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# An Axiomatic Design interpretation for the synthesis of dimensional tolerances

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**Abstract**

Synthesizing dimensional tolerances of mechanical systems involves finding the values of the allowable variation of the functional dimensions. According to Axiomatic Design, the synthesis of mechanical tolerances can be described as a redundant decoupled design. This is why experienced engineers are proficient in solving this kind of problems, even if they never heard about Axiomatic Design and about the independence axiom. This paper presents an Axiomatic Design interpretation for the synthesis of dimensional tolerances. This interpretation shows that the traditional method for solving this class of problems implicitly obeys the independence and the minimum information axioms.


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*Keywords:* *decoupled designs; tolerances; redundant designs*

## 1. Introduction

Mechanical systems are made of discrete parts. These parts may be manufactured through distinctive approaches, not only at sites that are geographically separated, but also through different production processes with dissimilar capabilities. This may involve developing distinct detail designs dedicated to each one of the aforesaid approaches.

This challenge is typically outdone by using manufacturing setups that are based on the details of each approach, which includes the knowledge about the capability of the possible manufacturing processes. For each approach, designers must define dimensions and tolerances of the parts to ensure the required fits, taking into account the capabilities of the likely production processes. Notice that the tolerance allocation plays an important role in the development of mechanical products that can operate at increasingly better performance at affordable costs.

Traditionally, designers deal with the tolerance issue by identifying independent dimensional chains. Hence, they quietly use an important principle of Axiomatic Design (AD): keeping the independence of the dimensional chains, in which fits are seen as the functional requirements, while dimensional tolerances are regarded as the design parameters.

The allocation of values to the tolerances of dimensions that form each dimensional chain is known as “synthesis of tolerances”. This paper discusses the synthesis of tolerances and uses the example of a simple mechanical system to make an interpretation of it in light of Axiomatic Design.

**Nomenclature**

| | |
|---|---|
| $f_i$ | Fit of the $i^{th}$ dimensional chain |
| | $f_i > 0$ Means clearance |
| | $f_i < 0$ Means interference |
| $\Delta f_i$ | Tolerance of the $i^{th}$ fit |
| $a, .., g$ | Dimensions of the parts |
| $\Delta a, .., \Delta g$ | Tolerances of the dimensions $a, .., g$ |

All dimensions in millimetres.


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## 2. The synthesis of tolerances

The precise knowledge about dimensional tolerances of products is essential for their manufacture and the inspection that should follow. Tolerances impact the selection of manufacturing processes and the assembly methods, which, in turn, influence the functional quality of the products as well as their production costs. Thus, the specification of tolerances represents a key connection between design and production.

It is well known that unduly tight tolerances lead to unacceptable production costs, as shown in Fig. 1.

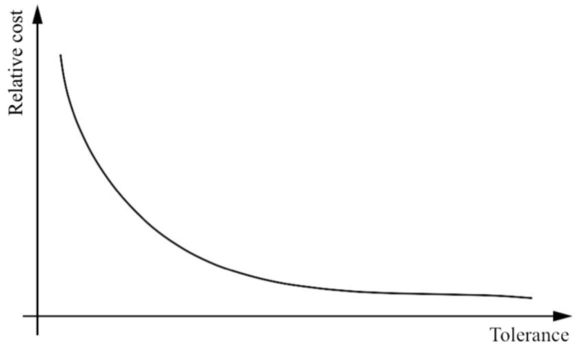


Fig. 1. The impact of dimensional tolerances on production cost

On the other hand, high performance usually requires tight tolerances, as depicted in Fig. 2.

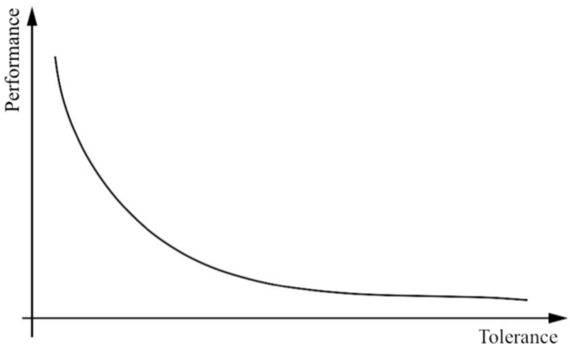


Fig. 2. The impact of dimensional tolerances on performance

Fig. 1 and Fig. 2 illustrate what one can call "the designer's dilemma": high performance is attained through tight tolerances, but this increases the production costs.

