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DESIGN PARAMETER SELECTION FOR RECTANGULAR DESIGN MATRICES

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ABSTRACT

Design matrices that are derived from physical laws are, in general, rectangular matrices with a larger number of design parameters than functional requirements. This paper explores some algebraic properties of such matrices and uses them in order to find a diagonal square matrix, which is the ideal design required by the Independence and Information Axioms. Based on these properties, a measure of the distance to the ideal design is proposed. Uncoupled, decoupled and coupled design matrices are explored. Finally, a rule for selecting the best design parameters for achieving a square design matrix is proposed.

Keywords: design matrix, adjustment directions, ideal design, diagonalization theorem.

1 INTRODUCTION

Axiomatic Design [Suh, 1990; 2001] provides a solid structure for mathematically characterizing the design matrix associated with the best design. In addition to Axiomatic Design, the design matrices are subject to the laws of algebra and must be derived from physical laws. Hence, at a first glance, Axiomatic Design, Algebra and Physics are the tools that the engineer has for achieving the best design.

On one hand, physics is a rigid mathematical framework with a fixed set of physical laws. Normally, the number of equations derived from the laws of physics is much, much, lower than the number of variables that must be used for describing a determined solution for a design problem. Hence, the design equations that are used by the designers have a lot of parameters to be explored, and a question about what are the best parameters to be selected in first place appears. For reducing the impact of this resource-consuming task, engineers require a criterion for doing that selection as quick as possible. On the other hand, the algebra is a rigid mathematical framework that allows the designer to extract information about the structure of the design matrix. In this case, the difficult question to be solved is how to extract the required information. This paper proposes a criterion for this purpose. The criterion presented for selecting design parameters will be founded on Axiomatic Design Theory, and on Algebra, taking into account that the matrices derived from physical considerations are rectangular matrices.

Although Axiomatic Design establishes a general procedure for obtaining an ideal design, the mathematical relationships that are embedded in the ideal design cannot always be implemented as a physical solution or device. In general, the result is a design that must satisfy r functional requirements and that have q with $q > r$ design parameters. However, Axiomatic Design establishes that only r design parameters must be selected as true design parameters and the other $q - r$ must be discarded or frozen. Without additional information, there are a large number of possibilities for this selection, but it is expected that only one set of r design parameters will be the best. Note that the number of possibilities is given by the combinatorial number $N = q!/(q-r)!/r!$ which increases when q increases. For this reason, if the best set of design parameters is not selected at the very moment of writing the design matrix the cost derived from a later iteration could be huge. As said, the aim of this paper is to propose a criterion for making this task easier.

The paper is structured as follows. First, the design equations are presented in the framework of a design environment. Second, a mathematical characterization of the best design is given by using the obtained design equations and Axiomatic Design. Third, the algebraic properties of a rectangular design matrix are presented. Fourth, based on these properties a criterion for selecting the best set of parameters is proposed. Then, the criterion is used for comparing uncoupled, decoupled and coupled designs. Finally, an example showing how the criterion discards a design parameter is presented.

2 TRANSFER FUNCTION AND DESIGN MATRIX

In engineering design problems, it is common to find a great variety of needs, specifications or requirements that can be described as variables whose value must belong to an allowed range. For example, we can think on the position of a given part, the concentration of an additive, the temperature of an infrared sensor, etc. In addition, for a great variety of specifications, this allowed range can be identified with an interval. Thus, a large number of engineering needs or specifications can be defined by using only two values: the minimum allowed value and the maximum allowed value. Suppose that for a given design problem there are r needs that can be specified by a set of allowed intervals that define the hyper-volume of acceptation D as:

$$D = [\underline{l}_1, \overline{l}_1] \times [\underline{l}_2, \overline{l}_2] \times ... \times [\underline{l}_r, \overline{l}_r] \subset \mathbb{R}^r \quad (1)$$

then we can establish the success condition for the design process as $l = (l_1, l_2, ..., l_r) \in D$ and the fail condition as $l \notin D$. In addition, the variables $l_1, l_2, ..., l_r$ associated with the needs or specifications are considered to be a set of functional requirements such as are defined by Suh [1990]: the functional requirements are the smallest set of independent requirements that completely characterize the design objectives for a specific need.

