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A procedure based on robust design to orient towards reduction of information content

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Abstract

To manage the design matrix is an apparently easy thing to do. Discovering incongruences and converging at least towards a decoupled structure could suggest to designers that they have reached a sufficiently good starting point for the product under development. This is not sufficient. To be able to evaluate the information content of that solution is on the contrary an almost difficult activity because many relations between Functional Requirements and Design Parameters may not be completely defined deterministically. Eliminating bias and reducing variance remain the objective to be pursued. The paper discusses a procedure based on the Taguchi method to orient designers when verifying the influence that each design parameter has on the functional requirements. After the association of an Objective Function with one functional requirement or with a macro-functional requirement, the relation between the Objective Function and a set of design parameters can be identified from the Design Matrix. This can allow the designer to discern the best range of each parameter, analysing the Mean Value of the Objective Function and Signal to Noise Ratio. In the case of a conjoint influence of many design parameters on the functional requirement, it is important to verify the mutual interaction among the design parameters and evaluate the kind and level of interaction.

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Keywords: *Robust Design; Information Axiom; Mean Values and SNR.*

1. Introduction

The measure of the robustness of an engineering device can be computed knowing the probability of satisfaction of its main characteristics. The information axiom [1] has been introduced, on the basis of the Shannon assumption [2], to evaluate the level of robustness of a design solution. The main difficulty during the product design phases is gathering this kind of data, in general at least a prototype being required on which to evaluate these performances and to measure the corresponding probabilities. Along all the phases of product development it is extremely useful to have data on which to base the right choice of the most promising design solution. Computer simulation by means of CAE systems gives designers many insights into the product behavior and can assist the right choices.

A further task would be to compare two or more alternative solutions, in order to decide which solution has the greater chance of solving the problem for which it has been developed. This latter problem is more complex, since not only technical data but also economic, marketing, and technological evaluations are involved. Multi-Criteria Decision Making techniques are generally employed for this kind of evaluation and many more people or stakeholders must share the responsibility for the final choice. Among the wide literature on this topic [3-5] can be indicated.

The main task of the design phase is to assure that each design alternative has reached the best configuration. Both axioms of Axiomatic Design can guide designers to improve a design solution. Both axioms must be managed with care. The first axiom is strictly related to the form of the design matrix, and the latter must be continuously updated and verified. However, it is not generally evident which kind of form it assumes, because many of the relations between functional requirements and design parameters might not be declared. The second axiom is really more difficult to apply

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when the design matrix has a triangular form and the terms outside the diagonal are not identified by deterministic relations.

The search for a robust solution requires verification that the performance of the design system must be contained inside the design range. This requires that the eventual bias between the mean values of domain range and system range must be removed, or at least reduced, and that the variance of the probability density function (pdf), which describes the performance of a design alternative, becomes smaller than the design range. The smaller the variance the more robust the system.

In order to guarantee these conditions designers must pursue a strategic design activity that integrates robust design into axiomatic design. Establishing a proper objective function associated with the main functional requirement or to a macro-functional requirement the design of an experiment, planned in terms of the Taguchi method, can be organized identifying the design parameters involved in the analysis. Having previously studied the design solution by axiomatic design this type of identification is done almost straightaway. The discussion of the results highlights the type of interaction between design parameters and the functional requirements associated with the objective function and suggests the nature of the design matrix, which could be unknown from the beginning.

In a certain sense the discussion of the results of a design of experiment gives many insights into the nature of the design matrix. The employment of the Taguchi method offers a more flexible investigative tool, in that the comparison between the influence of each design parameter on the Objective Function and the associated values of the Signal to Noise Ratio allows designers to understand the kind of relation among design parameters and functional requirements [6-9]. This kind of study details better the kind of design matrix structure because the presence of interaction among design parameters suggests the presence of off-diagonal terms.

The paper describes a procedure that introduces how to take into account the results obtained by the Taguchi method for the reduction of the information level of design solution and at the same time identify better the form of the design matrix. The main intent is to give the designer a tool by which to study the nature and the behavior of one design solution and to lower the information level that characterizes it.