Tolerancing is twofold: analytic and synthetic. In analysis, individual tolerances are known and the functional condition (as expressed by a fit) is checked. In synthesis, the individual tolerances of the components are assigned, in order to ensure a predefined functional condition.

Notice that in any mechanical multi-part system, the number of parts must be greater than the number of functional interfaces, which are pairs of mating surfaces. Otherwise the fine-tuning of the fits is not controllable because they are not independent. Additionally, the best solutions are the ones where the various tolerances are equally difficult to attain [1].

Many approaches to deal with analysis (models) and synthesis (methods) are known. The result of tolerance analysis is conditioned by the adopted mathematical model. Polini [2], classifies the major models for tolerance analysis as: Vector loop, Variational, Matrix, Jacobian, Torsor, Jacobian-Torsor and "T-maps". As for the available techniques for tolerance synthesis, Ye and Salustri [3] classifies them as traditional methods, methods focusing on manufacturing and methods focusing on quality. Nevertheless, the goal of this work is not about synthesis methods, rather presenting an AD interpretation of synthesis itself.

## 3. Independence in Axiomatic Design

Axiomatic Design (AD) is a general design framework based on two axioms: the independence axiom and the information axiom, and no counterexample has been found so far. At least, eight corollaries and some tenths of theorems were derived from these axioms. All of them constitute part of the Axiomatic Design framework.

Axiomatic Design was created by Nam P. Suh in 1978 [4] and was spread along the worldwide engineering community through his seminal book "The Principles of Design" [5].

The independence axiom might be stated as:

Axiom 1 (*The Independence Axiom*): Maintain the independence of the functional requirements (FRs)

In 2007, Park [6], presented two alternative statements for the same axiom:

Alternative Statement 1: An optimal design always maintains the independence of FRs.

Alternative Statement 2: In an acceptable design, design parameters (DPs) and functional requirements (FRs) are related in such a way that a specific DP can be adjusted to satisfy its corresponding FR without affecting other functional requirements.

The relationship between FRs and DPs is represented by the design equation

$$\{FR\} = [A]\{DP\},\ A_{ij} = \frac{\partial FR_i}{\partial DP_j},\ i = 1..,m,\ j = 1,..,n \qquad (1)$$

where $\{FR\}$ is the vector of functional requirements, $[A]$ is the design matrix and $\{DP\}$ is the vector of design parameters. The total numbers of FRs and DPs are expressed by $m$ and $n$, respectively.

The independence of FRs is guaranteed whenever the design matrix is diagonal (uncoupled designs). It is also assured when the design matrix is triangular (decoupled designs). Otherwise, independence is not achieved (coupled designs). According to theorem 4, in an ideal design, the number of DPs is equal to the number of FRs and FRs are always maintained independent of each other [7].

Some papers about using AD in the synthesis of tolerances have been published in the past: Campatelli [8] showed a way to include synthesis in the classical AD framework by introducing the concept of Tolerance Design Matrix (TDM), which describes the relationships between the DPs of distinct levels of the design decomposition, as a means to facilitate the optimization of costs; Henriques *et al*. [1] proposed a method to increase the sigma level of a mechanical assembly through the appropriate allocation of tolerances, i.e. the values for the DPs.

However, in a strict AD viewpoint on the synthesis of tolerances, the variation of the fits should be taken as functional requirements, while the tolerances of components should be taken as design parameters, and the independence axiom should be followed. Notice that this is the case of the 1-FR problem studied by Henriques *et al*., which is uncoupled by nature. As it was already indicated, the number of parts of any mechanical assembly must be greater than the number of interfaces (n-Fr problem). Thus, according to AD's theorem 3 [7], tolerance synthesis always represent a redundant design. Therefore, after checking that the design solution is uncoupled or decoupled, designers should select suitable tolerances in order to conform to the required functionally. Otherwise, the solution must be revised.

The next section details the case of the synthesis of a decoupled design.