Because a design solution must be implemented in the physical domain [Suh, 1990; 2001], the design equations must relate the functional requirements to a set of physical parameters. This set of physical parameters has to include all the physical constants (such as material properties), descriptive parameters (such as geometrical dimensions), and operational parameters (such as rotational speeds, temperatures, and voltages). The designer has no reason for not using all these variables in the process of seeking an adequate design point. From this point of view, all these variables can be considered as design parameters. It is interesting to note that, defined in this way, the number of design parameters is normally larger than the number of functional requirements to be satisfied. Let q (with $q > r$) be the number of design parameters. In addition, as it has been argued for the functional requirements, suppose that the design parameters can be defined by the interval where they can be established. Suppose that for a given design solution there are q design parameters that can be specified by a set of allowed intervals that define the hyper-volume of variation C as:

$$C = [\underline{m}_1, \overline{m}_1] \times [\underline{m}_2, \overline{m}_2] \times ... \times [\underline{m}_q, \overline{m}_q] \subset \mathbb{R}^q \quad (2)$$

then we can establish the design range as $m = (m_1, m_2, ..., m_r) \in C$.

It is useful to define the center of the hyper-volumes D and C as the following vectors:

$$l_o = \left(\frac{\overline{l}_1 + \underline{l}_1}{2}, \frac{\overline{l}_2 + \underline{l}_2}{2}, ..., \frac{\overline{l}_r + \underline{l}_r}{2} \right) \in \mathbb{R}^r \quad (3)$$

$$m_o = \left(\frac{\overline{m}_1 + \underline{m}_1}{2}, \frac{\overline{m}_2 + \underline{m}_2}{2}, ..., \frac{\overline{m}_q + \underline{m}_q}{2} \right) \in \mathbb{R}^q \quad (4)$$

The engineer implements the laws of physics that relate the vector of functional requirements to the vector of design parameters in the following function:

$$f : C \to \mathbb{R}^r \quad (5)$$

This is the map that transfers the decisions adopted by the designer in the space C (parameters of design) to the space D (functional requirements). For this reason it can be considered a transfer function. Function *f* will be considered a differentiable function, and hence, by applying the Taylor theorem, we can write:

$$l = l(m_o) + J(m_o)(m - m_o) + ... \quad (6)$$

The structure of Eqs. (1), (2) and (6) advises the following changes of variable [Benavides, 2012]:

$$y_j = \frac{l_j - \frac{\overline{l}_j + \underline{l}_j}{2}}{\frac{\overline{l}_j - \underline{l}_j}{2}}; \; j = 1, 2, ..., r \quad (7)$$

$$x_j = \frac{m_j - \frac{\overline{m}_j + \underline{m}_j}{2}}{\frac{\overline{m}_j - \underline{m}_j}{2}}; \; j = 1, 2, ..., q \quad (8)$$

As a result of these changes of variable, the hyper-volumes D and C transform respectively to:

$$E_r = [-1,1] \times [-1,1] \times ... \times [-1,1] \subset \mathbb{R}^r \quad (9)$$

$$E_q = [-1,1] \times [-1,1] \times ... \times [-1,1] \subset \mathbb{R}^q \quad (10)$$

The substitution of Eqs. (7) and (8) into Eq. (6) leads to:

$$y(x) = y(0) + Ax + ... \quad (11)$$

In this expression, the matrix A is a rectangular matrix of size $r \times q$:

$$A = \begin{pmatrix} A_{11} & \cdots & A_{1q} \\ \vdots & \ddots & \vdots \\ A_{r1} & \cdots & A_{rq} \end{pmatrix} \quad (12)$$

$$A_{ij} = \frac{\overline{m}_j - \underline{m}_j}{\overline{l}_i - \underline{l}_i} J_{ij} = \frac{\overline{m}_j - \underline{m}_j}{\overline{l}_i - \underline{l}_i} \frac{\partial f_i(m_o)}{\partial m_j} \quad (13)$$

This expression of an element of the design matrix was deduced by Benavides [2012] and gives a rational way for obtaining dimensionless design matrices.