2. The related literature

Many researchers have investigated the relation between axiomatic design and Robust Design [10-15] suggesting coherent strategies to support the identification of the best design solution. Bras and Mistree [10] introduced the compromise Decision Support Problem as a method to combine Axiomatic and Robust design by the Taguchi approach. They demonstrated how to determine the most suitable values and tolerances for a given set of parameters, and to identify the most suitable principal design parameters. This approach requires the definition of all the relations among Functional Requirements and Design Parameters. They also used reangularity and semangularity to establish the degree of independence of a design solution, even though it must be underlined that these two quantities were no longer employed by Suh after 2001. Gu et al [11] integrated the analysis of independence of Axiomatic design with Robust design and used the condition number of the sensitivity matrix, related to the design matrix, as a means to evaluate the degree of independence of a design solution. The design matrix, also in this case, must be fully determined in each component. Xiao and Cheng [12] developed an analytic approach to demonstrate the relation between Axiomatic Design and Robust Design. They used the new insight by Suh [1] and studied the uncoupled and decoupled design matrices. They demonstrated on the basis of some case studies, and with properly probability density functions, why an uncoupled design is more robust than a decoupled design and why the latter is better than a coupled one. More recently Lijuan et al. [13] used the concept of optimization framework to integrate axiomatic design, robust design and reliability-based design, even though they needed to use again reangularity and semangularity to configure the optimization framework. Kar [14] underlined the strict relation between axiomatic design and the Taguchi method. Frey et al. [15] proved that simple summation of information levels cannot be performed for decoupled designs and offered a method for computing the information content. They also suggested that decoupled designs can have a higher probability of success with respect to uncoupled designs and that a decision made only on the basis of the first axiom might not necessarily be a guide towards the solution with the lowest information content.

Suh in [1], and mainly in Chapters 2 and 3, explored more contexts and suggested many ways to guide designers towards elimination of bias and reduction of variance, in order to arrive at design solutions that can be considered robust for the purposes for which they were designed. The way followed in this paper is based on the employment of the Taguchi method and can be considered a reverse road map to be followed, without a previous deep knowledge of the relation among functional requirements and design parameters.

3. The information axiom through the insight of the Taguchi method.

The analysis carried out by means of the Taguchi method is performed comparing how each design parameter employed in the modeling has influenced the objective function, both in terms of the Mean Value (MV) and in terms of Signal to Noise Ratio (SNR).

The comparison of both results gives useful elements in order to correct the solution, modifying the range of variation of the design parameters, either trying to eliminate the bias or reducing the variance.

The evaluation of the performance in terms of Mean Value is obtained in strict relation to the law used to describe the objective function. This is associated with the loss function in one of the criteria: lower is better or higher is better. So the objective function must be maximized or minimized.

The search for a robust solution is guided by the SNR computed by the general form:

$$SNR = 10\, log_{10}\, \mu^2/\sigma^2 \quad (1)$$

where σ is the variance of the performance of the objective function when the design solution is simulated in a set of different operating conditions and μ is the mean value. The design is robust when the SNR is maximized.

The qualitative example that will be discussed reports the case when the Objective Function must be minimized.

In the case of bias reduction/elimination the performance of the design solution must be taken into account considering basically the range of variation of the design parameters and their influence on the Mean Value of the behavior. In Figure 2 both the Mean Value of the Objective Function and the associated SNR related to the design parameter DPi are represented. The Mean Value is shown with a blue line, with the label on the left and the SNR is shown with a green line, with label on the right.

The System Range must be moved towards the Design Range. Considering the data reported in Fig. 2a, it must be verified if adjusting the range of variation of the design parameter Dpi toward higher values, because in that direction the performances are good, really the behavior of the objective function becomes better, independently of the SNR. In Fig. 2b this situation is reached because in correspondence with the new value of the lower level of the DPi both O.F. and SNR are satisfied.

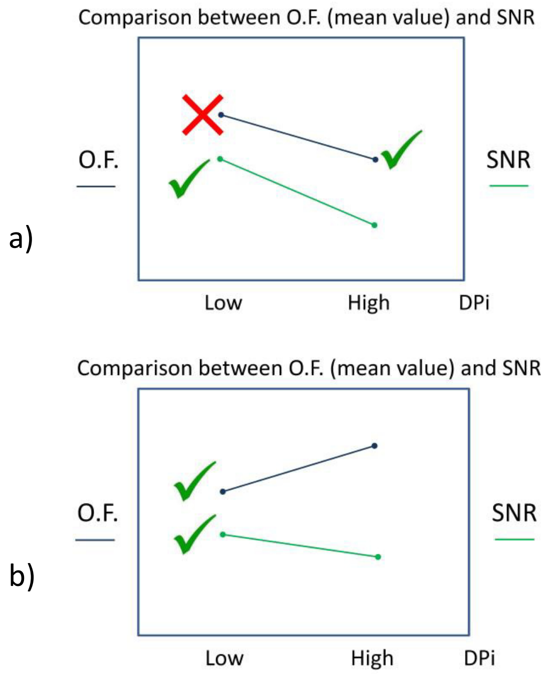

Fig. 2 Data related to bias reduction/elimination: a) before; b) after.