## 4. The synthesis of tolerances – a case study

This section presents an example of synthesis of dimensional tolerances of a mechanical system under the traditional point of view. Fig. 3 depicts that system, which has the four following dimensional chains.

Chain #1 – Ensures the fit $f_1$ and it is composed by the dimensions *a*, *b* and *c*.

Chain #2 – Ensures the fit $f_2$ and it is composed by the dimensions *b*, *d* and *e*.

Chain #3 – Ensures the fit $f_3$ and it is composed by the dimensions *a*, *b*, *d* and *f*.

Chain #4 – Ensures the fit $f_4$ and it is composed by the dimensions *a*, and *g*.

All the fits are clearances with the following values:

$1 \le f_1 \le 2$

$0.5 \le f_2 \le 1.5$

$1.5 \le f_3 \le 2.5$

$1.5 \le f_4 \le 2.5$

In addition, one knows that $a = 120$ and $e = 55$. Dimension $b = 24$ and $0 \le \Delta b \le 0.2$, which relates to the width of the bearings shown in Fig. 3 and dimensions *c*, *d*, *f* and *g* must be appropriately set by the designer.

Considering the worst-case condition, the equations of the dimensional chains and of the related tolerances are

Chain #1 $$f_1 = a + 2b - c \quad (2)$$
$$\Delta f_1 = \Delta a + 2\Delta b + \Delta c \quad (3)$$

Chain #2 $$f_2 = b + d - e \quad (4)$$
$$\Delta f_2 = \Delta b + \Delta d + \Delta e \quad (5)$$

Chain #3 $$f_3 = f - a - b - d \quad (6)$$
$$\Delta f_3 = \Delta f + \Delta a + \Delta b + \Delta d \quad (7)$$

Chain #4 $$f_4 = a - g \quad (8)$$
$$\Delta f_4 = \Delta a + \Delta g \quad (9)$$

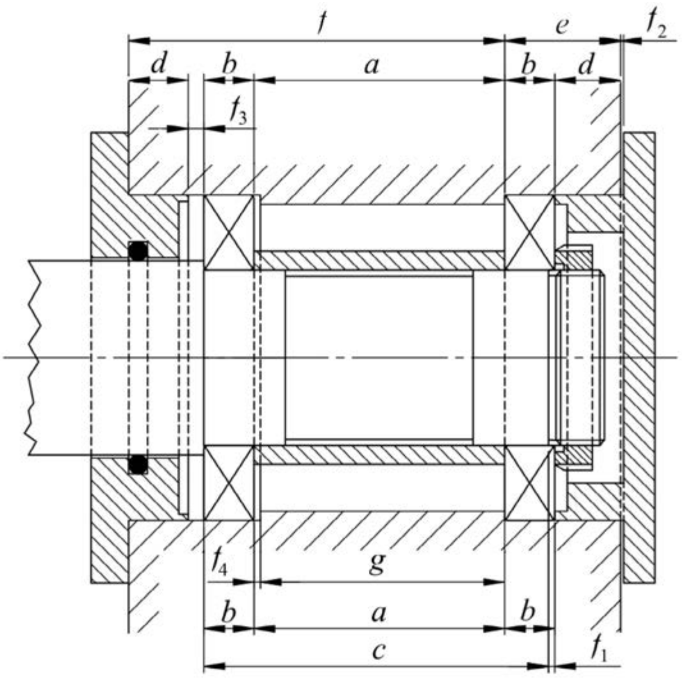


Fig 3. The mechanical system to be toleranced

The goal is to synthesize the tolerances, *i.e.*, to assign values to $\Delta a$, $\Delta b$, $\Delta c$, $\Delta d$, $\Delta f$ and $\Delta g$ in order to satisfy the functional requirements $\Delta f_i$. A typical approach is made of the following steps, based on the proportional approach to the allocation.

Chain #1

- Dimension $c$ is computed through Eq. 2 for the given values $a = 120$ and $b = 24$.
- $\Delta a$ and $\Delta c$ are interrelated trough the "standard tolerance unit", $i$

$$i = 0.45D^{1/3} + 0.001D \quad [\mu\text{m}] \tag{10}$$

where $D$ is the dimension in mm. Because both tolerances $\Delta a$ and $\Delta c$ should be equally difficult to attain, then the following relation holds

$$\frac{\Delta a}{\Delta c} = \frac{0.45a^{1/3} + 0.001a}{0.45c^{1/3} + 0.001c} \tag{11}$$

which allows to compute $\Delta a$ or $\Delta c$ trough Eq. 3.