3 IDEAL DESIGN

The conditions $x \in E_q$ and $y \in E_r$ assure that the maximum deviation of the functional requirement i can be written as:

$$y_i\big|_{\max} = y_i(0) + \sum_{j=1}^{q} \left| A_{ij} \right| \le 1 \quad (14)$$

$$y_i\big|_{\min} = y_i(0) - \sum_{j=1}^{q} \left| A_{ij} \right| \ge -1 \quad (15)$$

The substraction and the addition of Eqs. (14) and (15) lead respectively to:

$$\frac{y_i\big|_{\max} - y_i\big|_{\min}}{2} = \sum_{j=1}^{q} \left| A_{ij} \right| \le 1 \quad (16)$$

$$y_i(0) = \frac{y_i\big|_{\max} + y_i\big|_{\min}}{2} \quad (17)$$

Inequality (16) shows that not all the design matrices produce an acceptable design. Indeed, the restriction that the hyper-volume of acceptance imposes over the elements of the design matrix is even more exigent. This new restriction comes from the inequalities (14) and (15) and can be condensed in the following inequality:

$$\sum_{j=1}^{q}\left|A_{ij}\right| \leq \min\ 1-y_i(0), 1+y_i(0) \tag{18}$$

The range where this inequality is satisfied reaches a maximum when the following conditions are achieved:

$$y_i(0)=0 \tag{19}$$

$$A_{ij}\to 0 \quad j=1,2,...,q \tag{20}$$

Note that this is a mathematical formulation of the Information Axiom that states that the best design must have a minimum value of the information content, i.e., a maximum value of the probability of success [Suh, 1990]. Note also that condition (19) converts the inequality (18) into the inequality (16). On the other hand, the tendency given in (20) leads to the following tendencies [see Eq. (13)]:

$$\frac{\partial f_i(m_o)}{\partial m_j}\to 0 \tag{21}$$

$$\overline{m}_j-\underline{m}_j\to 0 \tag{22}$$

$$\overline{l}_i-\underline{l}_i\to \infty \tag{23}$$

Note that the tendency given by (23) is the mathematical formulation of the Corollary 6 given by Suh [1990]. However, the tendency given by (22) contradicts the tendency given by (23) because due to the hierarchy of the design process the design parameters of one level become the functional requirements of the following level [Suh, 1990]. Hence, when m_j is considered a design parameter of the first level, $\overline{m}_j-\underline{m}_j$ should be as low as possible [see Eq. (22)]; and when m_j is considered a functional requirement of the second level, $\overline{m}_j-\underline{m}_j$ should be as large as possible [see Eq. (23)]. For this reason, the tendency given by (22) will make an acceptable solution in the following level of the hierarchy of design impossible. In addition, the designer in any level of the hierarchy wants to get a formulation of the functional requirements that fulfill the condition (23). Therefore, it is an objective of the designer to increase as much as possible the intervals of acceptance for both the functional requirements and the design parameters. This allows us to write that the following tendency must be observed during the design process:

$$\overline{m}_j-\underline{m}_j\to \infty \tag{24}$$

Since the first functional requirement is fixed by the customer, the condition (23) cannot be completely satisfied, but the designer has to be creative enough for achieving the condition (24). If we assume that we have created the best design, which in this case is the one that increases as much as possible the length of the acceptance intervals for the next step, we can conclude that the following tendency is a necessary characteristic of the best design:

$$\frac{\overline{m}_j-\underline{m}_j}{\overline{l}_i-\underline{l}_i}\to \infty \tag{25}$$

