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ADAPTIVE TOLERANCE ALLOCATION TO ADJUST THE SIGMA LEVEL THROUGH MINIMIZATION OF THE INFORMATION CONTENT

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ABSTRACT

Nowadays, mass production companies usually employ the sigma level as the preferred metric to control their activities with the purpose of operational optimization. Six-sigma is usually taken as the minimum level to achieve the excellence of the organization. To increment the global sigma level of assembly operations it is necessary to assure a suitable level for the capability index, C_{pk}, in the manufacture of each one of the assembly components. C_{pk} depends on the design tolerances and on the characteristics of the manufacturing process and is usually attained by adjusting the manufacturing variables according to the data acquired through quality control. If these data were sent to the design department, then it would be possible to attain some prescribed sigma level at the assembling operation by selective adaptation of the tolerance allocation, without any intervention on the manufacturing processes. The tolerance values of the complete mechanical system are mutually dependent according to the adopted model of analysis, but they must assure the functionality of the assembled sets. However, the manufacturing of each component is independent from the others.

The knowledge about the manufacturing capability indices allows computing the information content, the best situation being to achieve the highest sigma level so that one could have the lowest information content for the overall manufacturing process. This paper presents a method that allows an adaptive tolerance allocation that leads to the required sigma level. The method is adaptive, for it can consider changes of the manufacturing system.

Keywords: process capability, information content, tolerance allocation, six-sigma.

1 INTRODUCTION

The lack of knowledge of the actual capabilities of manufacturing processes, associated to required tight tolerances, may yield to great difficulties to make products with the specifications required by design.

Manufacturing processes are exposed to several disturbances that may cause dimensional and geometrical variations of the manufactured part. These variations should be kept within the specified design tolerances, which accomplishment is translated into quality of the manufacturing process. If the product is an assembly of several components, the situation becomes more complex, since the variation of the quality characteristic of the assembly depends on the contributions of the components' variations.

The use of quality control methods, such as Statistical Process Control (SPC) is a industry standard, particularly in the automotive supply chain, where it is used for monitoring and controlling the processes with the aim of ensuring that they operate at their full potential [Dietrich & Schulze, 1998]. Statistical methods like SPC usually generates large amounts of data that accurately characterize the processes, which allow achieving the values for the capability indices of the manufacturing tasks.

Nowadays, mass production companies usually employ the sigma level as the preferred metric to control their actions. Their purpose is the operational optimization, specifically when the product is an assembly, which overall quality level depends on the quality of each component. Six-sigma is usually taken as the minimum overall quality level to achieve excellence in every production company.

Six-sigma is a set of practices for process improvement that seeks to improve the quality of the processes' outputs. In terms of the Axiomatic Design (AD) theory, the conceptual frameworks of six-sigma implementation do not take into account the potential contribution of the design parameters (*DP*s) to the whole solution [El-Haik, 2005]. In some cases, this leads to neglect the need for design changes when the candidate solution for manufacturing in not satisfactory or is not the most rational one. As a result, the traditional approach of just readjusting the process variables (*PV*s) may not be enough to solve a specific problem, or may be a more costly solution.

The adjustment of the nominal values to new targets within the specified tolerances, or changing the tolerance ranges keeping the nominal values, or both, are considered soft changes. A change is considered hard when it involves the elimination or the addition of *DP*s or *PV*s in the relevant mapping [El-Haik, 2005].

All the potential improvements should be considered when a six-sigma solution must be accomplished. In the second member of Equation 1, the first term represents a design change, that is, changing the functional requirements (*FR*s), while the second relates to a process change. Therefore, an efficient axiomatic quality strategy should be characterized by the use of both terms [El-Haik, 2005].

$$\frac{\partial FR_i}{\partial PV_k} = \frac{\partial FR_i}{\partial DP_j}\frac{\partial DP_j}{\partial PV_k} \qquad (1)$$

In the industry, particularly in manufacturing plants, it is usual to readjust the *PV*s to find better values when the capability indices are below the targets. This is accomplished using statistical data acquired by the quality control, namely, the capability index, C_{pk}, which depends on the design tolerances and on the characteristics of the manufacturing process.

