; $\boldsymbol{PV}$ is an n-vector which includes all established PVs. On differentiation, the matrix $[\boldsymbol{B}]$--which relates the $\boldsymbol{DP}$ vector to the $\boldsymbol{PV}$ vector of the manufacturing process-- is, in fact, the sensitivity process design matrix with entries given by the Eq. (15);

$$B_{jk} = \frac{\partial \boldsymbol{DP_j}}{\partial \boldsymbol{PV_k}} j = 1, \dots, p\,;\, k = 1, \dots, n \tag{15}$$

Therefore, the Eq. (16) follows;

$$\Sigma_{\boldsymbol{DP}} = \mathrm{Var}(\boldsymbol{DP}) = \mathrm{Var}([\boldsymbol{B}].\boldsymbol{PV})$$

$$= [\boldsymbol{B}].\Sigma_{\boldsymbol{PV}}.[\boldsymbol{B}]^{\mathbf{T}} \tag{16}$$

Where; $\Sigma_{\boldsymbol{PV}}$ represents the "Variance-Covariance Matrix" of the vector $\boldsymbol{PV}$, VCM$_{\boldsymbol{PV}}$. Also, the $\Sigma_{\boldsymbol{PV}}$can be expressed as the Eq. (17);

$$\Sigma_{\boldsymbol{PV}} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & . & \ddots & \vdots \\ \sigma_{1n} & \sigma_{2n} & \cdots & \sigma_{nn} \end{bmatrix} \tag{17}$$

Thus, VCM$_{\boldsymbol{DP}}$ is;

$$\Sigma_{\boldsymbol{DP}} = \begin{bmatrix} B_{11}.B_{11}\sigma_{11} & B_{11}.B_{22}\sigma_{12} & \cdots & B_{11}.B_{nn}\sigma_{1n} \\ B_{22}.B_{11}\sigma_{12} & B_{22}.B_{22}\sigma_{22} & \cdots & B_{22}.B_{nn}\sigma_{2n} \\ B_{33}.B_{11}\sigma_{13} & B_{33}.B_{22}\sigma_{23} & \cdots & B_{33}.B_{nn}\sigma_{3n} \\ \vdots & . & \ddots & \vdots \\ B_{nn}.B_{11}\sigma_{1n} & B_{nn}.B_{22}\sigma_{2n} & \cdots & B_{nn}.B_{nn}\sigma_{nn} \end{bmatrix} \tag{18}$$

Therefore, on the basis of the Eq. (18), for any pair of design parameters, the covariance between DP_i and DP_j (i,j=1,2, …, n ; i ≠ j) can be written as the Eq. (19);

$$\mathrm{Cov}\left(DP_i, DP_j\right) = B_{ii}.B_{jj}.\sigma_{ij} \quad (\mathrm{i,j} = 1,2, \dots \mathrm{n}\,; \mathrm{i} \neq \mathrm{j}) \tag{19}$$

Where, Cov $\left(DP_i, DP_j\right)$ stands for the covariance between DP_i and DP_j (i,j=1,2, …, n ; i ≠ j). Also, the σ_{ij} represents the covariance between PV_i and PV_j (i,j=1,2, …, n ; i ≠ j). Obviously, Cov (DP_i, DP_j) will be zero if and only if either B_{ii} or B_{jj} is equal to zero.

Hence, according to the Eq. (19), any kind of correlation between any given pair of the DPs can be directly derived from the correlation between its corresponding PVs. Moreover, generalizing the Eq. (12), it is clearly concluded that any causal correlation/relationship between a certain pair of PVs will result in developing a causal correlation/ relationship between their corresponding DPs as well. This indicates violation of the Independence Axiom of the AD theory in process design of the product even though the mapping practices are accomplished in an uncoupled/ decoupled pattern. In addition, because of the interplay between the physical and functional domain of the product, it is concluded that the developed causal correlation/relationship between a certain pair of the DPs will subsequently result in developing a causal correlation/relationship between their corresponding FRs as well. Similarly, this indicates violation of the Independence Axiom of the AD theory in physical design of the product even though the mapping practices are performed in an uncoupled/ decoupled pattern.