In the case of variance reduction the performance of the design solution must be taken into account considering basically the range of variation of the design parameters and their influence on the SNR of the behavior.

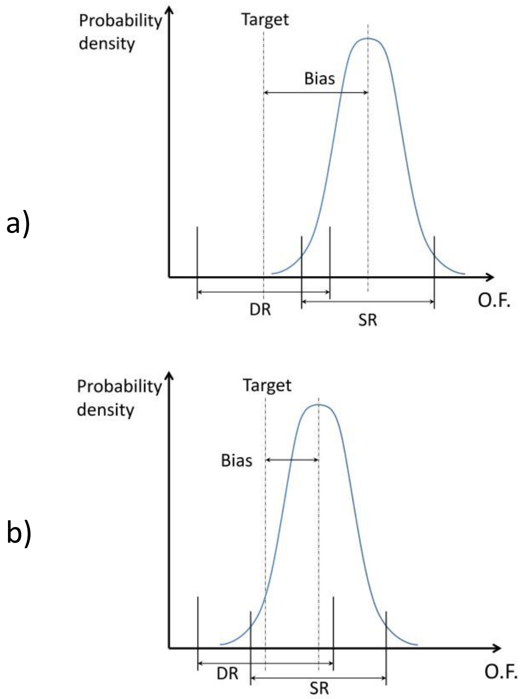

Fig. 1 Bias reduction in terms of pdf position: a) before ; b) after.

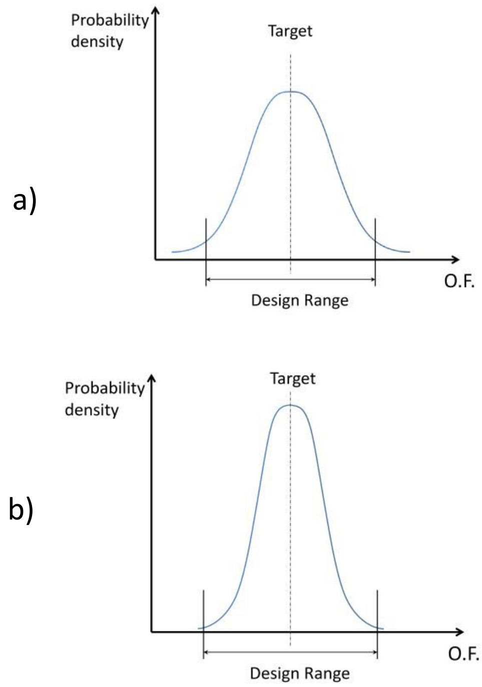

Fig. 3 Variance reduction in terms of pdf form: a) before ; b) after.

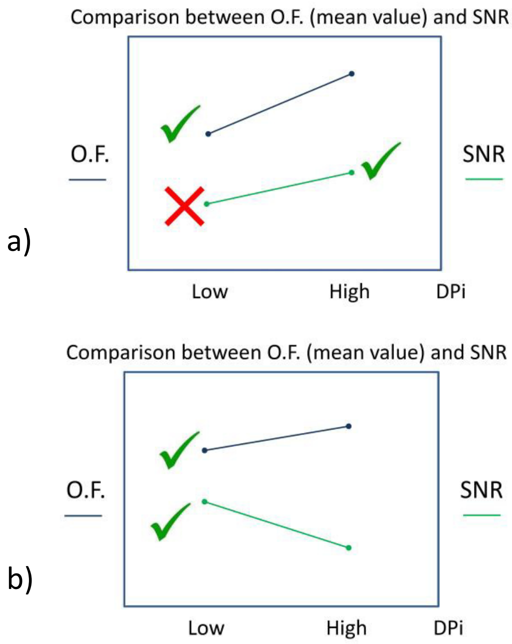

Fig. 4. Data related to variance reduction: a) before ; b) after.

The probability density function must be contained at most inside the Design Range. Considering the data reported in Fig. 4a, it must be verified if adjusting the range of variation of the design parameter DPi toward higher values, because in that direction the performances of the SNR are well, really the behavior becomes better. In any case it is important to verify that also the MV of the O.F. continues to remain low, as required by the law associated with the Objective Function.