On the other hand, the tendency given by (21) cannot represent a real physical device. In effect, if all the derivatives in the design matrix are zero, there would not be any relationship between the functional requirements and the design parameters. For this reason at least one derivative cannot be zero:

$$\frac{\partial f_i(m_o)}{\partial m_j}=Kte\neq 0 \tag{26}$$

The conditions (25) and (26) lead to

$$A_{ij}\to\infty \tag{27}$$

for some *j*. This contradicts the condition (20), and hence the inequality (18) cannot be fulfilled. Thus, the designer must seek that the condition (20) holds for the major number of elements in one row of the design matrix. On the other hand, the designer must try to obtain the condition (27) for at least one element of the row, but this fact is forbidden by inequality (18). In addition, Eq. (19) must be imposed by the designer in Eq. (18), and hence the maximum allowable value on the right hand side of that inequality is 1. Putting all this information together (i.e., $\sum_{j=1}^{q}\left|A_{ij}\right|\leq 1,\ A_{ij}\to 0$ for almost all the elements, and $A_{ij}\to\infty$ for at least one element) and taken into account the Independence Axiom (and, if necessary, permuting rows and permuting columns) we obtain the following formulation for the design matrix of the best design (i.e., the design matrix of the ideal design):

$$y(0)=0 \tag{28}$$

$$A_{ij}=\delta_{ij}=\begin{cases}0 & i\neq j\\ 1 & i=j\end{cases} \tag{29}$$

4 QUANTITATIVE STUDY OF THE DESIGN MATRIX

In general, the design matrix obtained by the designers during the creative process is not the ideal one. So, it is convenient to find a general procedure to convert the non-ideal design into an ideal design. A general description of the algebraic properties of this matrix can be found in Benavides [2012]. This section provides the minimum required algebra for doing this task.

Let us establish a set of r functional requirements as a vector in $\mathbb{R}^r$ using its coordinates in the canonical basis. Let us establish a solution characterized by a set of q design parameters that can be varied independently. As seen in the previous section, the design parameters can be identified using the coordinates of the vector $x\in\mathbb{R}^q$. As discussed in the previous sections, $q\geq r$ holds. In addition, the rank of the design matrix $A\in \mathrm{M}_{r\times q}$ must be r [see Eq. (29)] and hence, its row vectors $a_1{}^t,...,a_r{}^t$ must be linearly independent. For the same reason, the vector set $Aa_1,...,Aa_r$ is a basis of $\mathbb{R}^r$. This set of vectors can be written in matrix notation as $AA^t\in\mathrm{M}_{r\times r}$, which is invertible. Thus, $I=AA^t(AA^t)^{-1}\in\mathrm{M}_{r\times r}$ holds. Therefore, the column vectors in the matrix $A^t(AA^t)^{-1}\in\mathrm{M}_{q\times r}$ are a combination of design parameters that enable us to vary the functional requirements independently. The kernel of the linear map A is the subspace generated with the column vectors of the matrix $B \in \mathrm{M}_{q\times(q-r)}$, which has to verify $AB = 0$. Let us define an arbitrary matrix $\beta \in \mathrm{M}_{(q-r)\times r}$, and construct the matrix

$$X = A^t(AA^t)^{-1} + B\beta \in \mathrm{M}_{q\times r} \tag{30}$$

This matrix contains in its columns all the combinations of the linear parameters that keep the functional requirements independent. For this reason they are called adjustment directions [Benavides, 2012]. The arbitrary matrix β can be chosen for eliminating the influence of a design parameter (or a linear combination of design parameters). Because matrix β has $q-r$ column vectors, designer can remove the influence of $q-r$ design parameters (or specified directions). Let designer define a matrix $X' \in \mathrm{M}_{q\times(q-r)}$ whose column vectors are the directions in the space of the design parameters that the designer wants to remove. The removing of these directions requires to solve the linear system $X'^t X = 0$, which leads to

$$\beta = -(X'^t B)^{-1} X'^t A^t (AA^t)^{-1} \tag{31}$$

Note that, as it is remarked in Benavides [2012], the matrix $X'^t B \in \mathrm{M}_{(q-r)\times(q-r)}$ could not be invertible. The substitution of β leads to

$$X = \left[I - B(X'^t B)^{-1} X'^t \right] A^t (AA^t)^{-1} \tag{32}$$

This result lets us assume that there is a vector $e \in \mathbb{R}^r$ that represents a new set of design parameters. In effect, if this is assumed, then the transfer function can be written as:

$$\begin{aligned} y &= A\left[I - B(X'^t B)^{-1} X'^t \right] A^t (AA^t)^{-1} e + \dots \\ &= e + \dots \end{aligned} \tag{33}$$