Therefore, regarding the physical design of the product, to clarify the magnitude of the (causal) correlations between a given pair of the FRs in terms of degree of the correlation between their corresponding DPs, the Eq. (20), like the Eq. (19), can be written;

$$\mathrm{Cov}\left(FR_i, FR_j\right) = A_{ii}.A_{jj}.\mathrm{Cov}\left(DP_i, DP_j\right)(\mathrm{i,j} = 1,2, \dots \mathrm{n}\,; \mathrm{i} \neq \mathrm{j}) \tag{20}$$

Where, A_{kk} is the entry (k,k) of the physical design matrix $[\boldsymbol{A}]$. In addition, let's assume that $[\boldsymbol{C}] = [\boldsymbol{A}].[\boldsymbol{B}]$. Hence, the Eq. (18) can be expressed as the Eq. (21);

$$\mathrm{Cov}\left(FR_i, FR_j\right) = C_{ii}.C_{jj}.\sigma_{ij} \quad (\mathrm{i,j} = 1,2, \dots \mathrm{n}\,; \mathrm{i} \neq \mathrm{j}) \tag{21}$$

Where, Cov $\left(FR_i, FR_j\right)$stands for the covariance between FR_i and FR_j (i,j=1,2, …, n ; i ≠ j), and σ_{ij} represents the covariance between PV_i and PV_j (i,j=1,2, …, n ; i ≠ j). Obviously, Cov (FR_i, FR_j) will be zero if and only if either C_{ii} or C_{jj} is equal to zero.

Therefore, according to the Eq. (21), any strong causal relationship between any given pair of PVs will subsequently result in developing some type of causal relationships (coupling) between their associated FRs as well.

3. Deriving A Mathematical Relationship between Degree of the PVs Dependencies and Amount of Loss that Stakeholders Have to Incur

With respect to an uncoupled design for physical mapping between functional and physical domain of the intended product design, let consider $\boldsymbol{T}^T = [T_1 \quad T_2 \quad \dots \quad T_m]^T$ and $\boldsymbol{FR}^T = [FR_1 \quad FR_2 \quad \dots \quad FR_m]^T$ be the "Target Value" and the identified "***FR***" vector, respectively. In addition, let consider **L (*T, FR*)** be the product physical design quality loss function [22-25]. Expanding **L (*T, FR*)** at ***FR=T*** yields the Eq. (22);

$$\mathbf{L}(\boldsymbol{T},\boldsymbol{FR}) = \mathbf{L}|_{\boldsymbol{FR}=\boldsymbol{T}} + \nabla\mathbf{L}|_{\boldsymbol{FR}=\boldsymbol{T}}\{\boldsymbol{FR}-\boldsymbol{T}\} + \frac{1}{2}\{\boldsymbol{FR}-\boldsymbol{T}\}^{\mathrm{T}}.\boldsymbol{H}|_{\boldsymbol{FR}=\boldsymbol{T}}\{\boldsymbol{FR}-\boldsymbol{T}\} + \cdots \quad (22)$$

Where; ***H*** is the Hessian matrix. It is clear that **L (*T, FR*)** would be minimal when ***FR*** is equal to ***T***. Therefore, $\mathbf{L}|_{\boldsymbol{FR}=\boldsymbol{T}} = \mathbf{0}$. If the product operates around the vector ***FR*** which is equal to ***T***, the quadratic term of the Eq. (22) may dominate the expansion and, hence, we can express the Eq.(22) as Eq.(23);

$$\mathbf{L}(\boldsymbol{T},\boldsymbol{FR}) \cong \frac{1}{2}\{\boldsymbol{FR}-\boldsymbol{T}\}^{\mathrm{T}}.\boldsymbol{H}|_{\boldsymbol{FR}=\boldsymbol{T}}\{\boldsymbol{FR}-\boldsymbol{T}\} \quad (23)$$