In Fig. 4b this situation is reached because in correspondence with the new value of the lower level of the DPi both O.F. and SNR are satisfied.

4. The independence axiom through the insight of the Taguchi method.

When modeling a design problem in terms of axiomatic design, it is important to know the type of design matrix. After the first draft of the identification of the design parameters and the associated functional requirements, it is important to design the experiment in terms of the Taguchi method, to search for a macro-Functional Requirement that is determined by the set of Design Parameters. This is similar to the relation between the "One FR" and the associated set of design parameters, discussed by Suh in the chap. 2 of [1]. The method proposed here consists in the association of one Objective Function with the macro-FR, the identification of the "loss function" and the design of the experiment employing an orthogonal array. The main assumption at the basis of this approach is to consider the problem initially as uncoupled, in which each DPi independently contributes to the performance of the macro-Functional Requirement FRi, as represented in Figure 5.

		...	Dpi							...
			DPi1	DPi2	DPi3	DPi4	DPi5	DPi6	DPi7	
...		X								
FRi	FRi1		X							
	FRi2			X						
	FRi3				X					
	FRi4					X				
	FRi5						X			
	FRi6							X		
	FRi7								X	
...										X

Fig 5. Example of a sub-domain structured as an uncoupled DM. Case of seven DPi influencing the macro-FRi.

A second step in the experiment designed by Taguchi method is focused on:

- The identification of the most influencing design parameters on the behavior of the design solution;
- The presence of interrelation between design parameters and of which type.

The first point highlights, among all the *m* design parameters the designers have considered in the study, only those really relevant. For this purpose the Analysis of Variance (ANOVA) can be used to differentiate among them, and considering that all design parameters have the same mean value, the ANOVA can be expressed as:

$$SSTot = SS\ (DPi1) + \ldots SS\ (DPik) \ldots + SS\ (DPim) \qquad (2)$$

where $SSTot$ is the Total Sum of Square and $SS\ (DPik)$ is the Sum of the Square of each k-th Design Parameters.

Figure 6 shows the section of the design matrix related to the macro-FRi and, after ANOVA, the importance of each DPi. The most influential Design Parameters related to FRi continue to be represented by capital X, whereas those less influential are represented by lower case x.

The most influential Design Parameters must be analyzed also in term of interaction [16].

		...	Dpi							...
			DPi1	DPi2	DPi3	DPi4	DPi5	DPi6	DPi7	
...		X								
FRi	FRi1		x							
	FRi2			X	*x*			*x*		
	FRi3				X			*x*		
	FRi4					x				
	FRi5						x			
	FRi6							X		
	FRi7								x	
...										X

Fig 6. The nature of the sub-domain structured as a decoupled DM, with the interaction of the most relevant design parameters, i.e. DPi2, DPi3 and DPi6.

Considering Fig. 6 it is interesting to study the pairwise interaction between DPi2 and DPi3, DPi2 and DPi6, DPi3 and DPi6, represented by the italic x in the Design Matrix.

The presence of interrelation between design parameters is the first signal that the nature of the design matrix is at least changing from uncoupled to decoupled. The pruning of the design parameters only to the most influential, gives the designers the possibility of investigating only a reduced number of combinations, these being *n*(n-1)/2*.

The general expression that represents the multiple influence of many Design Parameters DP_i on the same Functional Requirement FR_i, in a upper triangular matrix, is:

$$FR_i = \sum_{j=k}^{m} \frac{\partial FR_i}{\partial DP_j} DP_j + \sum_{j=k}^{m} \sum_{n=k+1}^{m} \frac{\partial^2 FR_i}{\partial DP_j \partial DP_n} DP_j\, DP_n \quad (3)$$

where i is the i-th Functional Requirement FRi that is influenced by m design parameters DPi; j counts the position of the diagonal terms and n indicates the position of the off-diagonal terms.

This equation allows the designer to evaluate the direct influence of each design parameter on the macro-FR (the first term on the left hand side), and at the same time verify the existence and the type of interrelation between design parameters (the second term). The latter are analyzed by means of the data contained in the performed experiments of robust design. The interrelation, represented by the off-diagonal terms of the Design Matrix, can be of the type shown in Figure 7. In Figure 7a the design parameters DPi2 and DPi3 cooperate, both having the same behavior. This configuration is the most interesting because no problems are added for the designers. In Figure 7b the design parameters interact and designers are alerted to finding the solution to the conflict and the opposite influence that both parameters have on the functional requirement.