The lack of communication between those responsible for the design and for the production, in advanced stages of manufacturing, leads to missing the opportunity of matching the design to the actual process capabilities. In the case of mechanical tolerances, there are other reasons that usually lead to a process change, instead of a design change, basically when: i) the quality of the manufacturing operation of each component of an assembly set is independent of what happens with others; ii) the tolerances of the components, which once assembled confers the set´s functionality, are not independent, for they must satisfy an analytical model (worst case or statistical). This implies that any adjustment of the first term of Equation 1 can only be made if is possible to act coherently on other components according to the required functionality of the system as whole. Therefore, if the design team have data of quality control from all manufacturing operations, then it is able to change the tolerance allocations for each component, in strict accordance with the required functionality of the mechanical system. Minimizing the information content makes this change, given that the tolerance allocation of each component yields to a better capability index for its manufacture. Thus, one can attain a better sigma level to the assembly, which means a better resilience for the overall process through adaptive tolerance allocation. This approach seems to be appropriate to effectively upgrade the product specifications in accordance with the evolution of the actual capabilities of the production, or to the cases where the manufacturing systems are subjected to substantial variations.

2 TOLERANCE ALLOCATION AND CAPABILITY INDICES

A right allocation of mechanical tolerances must ponder the following issues: a) the variation of each dimension of a mechanical assembly´s component must guarantee the functionality of the set; b) the (low) manufacturing cost of each component; c) the ease of the assembling operations; d) the ease of the metrological control operations.

The allocation of tolerances must be made using synthesis methods according to an analytical model [Chase & Greenwood, 1988; Wu *et al*., 1988; Zhang, 1997]. As for the analytical model, there are basically two: the worst-case model and the statistical model. The first one takes into account the whole range of tolerance for each dimension, so that its application is appropriate to the cases of small production runs. As for the second, it is more appropriate for mass production. In the latter, the stacking method is based on the fact that the manufacturing processes are characterized by variability, and on the low probability of assembling two sets of components with the same extreme deviations. According to Spotts [1973], in a model with a normal distribution, the overall tolerance, t, of a stacked assembly with n components, relates to the tolerances of the components, t_i, i=1.. n, through

$$t = \sqrt{\sum_{i=1}^{n} t_i^2}\,. \qquad (2)$$

The process capability is a measurable property that can be expressed as a ratio between the tolerance range and the process variability, and it is often represented by the process capability index, C_{pk} [Montgomery & Runger, 2014]. C_{pk} is used to deal with biased distributions. Letting the lower and the upper specification limits be LSL and USL, respectively, then the capability index of process i is given by

$$\left(C_{pk}\right)_i = \min\left\{\frac{USL_i-\mu_i}{3\sigma_i}, \frac{\mu_i-LSL_i}{3\sigma_i}\right\}, \qquad (3)$$

where μ_i and σ_i are the process mean value and the standard deviation.

For the same process i, the probability of not occurring X is the sum of the lateral nonconforming probabilities given by

$$\left(p_f\right)_i = P\left(X < LSL_i\right) + P\left(X > USL_i\right). \qquad (4)$$

The nonconforming probability of the process can also be computed through

$$\left(p_f\right)_i = \int_{-\infty}^{LSL_i} f_i(x)dx + \int_{USL_i}^{+\infty} f_i(x)dx\,, \qquad (5)$$

where x is a normal random variable which probability density is given by

$$f_i(x) = \frac{1}{\sigma_i\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_i}\right)^2}. \qquad (6)$$

Equation 5 can only be solved numerically but a standard normal distribution table (Z table) allows computing the value of the nonconforming probability through the equation

$$\left(p_f\right)_i = P\left(Z < \frac{LSL_i-\mu_i}{\sigma_i}\right) + P\left(Z > \frac{USL_i-\mu_i}{\sigma_i}\right) \qquad (7)$$

The total probability of functional nonconformity is the probability of an assembly not satisfying its specification, which can be computed through

$$\left(p_f\right)_{Total} = 1 - \prod_{i=1}^{n}\left(1-\left(p_f\right)_i\right). \qquad \neg\ (8)$$

The number of defects per million opportunities (*DPMO*) is obtained from the probability of occurrence of a nonconforming assembly, and is given by

$$DPMO = \left(p_f\right)_{Total} \times 10^6 \qquad (9)$$

The output of industrial processes can vary over time and their sigma level can be computed through

$$SigmaLevel = \Phi^{-1}\left(1-\frac{DPMO}{10^6}\right)+1.5, \quad (10)$$

where Φ^{-1} is the normal inverse distribution function [Evans & Lindsay, 2008]. The 1.5 offset in the equation allows for the aforesaid variation.