Let μ_{FR_i} and $\sigma^2_{FR_i}$ represent the mean and the variance of FR_i , i=1, 2, …, m, respectively. Therefore, the expected value for the product's quality loss in the Eq. (23) can be expressed as the Eq. (24);

$$\mathrm{E}[\mathrm{L}(FRs,Ts)] = \sum_{i=1}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i^2}|_{FR_i=T_i}\left[\sigma^2_{FR_i} + \left(\mu_{FR_i}-T_i\right)^2\right] + \sum_{i=1}^{m-1}\sum_{k=1+i}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i\,\partial FR_k}|_{\substack{FR_i=T_i\\FR_k=T_k}}[\mathrm{Cov}\,(FR_i,FR_k) + (\mu_{FR_i}-T_i).(\mu_{FR_k}-T_k)] \quad (24)$$

Where; $\mathrm{Cov}\,(FR_i,FR_k)$ denotes the covariance between FR_i and FR_k (i,k=1,2, …, m ; i ≠ k).

Hence, according to the Eq. (24), the product quality loss function consists of three ingredients including "variability of each *FR*", "mean adjustment of each *FR* to its associated target value", and "covariance between the individual FRs".

Let Y_{ij} ,i = 1, . . . , m; j = 1, . . ., m, be a binary variable indicating the existence of the mapping between FR_i and DP_j as the Eq. (25) expresses;

$$Y_{ij} = \begin{cases}1 & \mathrm{FR_i\,is\ Mapped\ into\ DP_i}\\ 0 & \mathrm{Otherwise}\end{cases} \quad (25)$$

Also, let μ_{DP_i} and $\sigma^2_{DP_i}$ be the mean and the variance of the design parameter DP_{i}. Therefore, FR_{i} can be expressed as the Eq. (26);

$$FR_i = \sum_{k=1}^{m} Y_{ik}\left(\frac{\partial FR_i}{\partial DP_k}\right)_{DP_k=\mu_k}.DP_k \quad (26)$$

And, FR_{j} can be expressed as the Eq. (27);

$$FR_j = \sum_{l=1}^{m} Y_{jl}\left(\frac{\partial FR_j}{\partial DP_l}\right)_{DP_l=\mu_l}.DP_l \quad (27)$$

Now, let $L_{\mathrm{Corr.}}$ be the quality loss due to DPs correlation. Then, we may have;

$$\begin{aligned}\mathrm{E}[\mathrm{L_{Corr.}}(FRs,Ts)] &= \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i\,\partial FR_j}_{|\substack{DP_i=\mu_i\\DP_j=\mu_j}} .\ \mathrm{Cov}\left(\begin{matrix}\sum_{k=1}^{m} Y_{ik}\left(\frac{\partial FR_i}{\partial DP_k}\right)_{DP_k=\mu_k}.DP_k,\\ \sum_{l=1}^{m} Y_{jl}\left(\frac{\partial FR_j}{\partial DP_l}\right)_{DP_l=\mu_l}.DP_l\end{matrix}\right)_{\substack{Y_{ik}=Y_{jl}=1\\ j\neq l}}\\ &= \sum_{i=1}^{m-1}\sum_{j=1+i}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i\,\partial FR_j}_{|\substack{DP_i=\mu_i\\DP_j=\mu_j}}.\sum_{l=1}^{m-1}\sum_{k=l+1}^{m} Y_{ik}.Y_{jl}.\left(\frac{\partial FR_i}{\partial DP_k}\right)_{DP_k=\mu_k}.\left(\frac{\partial FR_j}{\partial DP_l}\right)_{DP_l=\mu_l}.\mathrm{Cov}\,(DP_k,DP_l)\end{aligned} \quad (28)$$