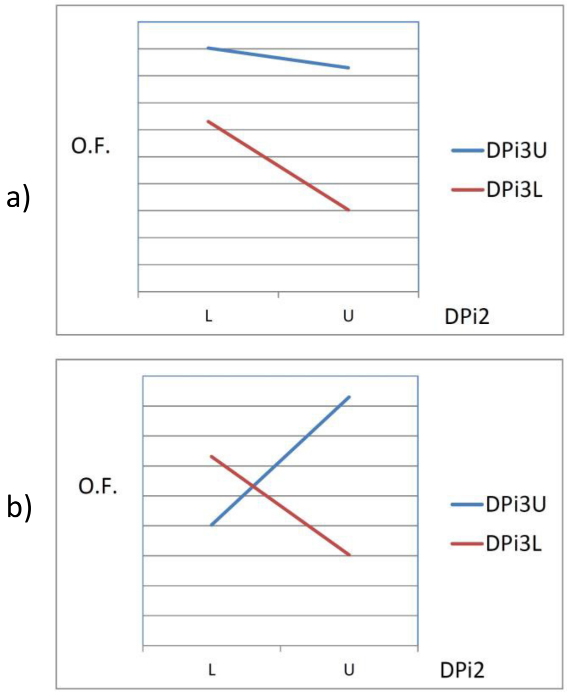

Fig 7. The interrelation of the design parameters: a) collaboration; b) intersection.

5. The procedure to manage the Taguchi method in the context of Axiomatic Design.

The Appendix reports the flow chart of the procedure that aids the designers to manage the process of product design following the Axiomatic Design approach. The main difficulty in the analysis is the search for an Objective Function associated with the macro-Functional Requirement. During the design process the evaluation of the performance of the device in the course of design is done by means of CAE systems, involving the physical phenomena that will occur when the device operates.

The procedure is in ten steps:

1. Identify the Objective Function (O.F.) associated with the macro-FR.
2. Identify the Control Factors (Design Parameters) and the Noise related to the experiment.
3. Establish the ranges for each Control Factor.
4. Plane the experiment. Choose the orthogonal array.
5. Perform the experiments and arrange the results in terms of Mean Values and Signal to Noise Ratio.
6. Identify the best design parameters configuration able to satisfy the Objective Function.
7. Compute the SNR and evaluate it comparatively in conjunction with Mean Values.
8. Perform the Analysis of Variance (ANOVA).
9. Study the interactions between design parameters and their typology.
10. Apply and verify.

During the first attempt it is sufficient to use experiments in which the parameters vary between two values, the lower and higher limit of the design range. This allows experiments with the lowest number of tests to be arranged.

If the effects of all parameters conjointly lead to satisfying the loss function associated with the Objective Function and to maximizing the SNR the procedure quickly arrives at the end, because the design of parameters is reached. It remains to perform the ANOVA just to have information about the most relevant parameters. This step is encouraged so that the designer familiarizes himself/herself with the phenomenon under investigation.

The problems begin to surmount the normal design activity when some of the parameters do not behave as supposed.

At this point it is necessary to classify the kind of answer on the basis of the variation that the Mean Value of the Objective Function and the SNR has been recognized in the experiments. The four combinations are reported in Table 1.

Table 1. Conjoint variation of Mean Value of O.F. and SNR.

	SNR	
Mean Values of O.F.	Low - Low	Low - High
	High - Low	High - High

The H-H case of variation of O.F.-SNR must be solved in the way treated in Section 3, because the problem can be associated with bias elimination or variance reduction.

The L-L case of variation of O.F.-SNR does not create a problem, even though in this context it confirms that the design parameter is not influential in the performance under examination.

In the L-H case of variation of O.F.-SNR the design parameter must be chosen at the level associated with maximum value of SNR.

In the H-L case of variation of O.F.-SNR the design parameter must be chosen at the level associated with the satisfaction of the loss function related to the Objective Function.

After this selection, a first refinement is performed: the design range of these parameters must be calibrated moving it towards the direction more promising for the satisfaction of the Objective Function. A second experiment must be pursued.

The case of intersection between design parameters requires further insight, because this contradiction in the design matrix must be eliminated. In order to do this, it is necessary to investigate the problem with a new type of plane of experiment, with higher number of levels. In a second refinement, the design range of these parameters must be also calibrated moving it towards the direction more promising for the satisfaction of the Objective Function, and treated at three level, adding the mean value of the range. The analysis is performed on a reduced number of parameters, those really relevant and defined on the basis of ANOVA. This kind of investigation is time consuming and it is reasonable to reduce it to only the essential. This step highlights the nature of the design matrix, with the identification of non-zero off-diagonal terms.