Note that in this equation the designer has reduced the number of design parameters from q to r and has achieved an ideal design. This result was used by Benavides [2012] to prove the diagonalization theorem that states that the ideal design always exists. For other interesting algebraic results, such as the spectral decomposition of the design matrix, please refer to Benavides [2012]. This expression shows also that, if the designer acts on the design parameters by following the strategy of varying several of them at the same time, as indicated by the column vectors in matrix X, it is always possible to maintain the independence between the requirements. By taking the column vectors of X as a basis, the linear map takes the form of the ideal design given by Eq. (29).

Eq. (33) shows that the existence of the ideal design comes from the following property of the design matrix:

$$AX = I \tag{34}$$

In addition, Eq. (34) shows that all the relevant information for obtaining an ideal design from a given (rectangular or not) design matrix is collected in the matrix X defined by Eq. (32) which defines the adjustment directions.

5 MEASSURE OF THE GOODNESS OF THE DESIGN MATRIX

The vector X collects the relevant information from the design matrix required for transforming a general design into an ideal one. Eq. (34) states that the column vectors of the matrix X collect the values of the design parameters that move the functional requirements to the point 1.0, which is the maximum value accepted by the customer. But because the ideal design matrix is the identity matrix, it states also that each column vector of X moves one and only one functional requirement from the value 0.0 to the value 1.0.

From Eq. (30) we can obtain the following matrices:

$$X^t = (AA^t)^{-1}A + \beta^t B^t \in \mathrm{M}_{r\times q} \tag{35}$$

$$X^t X = (AA^t)^{-1} + \beta^t B^t B\beta \in \mathrm{M}_{r\times r} \tag{36}$$

Eq. (36) shows that the condition for the ideal design is $X^t X = I$ (note that when $A = I$ holds, $B = 0$ also holds). However, in general, this condition cannot be reached and hence, it is convenient to define the matrix:

$$E = (AA^t)^{-1} + \beta^t B^t B\beta - I \tag{37}$$

Note that E is a symmetrical matrix that should be identical to the zero matrix for the ideal design. If any element in the matrix E is not zero, then the norm of the respective column vector will not be zero. This fact allows us to construct a real positive number that measures how much the matrix E deviates from the zero matrix. This number is:

$$\varepsilon^2 = \mathrm{trace}(E^t E) = \mathrm{trace}(E^2) \tag{38}$$

where E^2 is given by the following expression

$$\begin{aligned} E^2 &= (AA^t AA^t)^{-1} - I \\ &\quad + (AA^t)^{-1}\beta^t B^t B\beta + \beta^t B^t B\beta (AA^t)^{-1} \\ &\quad + (\beta^t B^t B\beta)^2 \end{aligned} \tag{39}$$

Therefore, the ideal design ($A = I$) meets the condition ε =0. The calculation of this deviation is quite hard because the designer should explore all the possible values of the matrix β. Eq. (30) gives the adjustment directions for a given β. When β is calculated with Eq. (31) the calculation of ε is reduced to the adjustment directions that result from removing existing design parameters. In any case, the adjustment directions that produce the minimum value of ε constitute the new set of design parameters that achieves the ideal design. However, as it is well discussed by Suh [1990], these new parameters are not always feasible in the real world because there could have some limitations, for example creativity, that avoid such implementation. When design parameters cannot be combined and the adjustment directions cannot be followed, a more practical criterion exits. This is the one where the designer checks if the column vectors of X have a maximum component with an absolute value close to 1.0 and the other components remains between 0 and +1. In this case, the deviation function given by Eq. (38) could be substituted by:

$$D=\sqrt{\sum_{j=1}^{r}\left(\max_{i}\left|x_{ij}\right|-1\right)^{2}} \qquad (40)$$