In addition, since the DPs are correlated, the error propagation formula of $\mathrm{FR_i}$ can be given as the Eq. (27);

$$\sigma^2_{FR_i} \cong \sum_{k=1}^{m}\sum_{l=1}^{m}\left(\frac{\partial FR_i}{\partial DP_k}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.\left(\frac{\partial FR_i}{\partial DP_l}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.\mathrm{Cov}(DP_k,DP_l) \quad (29)$$

Therefore, according to the Eq. (19), Eq. (28), & Eq. (29), the Eq. (24) can be expressed as the Eq. (30). The Eq. (30), in fact, mathematically relates the degree of dependency (correlation) between any given pair of PVs to the amount of loss (cost) the stakeholders (customers) have to incur. To be more specific, on the basis of the Eq. (30), we can predict amount of the loss the stakeholders will incur due to the product's FRs dependencies/couplings that originate from the PVs integration.

$$\begin{aligned}\mathrm{E}[\mathrm{L}(FRs,Ts)] &= \sum_{i=1}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i^2}|_{FR_i=T_i}\left\{\sum_{k=1}^{m}\sum_{l=1}^{m}\left(\frac{\partial FR_i}{\partial DP_k}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.\left(\frac{\partial FR_i}{\partial DP_l}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.B_{kk}.B_{ll}.\sigma_{kl} + \left[\sum_{k=1}^{m} Y_{ik}\left(\frac{\partial FR_i}{\partial DP_k}\right)_{DP_k=\mu_k}.\mu_k - T_i\right]^2\right\}\\ &+ \sum_{i=1}^{m-1}\sum_{j=1+i}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i\,\partial FR_j}_{|\substack{DP_i=\mu_i\\DP_j=\mu_j}}.\sum_{l=1}^{m-1}\sum_{k=l+1}^{m} Y_{ik}.Y_{jl}.\left(\frac{\partial FR_i}{\partial DP_k}\right)_{DP_k=\mu_k}.\left(\frac{\partial FR_j}{\partial DP_l}\right)_{DP_l=\mu_l}.B_{kk}.B_{ll}.\sigma_{kl}\\ &+ \sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\frac{\partial^2\mathrm{L}}{\partial FR_i\,\partial FR_j}_{|\substack{FR_i=T\\FR_k=T_j}}.\left\{\sum_{k=1}^{m}[Y_{ik}.Y_{kj}.\left(\frac{\partial FR_i}{\partial DP_k}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.\left(\frac{\partial FR_k}{\partial DP_j}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.\mu^2_{DP} - \mu_{DP_j}.[Y_{ik}.\left(\frac{\partial FR_i}{\partial DP_k}\right)_{\substack{DP_k=\mu_k\\DP_l=\mu_l}}.T_k + Y_{kj}.\left(\frac{\partial FR_i}{\partial DP_j}\right)_{\substack{DP_j=\mu_{DP_j}\\DP_l=\mu_l}}.T_i]] + T_i.T_k\right\}\end{aligned} \quad (30)$$

According to the Eq. (30), it is concluded that the stronger the PVs dependencies are, the heavier the loss will be incurred. On the basis of such a conclusion, the Eq. (30) is considered to be a sound quantitative criterion for comparing any two different design decisions on integrating the PVs in one single process unit and, hence, choose the one with relatively lower loss.

4. "Partial & Semi-partial Correlation Analysis" as a Statistical Solution to Identify the Most Appropriate Choice of the Process (Manufacturing) Integration

On the basis of what we argued in sections above, it is concluded that to provide an appropriate design for the manufacturing system associated with the product of interest, we have to detect and analyze the statistical (causal) correlations which may arise among a subset of the PVs integrated in one single process unit. For this reason, type and significance of the relationships among the integrated PVs are of particular interest of the present study. In practice, we seldom find PVs perfectly correlated or cases that do not contain any stochastic error component. Hence, providing some effective diagnostic tool for detecting "correlation" among a sub-set of the PVs is considered a significant step toward achieving good designs for the manufacturing system of the product.