Having performed this new experiment the discussion must be moved on the design solution. The presence of divergence also for these new results reveals that something is not properly done, probably in the identification of the design parameters or the nature of the Objective Function chosen to study the design solution.

6. Conclusion.

In the paper a new interpretation of the results obtained by the application of the Taguchi method is presented for the identification of the best design parameters for a problem described in term of the Axiomatic Design approach.

Consulting the variations of an Objective Function, related to the Functional Requirement, in term of the Mean Values, Signal to Noise ratio and interaction among parameters, the designers have the possibility of reasoning about the design solutions on which they are working.

The nature of the Design Matrix, or a part of it, can be disclosed after this kind of study. Starting from the first assumption of an uncoupled Design Matrix, the interaction between several pairs of Design Parameters can emerge. This clearly transforms the Design Matrix into a decoupled form. The presence of a non-zero off-diagonal term could lead to a coupled form and this must be avoided checking the nature of the intersection between of the design parameters involved.

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Appendix

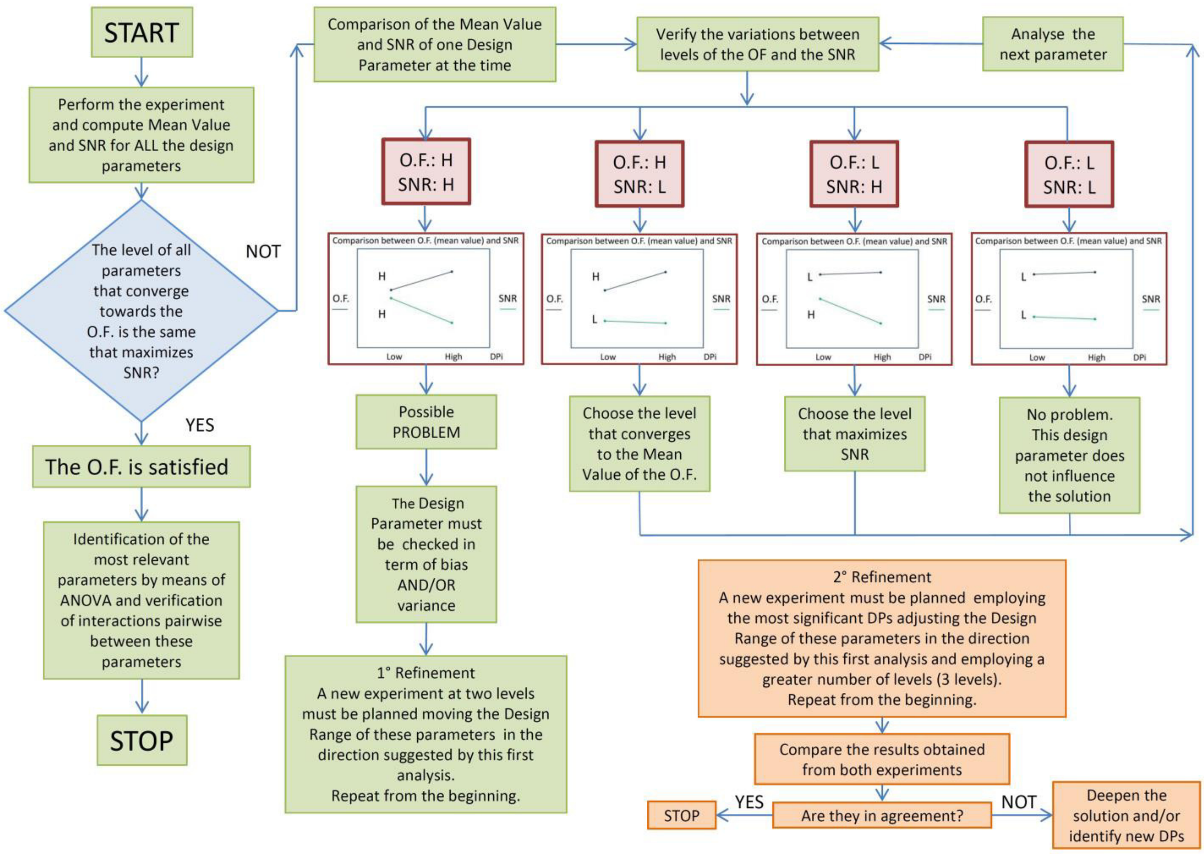

Fig. 8 Flow chart of the procedure.