- The tolerances of $\Delta a$ and $\Delta c$ are computed considering the tolerance zone $h$. Since $a$ can be regarded as a shaft, then $a_{max} = a$, and $c_{max}$, as well as $c_{min}$, are computed trough Eq. 2.

Chains #2, #3 and #4

- Solving Chains #2, #3 and #4 follow the same procedure that was used for Chain #1.

Notice that, in Chain #2, $d$ and $e$ are external and internal dimensions, respectively. Thus, the following relation holds

$$\frac{\Delta d}{\Delta e} = \frac{1}{1.58} \cdot \frac{0.45d^{1/3} + 0.001d}{0.45e^{1/3} + 0.001e} \tag{12}$$

Therefore, the values of $d$, $e$, $\Delta d$ and $\Delta e$ are found by solving Chain #2 through Eq. 4 and 5, taking into account Eq. 12. Then, Chain #3 allows determining the values of $f$ and $\Delta f$ through Eq. 6 and 7. At last, solving Chain #4 allows getting the values of $g$ and $\Delta g$ through Eq. 8 and 9.

In the next section, tolerance synthesis for this case is interpreted under an Axiomatic Design viewpoint.

## 5. An AD interpretation of the synthesis of tolerances

Eq. 13 derives from Eq. 3, 5, 7 and 9. It depicts the AD's design equation for the system of Fig. 3, and one can see that the corresponding design matrix is of rank 4x7,

$$\begin{Bmatrix} \Delta f_1 \\ \Delta f_2 \\ \Delta f_3 \\ \Delta f_4 \end{Bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \Delta c \\ \Delta d \\ \Delta e \\ \Delta f \\ \Delta g \end{Bmatrix} \tag{13}$$

subjected to the constraints represented by Eq. 11 and 12. With its right-trapezoid design matrix, Eq. 13 depicts a redundant decoupled design as per AD's theorem R1 [9].

From Eq. 3, one can write

$$\{\Delta f_1\} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \Delta c \end{Bmatrix} \tag{14}$$

Taking into account Eq. 11, one has

$$\Delta c = \frac{\Delta a}{k_1}, \quad k_1 = \frac{0.45a^{1/3} + 0.001a}{0.45c^{1/3} + 0.001c} \tag{15}$$

and Eq. 14 becomes

$$\{\Delta f_1\} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \dfrac{\Delta a}{k_1} \end{Bmatrix} = \begin{bmatrix} \left(1 + \dfrac{1}{k_1}\right) & 2 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \end{Bmatrix} \tag{16}$$

A similar reasoning can be used for Chains #2, #3 and #4. For Chain #2 one has

$$\{\Delta f_2\} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \Delta b \\ \Delta d \\ \Delta e \end{bmatrix} \tag{17}$$

Next, taking into account Eq. 12, one has

$$\Delta e = \frac{\Delta d}{k_2}, \quad k_2 = \frac{1}{1.58i} \tag{18}$$

Using Eq. 18 to eliminate $\Delta e$ in Eq. 17 yields to

$$\{\Delta f_2\} = \begin{bmatrix} 1 & \left(1 + \dfrac{1}{k_2}\right) \end{bmatrix} \begin{bmatrix} \Delta b \\ \Delta d \end{bmatrix} \tag{19}$$

Concerning to Chains #3 and #4, one can write

$$\{\Delta f_3\} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \Delta d \\ \Delta f \end{Bmatrix} \tag{20}$$

$$\{\Delta f_4\} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta g \end{Bmatrix} \tag{21}$$

Now, combining Eq. 16, 19, 20 and 21 we can rewrite Eq. 13 as an equation with a design matrix of rank 4x5.