4.1. Detection of Correlation among the PVs

To detect the "correlation" among the PVs, here we propose use of "partial and semi-partial correlation analysis", which is frequently helpful for identifying correlation among the PVs [26]. A "partial correlation" is a correlation between two specific PVs from which the linear relations, or effects, of another PV(s) have been removed. The "order" of the partial correlation coefficient is indicated by the number of PVs that are being controlled. A simple correlation is sometimes referred to as a "zero-order" correlation while a correlation between two PVs controlling for a third would be a "first-order" correlation in case of dealing with three PVs (Fig. 2) [26];

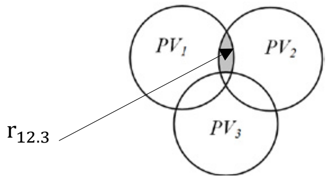

Fig.2. First Order Correlation

For instance, in case of studying three different PVs (PV_1-PV_3); the first-order partial correlation between PV_1 and PV_2 where the effect of PV_3 is removed ($r_{12.3}$) can be expressed as Eq. (31);

$$r_{12.3} = \frac{r_{12} - r_{13}.r_{23}}{\sqrt{1 - r_{13}^2}.\sqrt{1 - r_{23}^2}} \quad (31)$$

Where, "1", "2", and "3" represent PV_1, PV_2, and PV_3, respectively. In addition, r_{ij} is to represent the statistical correlation between PV_i and PV_j (i,j=1,2,& 3 ; $i \neq j$).

Alternatively, as another effective way for accomplishing the partial correlation considerations, the multiple regression analysis can also be employed usefully. To be more specific, regarding this way of partial correlation analysis, we can pillar the squared partial correlation of interest on the coefficients of partial determination analysis. On the basis of this approach, it can be shown that the Eq. (31) can be expressed as the Eq. (32);

$$r_{12.3}^2 = \frac{R_{1.23}^2 - R_{1.3}^2}{1 - R_{1.3}^2} \quad (32)$$

Where;

- $r_{12.3}^2$; represents the coefficient of partial determination for measuring the marginal contribution of PV_2 to predicting $PV_1$where PV_3 is already included in the regression model.
- $R_{1.23}^2$; represents the coefficients of determination from the multiple regression model in which PV_2 and PV_3 are explanatory variables employed to predict the response variable PV_1.
- $R_{1.3}^2$; represents the coefficients of determination from the simple regression model in which $PV_3$is explanatory variables employed to predict the response variable PV_1.

The second-order partial correlation is the correlation between two PVs where the effects of two other PVs are removed (Fig.4). For example, in case of studying four different PVs (PV_1-PV_4); the second-order partial correlation between PV_1 and PV_2 where the effects of PV_3 and PV_4 are removed ($r_{12.34}$) can be obtained by Eq. (33);

$$r_{12.34} = \frac{r_{12.3} - r_{14.3}.r_{24.3}}{\sqrt{1 - r_{14.3}^2}.\sqrt{1 - r_{24.3}^2}} \quad (33)$$

Where, "1", "2", "3", and "4" represent PV_1, PV_2, PV_3, and PV_4, respectively. In addition, $r_{ij.k}$ stands for the coefficient of partial determination for measuring the marginal contribution of PV_j to predicting PV_i where PV_k is already included in the regression model (i,j, k=1,2,& 3 ; $i \neq j \neq k$).