Due to the independence of the manufacturing process of each component, the determination of the sigma level of the whole process allows monitoring the behaviour of the long-term process performance. Usually, the goal of the industrial processes is based on six-sigma for the long-term performance analysis, which means a *DPMO* lower than 3.4.

3 THE PROPOSED METHOD

In order to increment the global sigma level of an assembly operation, it is necessary to ensure an appropriate capability index to each one of the manufacturing operations (*i.e.,* of each one of the assembly´s components). The flowchart of the proposed method is outlined in Figure 1.

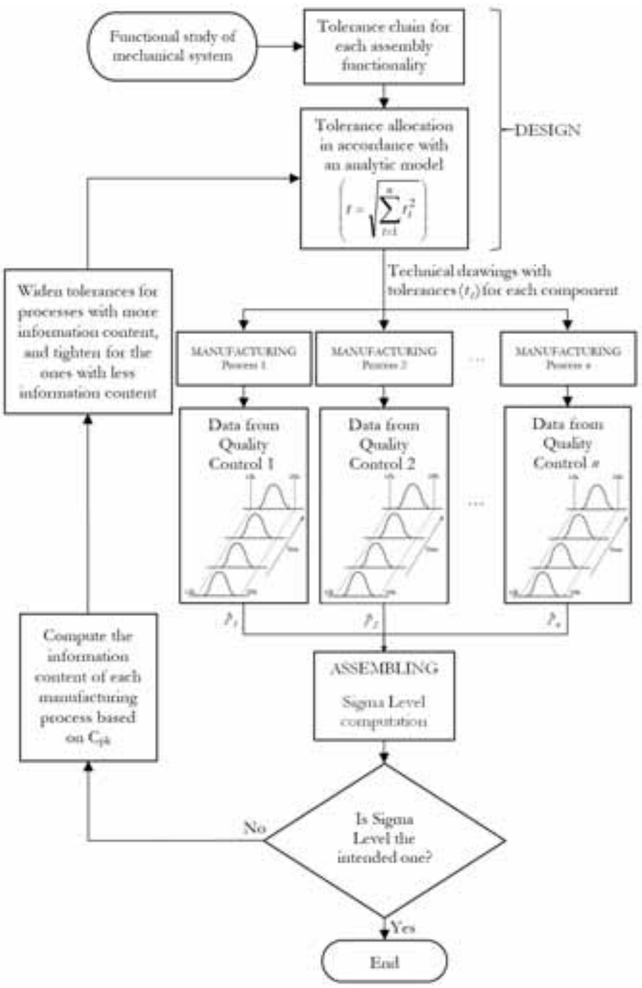

Figure 1. The flowchart of the **proposed adaptive** tolerance allocation to adjust **the** sigma level

The appropriate value for C_{pk} is usually attained by adjusting the manufacturing process by taking into account the data acquired through quality control.

If these data were sent to the design department, then it would be possible to attain a suitable value for the sigma level for the assembling operation through selective adaptation of the tolerance allocation to the different components, without any intervention on the manufacturing processes.

The method fosters the communication between design and quality throughout the production phase. As in many other sectors, real time communication is essential within any organization, because it allows a better perception of the difficulties and the capabilities of each department, as well as timely and more effective corrective actions.

The advantage of this method is that it allows achieving a higher sigma level in the overall process without any change in the processes. This increase is achieved by adapting the design specifications to the actual capabilities of the manufacturing processes, therefore not following the usual approaches of reducing the process variation to increase the overall quality of the manufacturing process.