This merit function was proposed, together with other additional measures of the degree of independence, by Benavides [2012] for detecting which one is the best set of parameters to be selected in a design matrix (an ideal design meets the condition $D=0$). The condition $D=0$ indicates if at least one design parameter has reached its maximum range of variation. For situations where the design parameters cannot be physically combined, D from Eq. (40) is more suitable than ε^2 from Eq. (38).

6 APLICATION TO UNCOUPLED, DECOUPLED, AND COUPLED DESIGNS

Suh [1990] clearly defines uncoupled, decoupled and coupled designs. Uncoupled and decoupled designs are those that have, respectively, a diagonal design matrix, and a triangular design matrix. Finally coupled designs are those that do not belong to the previous categories. In this section we will collect some simple examples of these categories in order to calculate the matrices and merit functions defined previously. These examples are illustrative and for this reason are kept as simple as possible: all the calculations [see Eqs. (37) and (38)] will be done for full-rank (B =0) square design matrices and for three functional requirements.

Table 1. Comparison between designs.

	Uncoupled	Decoupled	Coupled
A	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\1&1&1\end{bmatrix}$	$\begin{bmatrix}1&0&-1\\0&1&0\\1&1&1\end{bmatrix}$
X	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\-1&-1&1\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\0&1&0\\-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\end{bmatrix}$
E	$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\begin{bmatrix}1&1&-1\\1&1&-1\\-1&-1&0\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2}&0&0\\0&\frac{1}{2}&-\frac{1}{2}\\0&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$
ε^2	0	8	$\frac{5}{4}$
D	0	0	$\frac{1}{\sqrt{2}}$

This example shows that a decoupled design can be worse, in terms of the deviation ε, than a coupled design. The reason is that a decoupled design can have the adjustment directions near parallel. But both, the decoupled and the coupled designs, are worse than the uncoupled design, such as the ideal design requires.

7 APLICATION TO THE SELECTION OF DESIGN PARAMETERS

The first proposed example is a coupled design with the following rectangular matrix:

$$A=\begin{bmatrix}1&0&-1&1\\0&1&0&1\\1&1&1&0\end{bmatrix}$$

The main results for different values of the matrix X' are collected in the Table 2.

Table 2. Selection of design parameters.

X'	X	E	ε^2	D
$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-\frac{1}{6}&\frac{1}{2}&\frac{1}{6}\\-\frac{1}{3}&0&\frac{1}{3}\\\frac{1}{6}&\frac{1}{2}&-\frac{1}{6}\end{bmatrix}$	$\begin{bmatrix}-\frac{7}{12}&-\frac{1}{4}&\frac{1}{12}\\-\frac{1}{4}&-\frac{1}{4}&-\frac{1}{4}\\\frac{1}{12}&-\frac{1}{4}&-\frac{7}{12}\end{bmatrix}$	$\frac{145}{144}$	$\frac{\sqrt{3}}{2}$
$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$	Design matrix becomes singular			
$\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\0&0&0\\-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\0&1&0\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2}&-\frac{1}{2}&0\\-\frac{1}{2}&\frac{1}{2}&0\\0&0&-\frac{1}{2}\end{bmatrix}$	$\frac{5}{4}$	$\frac{\sqrt{2}}{2}$
$\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\0&0&0\\\frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{4}&-\frac{1}{4}&-\frac{1}{4}\\-\frac{1}{4}&-\frac{1}{4}&-\frac{1}{4}\\-\frac{1}{4}&-\frac{1}{4}&-\frac{1}{4}\end{bmatrix}$	$\frac{9}{16}$	$\frac{\sqrt{3}}{2}$
$\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$	$\begin{bmatrix}\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\0&1&0\\-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\0&0&0\end{bmatrix}$	$\begin{bmatrix}-\frac{1}{2}&0&0\\0&\frac{1}{2}&-\frac{1}{2}\\0&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$	$\frac{5}{4}$	$\frac{\sqrt{2}}{2}$