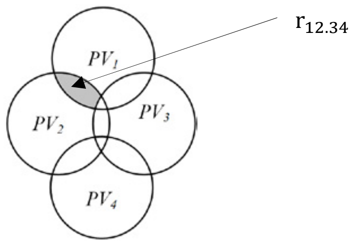

Fig. 4.Second Order Correlation

Similarly, here we can also use the multiple regression analysis to perform the partial correlation considerations. That is, the Eq. (33) can also be expressed as Eq. (34);

$$r_{12.34}^2 = \frac{R_{1.234}^2 - R_{1.34}^2}{1 - R_{1.34}^2} \quad (34)$$

Where;

- $r_{12.34}^2$; represents the coefficient of partial determination for measuring the marginal contribution of PV_2 to predicting $PV_1$where $PV_3$and PV_4 are already included in the regression model.
- $R_{1.234}^2$; represents the coefficients of determination from the multiple regression model in which PV_2 , PV_3, and PV_4 are explanatory variables employed to predict the response variable PV_1.
- $R_{1.34}^2$; represents the coefficients of determination from the simple regression model in which PV_3 and PV_4 are explanatory variables employed to predict the response variable PV_1.

The "semipartial correlation analysis" (also called as "part correlation analysis") can also be employed as another helpful tool for detecting the correlation among the product PVs, Specifically, using the semipartial correlation, we can remove the effects of additional PVs from one of the concerned PVs. For instance, in case of studying three different PVs (PV_1-PV_3); the first semipartial correlation between PV_1 and PV_2

where the effect of PV_3 is removed from PV_2 ($r_{1(2.3)}$) can be expressed as Eq. (35);

$$r_{1(2.3)} = \frac{r_{12} - r_{13}.r_{23}}{\sqrt{1 - r_{23}^2}} \quad (35)$$

Where, "1", "2", and "3" represent PV_1, PV_2, and PV_3, respectively. In addition, r_{ij} is to represent the statistical correlation between PV_i and PV_j (i,j=1,2,& 3 ; $i \neq j$).

Similarly, we can benefit from the multiple regression analysis to accomplish the semipartial correlation analysis. That is, the Eq. (35) can be expressed as the Eq. (36);

$$r_{1(2.3)}^2 = R_{1.23}^2 - R_{1.3}^2 \quad (34)$$

Where;

- $r_{1(2.3)}^2$; represents the coefficient of semipartial determination for measuring the marginal contribution of PV_2 to predicting $PV_1$where PV_3 is already included in the regression model.
- $R_{1.23}^2$; represents the coefficients of determination from the multiple regression model in which PV_2 and PV_3 are explanatory variables employed to predict the response variable PV_1.
- $R_{1.3}^2$; represents the coefficients of determination from the simple regression model in which PV_3 is explanatory variables employed to predict the response variable PV_1.

Such a statistical analysis can be usefully employed to examine all possible choices of the PVs integration and identify the most appropriate integration choice.

5. Conclusion and Discussion

Often, in developing an engineering system or a product based on the Axiomatic Design (AD)'s principles, it is argued that integration of the product's PVs on a single process unit is a good way for reaching system designs with relatively lower complexity. However, in this study, we argued that such a design practice may be true where there is no "noise factors" in the system. This work theoretically illustrated that, due to the presence of some active noise factors in manufacturing environments, the integration of PVs may unintentionally result in development of some significant statistical causal relationships among a specific subset of the PVs. This damages the inherent independency among the PVs and will result in violating the AD's First (Independence) Axiom in both process and physical design of the product even though uncoupled or decoupled mapping designs are apparently presented. However, at most of times, due to some technical/ physical/financial constraints, integration of a subset of PVs is inevitable. For this reason, we established a sound quantitative criterion to fairly compare any two non-ideal manufacturing system designs to help system designers identify a good design with relatively lower loss. Indeed, based on this criterion, the system designers are able to predict amount of the loss that the stakeholders will incur because of the PVs dependencies. Moreover, in order to explore and control any significant statistical causal relationship between any pair of the PVs, we proposed partial & semi-partial correlation analysis as a useful statistical solution to identify the most appropriate integration choices in which the PVs dependencies are minimal.

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