Business groups consisting of several production units that are responsible for large productions should pay great attention to the data exchange between the design and the quality control departments of the involved production units.

The proposed method is appropriate for the cases where the production of distinct components is committed to different factories. However, it cannot be regarded as a means to avoid the optimization of manufacturing processes of low capability.

4 AN APPLICATION OF THE METHOD

An organization is responsible for the design and production of a specific product. This one is a mechanical assembly with three components that must be mass-produced, which nominal dimensions are depicted in Figure 2. The manufacturing of the components is committed to three independent plants, each of them being responsible for the production of a different component of the assembly.

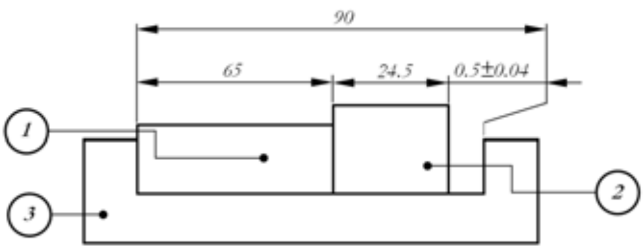

Figure 2. The mechanical assembly under study **(dimensions in mm)**

The top-level functional requirement of the mechanical assembly is the clearance of 0.5 mm ± 0.04 mm that is shown in the figure. By design, the existing three design parameters (*i.e.* the tolerances to the nominal dimensions of the three components) cannot be unified. Therefore, we have a redundant design with one *FR* and three *DP*s.

The manufacturing plants are not known at the phase of product development, so that the industrialization phase of the design is based on general data.

4.1 THE 1ST TOLERANCE ALLOCATION

It was assumed that the allocation of tolerances should be initially based on industry standards, as well as on the uniformity of quality condition. It was also assumed that the tolerance bands are symmetrical for all the components of the dimension chain.

Therefore, tolerance values of the ISO 286 IT8 grade were allocated to all the components, in order to ensure that the level of precision would be the same for all the dimensions of the tolerance chain.

The relevant nominal dimensions of the three components and of their initial tolerances are presented in Table 1.

Table 1. The first tolerance allocation

Component number	Nominal Dimension [mm]	Tolerance (IT8) [mm]
1	65	0.046
2	24.5	0.033
3	90	0.054

The tolerance of the assembly is 0.078 mm, as computed through Equation 2. Thus, the computed tolerance band is narrower than the specified value of 0.08 mm (*i.e.* ± 0.04 mm).

If the three manufacturing plants are independent, then it is easy to presume that the three processes have different capability indices.

According to the aforementioned records of quality control, the plant to which was committed the manufacture of component 1 has the lowest capability index and the fabricator of component 3 exhibits the highest C_{pk}.

The simulation of the manufacturing processes was carried out through Equation 8 using the quality control data and assuming the normal distribution condition. The results of the simulation are shown in Table 2.

Table 2. Values of the capability study relative to the first tolerance allocation

Process (dimension)	C_{pk}	Percentage outside specs	DPMO	Sigma Level
1 (65 mm)	1.38	3.44e-05	34.4	
2 (24.5 mm)	1.68	2.17e-07	0.22	
3 (90 mm)	1.91	9.05e-09	0.0091	
Overall		3.46e-05	34.6	5.48

Table 2 shows that the sigma level of the whole process is high, although lower than 6; nevertheless, the capability indices of the processes are irregular. The difficulties experienced by process 1 to produce within specifications, led to a high percentage of nonconforming parts. Thus, any process improvement of processes 2 and 3 to increase the overall sigma level will be limited by the comparative lower capability of process 1.

If the manufacturing plants try to do their best, than any capability improvement implies increased costs, specifically with the acquisition of equipment with a higher performance. However, this approach would not take into consideration the manufacture as a whole, each plant acting individually.

In an overall outlook, the approach should ponder the possibility of adjusting the design specifications of all the manufacturing operations.

The proposed model considers that the quality control data collected at each factory can be used by the design team to reallocate tolerances, taking into account the actual capabilities of all the involved manufacturing processes. This implies opening additional data flows from all the manufacturing plants to the design department.