The results in Table 2 show that, in the studied case, the initial design matrix does not allow obtaining an ideal design by removing design parameters. When the DPs can be combined to obtain new DPs, the table shows that the best selection for the design parameters is {DP1, DP2, DP4} (ε =9/16), which means that DP3 should be removed or frozen. The results also show that this option is better than not removing any design parameters. However, when the DPs cannot be combined, this is not the best option and the best selections will be {DP1, DP3, DP4} or {DP1, DP2, DP3} (D =1/2$^{1/2}$). It is also interesting that DP1 cannot be removed: it is an essential part of the design because it is a key element for maintaining the rank of the design matrix.

The second proposed example is the design of a faucet that must control the flow rate and the temperature of a liquid flow: {FR1, FR2}={*flow rate*, *temperature*} and {DP1, DP2, DP3, DP4, DP5}={*pressure1*, *pressure2*, *area1*, *area2*, *hot temperature*}. The design matrix for this problem is [Benavides, 2012]:

$$A = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$

This matrix is interesting because represents a real device with a coupled (select, for example, {DP1,DP2} as the design parameters) or decoupled (select, for example, {DP4, DP5}) design matrix that cannot be uncoupled by means of a straightforward procedure. Results for X^t, ε^2 and D are presented in Table 3 for different values of the matrix X'^t.

In this case, the best selection of the design parameters is {DP3, DP4} for both criteria, minimum ε and D. This means that: 1) because D is minimum, the option of controlling the areas is better than controlling the pressures or the temperature; and 2) because ε is minimum, the option of combining the areas is better for achieving an ideal design than combining the pressures and the temperature. Uncoupled physical solutions, obtained by doing this combination of areas, can be found in Suh [2001] and Benavides [2012].

8 CONCLUSION

It is possible to derive an indicator, based on the deviation of the design matrix from the ideal one, from the algebraic properties of the design matrix. This indicator allows the designer to select the best set of design parameters when the design matrix is not a square matrix. The indicator also establishes that reconfiguring the PDs could be more difficult for a decoupled design than for a coupled design.

9 REFERENCES

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Table 3. Selection of design parameters.

X'^t	X^t	ε^2	D
$\begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \end{bmatrix}$	$\begin{bmatrix} 0&0&0&2&1 \\ 0&0&0&0&1 \end{bmatrix}$	18	1
$\begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$	$\begin{bmatrix} 0&0&2&0&-1 \\ 0&0&0&0&1 \end{bmatrix}$	18	1
$\begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&0&1 \end{bmatrix}$	$\begin{bmatrix} 0&0&1&1&0 \\ 0&0&1&-1&0 \end{bmatrix}$	2	0
$\begin{bmatrix} 1&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \end{bmatrix}$	$\begin{bmatrix} 0&4&0&0&1 \\ 0&0&0&0&1 \end{bmatrix}$	258	3
$\begin{bmatrix} 1&0&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&1 \end{bmatrix}$	Design matrix becomes singular		
$\begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$	$\begin{bmatrix} 0&2&1&0&0 \\ 0&-2&1&0&0 \end{bmatrix}$	50	1.4
$\begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \end{bmatrix}$	$\begin{bmatrix} 4&0&0&0&-1 \\ 0&0&0&0&1 \end{bmatrix}$	258	3
$\begin{bmatrix} 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&0&1 \end{bmatrix}$	$\begin{bmatrix} 2&0&0&1&0 \\ 2&0&0&-1&0 \end{bmatrix}$	50	1.4
$\begin{bmatrix} 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$	Design matrix becomes singular		
$\begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$	$\begin{bmatrix} 2&2&0&0&0 \\ 2&-2&0&0&0 \end{bmatrix}$	98	1.4