In this condition, the design team can adjust the tolerance values, without any change in the manufacturing processes and preserving the relationship between them through the analytical model of Equation 2. The proposed criterion is based on the following assumptions: a) the best tolerance allocation is one that leads to the lowest information content of the system as whole; b) the manufacturing processes are independent; c) the minimization of the information content is achieved through the improvement of the process with the worst capability index, which is made possible through the levelling of the C_{pk} values of the different manufacturing processes.

4.2 THE RESULTS

The application of the proposed method is made iteratively, in accordance with the analytical model of Equation 2. The goal is upgrading the worst C_{pk} while levelling all the others concurrently. Indeed, this computing process is coupled, but the result can be applied independently.

Table 3 contains the result of the computing process, which satisfies the tolerance of the clearance that was imposed by design, since the new computed t is 0.08 mm.

Table 3. Tolerances resulting from adaptive allocation

Component number	Nominal Dimension [mm]	Tolerance [mm]
1	65	0.055
2	24.5	0.033
3	90	0.048

For these new specifications, a new simulation of the production of components 1, 2 and 3 was performed. This simulation was carried out with the same process variance, because it was assumed that process did not change. Table 4 shows the values that were obtained.

Table 4 **-** Values **of** the capability study **with adaptive** tolerance allocation

Process (dimension)	C_{pk}	Percentage outside specs	DPMO	Sigma Level
1 (65 mm)	1.67	5.80e-07	0.58	
2 (24.5 mm)	1.66	3.44e-07	0.34	
3 (90 mm)	1.72	2.60e-07	0.26	
Overall		1.18e-06	1.18	6.22

Now one can see that fine-tuning the design tolerances of the components 1 and 3 allowed increasing the C_{pk} of process 1. Actually, levelling the capability indices led to a better sigma level in the overall productive process, 6.22 instead of 5.48, which means an increase of 13.5%.

The raise of the sigma level of the entire production process represents an increase in the overall quality of the product, and it is worth stressing that this was achieved without rising costs, because no changes were made in the manufacturing processes.

4.3 VALIDATION THROUGH INFORMATION CONTENT

The independence of the manufacturing processes allows writing down the design equation 11 to represent the accomplishment of the design tolerances specified to each component.

$$\begin{Bmatrix} Tol.\,1 \\ Tol.\,2 \\ Tol.\,3 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \begin{Bmatrix} Manufacturing\ Process\ 1 \\ Manufacturing\ Process\ 2 \\ Manufacturing\ Process\ 3 \end{Bmatrix} \quad (11)$$

As one can see, this is an uncoupled design, for which the total information content, I_t, can be computed through the equation

$$I_t = \sum_{1}^{3} -\log_2 p_i \quad (12)$$

where p_i is the probability of each design tolerance being satisfied by the manufacturing process, which is given by

$$p_i = 1 - \left(p_f\right)_i \quad (13)$$

where p_f is the nonconforming probability that can be computed through Equation 7.

The information axiom states that the best design is the one with the smallest information content [Suh, 1990]. Therefore, let us compare the levelled solution that we have found with the one that was initially considered.

Using Equation 12, we can find the information content for the initial situation:

$$\left(I_t\right)_I = 4.99x10^{-5}\ bit \quad (14)$$

After the tolerance reallocation is made, and using again the same equation, the information content becomes:

$$\left(I_t\right)_F = 1.71x10^{-6}\ bit \quad (15)$$

Because

$$\left(I_t\right)_F < \left(I_t\right)_I \quad (16)$$

one can conclude that the tolerance reallocation yielded to a better design solution.

5 CONCLUSION

This paper presents a method that allows adaptive tolerance allocation using data previously acquired by quality control departments, and shows how this data can be used to adapt design specifications to the industrial reality.

The knowledge about the capability index of manufacturing operations allows computing the corresponding information content, and high sigma levels in the manufacturing of assemblies correspond to low information content.

The method makes it possible to increase the efficiency of manufacturing systems, even if their overall capability was already attained by the corresponding production systems.

As an overall conclusion, the presented case study describes an efficient axiomatic quality strategy through design change, as represented by the first term of Equation 1.

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