$$\begin{Bmatrix} \Delta f_1 \\ \Delta f_2 \\ \Delta f_3 \\ \Delta f_4 \end{Bmatrix} = \begin{bmatrix} \left(1+\frac{1}{k_1}\right) & 2 & 0 & 0 & 0 \\ 0 & 1 & \left(1+\frac{1}{k_2}\right) & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta b \\ \Delta d \\ \Delta f \\ \Delta g \end{Bmatrix} \tag{22}$$

At last, making

$$\Delta f_1^* = \Delta f_1 - 2\Delta b \tag{23}$$

$$\Delta f_2^* = \Delta f_2 - \Delta b \tag{24}$$

$$\Delta f_3^* = \Delta f_3 - \Delta b \tag{25}$$

one can rewrite Eq. 22 as an equation with a design matrix of rank 4x4

$$\begin{Bmatrix} \Delta f_1^* \\ \Delta f_2^* \\ \Delta f_3^* \\ \Delta f_4 \end{Bmatrix} = \begin{bmatrix} \left(1+\frac{1}{k_1}\right) & 0 & 0 & 0 \\ 0 & \left(1+\frac{1}{k_2}\right) & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta a \\ \Delta d \\ \Delta f \\ \Delta g \end{Bmatrix} \tag{26}$$

subjected to the constraints denoted by Eq. 11 and 12.

With its triangular design matrix, Eq. 26 clearly depicts the case of Fig. 3 as a squared matrix decoupled design, so confirming the conclusion that was already made through Eq. 13. However, Eq. 26 is much easier to use than Eq. 13.

First, Eq. 23, 24 and 25 are used to compute $\Delta f_1^*, \Delta f_2^*$ and $\Delta f_3^*$. This can be promptly done because one knows the values of $\Delta f_1$, $\Delta f_2$, $\Delta f_3$ and $\Delta b$. Then, Eq. 26 is used to compute $\Delta a$, $\Delta d$, $\Delta f$ and $\Delta g$. Next, Eq. 11 is used to compute $\Delta c$ and, at last, Eq. 12 allows computing $\Delta e$.

## 6. Discussion

Eq. 13 describes Fig. 3 and denotes a decoupled design because its design matrix is populated according to a right-trapezoid arrangement, thus conforming to theorem R1 [9]. In other words, well-solved problems of synthesis of tolerances implicitly obey the independence axiom.

Solving Eq. 13 implies making two early assumptions for the design parameters because the first line of the corresponding design matrix contains three non-zero elements.

The easiest assumption to make is taking $\Delta b = 0.2$, because for the selected bearings one knows that $0 \leq \Delta b \leq 0.2$. Now, taking into account Eq. 11, one can simultaneously compute the values of $\Delta a$ and $\Delta c$ from Eq. 13 because the value of $f_1$, is known. Eq. 11 implicitly uses the minimum information axiom because it prescribes that the difficulty of attaining the dimensions $a$ and $c$ (i.e., their probability of success) should be equal. Subsequently, one tackles the functional requirement $f_2$. The fulfilment of this functional requirement depends on finding the values of $\Delta d$ and $\Delta e$ because the value of $\Delta b$ is already known. However, $\Delta d$ and $\Delta e$ are interrelated through Eq. 12, which, once more, is an implicit call to the minimum information axiom that allows simultaneously finding the values of the aforementioned design parameters. At last, one can promptly find the value of $\Delta f$ that satisfies $f_3$ and the value of $\Delta g$ that satisfies $f_4$.

A much more straightforward procedure for solving the synthesis of tolerances of Fig. 3 is offered by Eq. 26, which results from Eq. 13 and contains a squared, triangular design matrix: first, Eq. 26 is used to find the values of $\Delta a$, $\Delta d$, $\Delta f$ and $\Delta g$ using the well-known technique for solving decoupled design equations; next, Eq. 11 is used to find $\Delta c$ and, at last, Eq. 12 allows finding $\Delta e$.

## 7. Conclusions

This paper presents an interpretation of the synthesis of mechanical tolerances in light of Axiomatic Design, on the basis of a case study.

The number of parts of any mechanical multi-part system should be greater than the number of functional interfaces, which always makes any dimensional synthesis problem a redundant design. This is because every mechanical fit, being it a clearance or interference, clearly depends on at least two dimensions

Our interpretation shows that the synthesis of tolerances involves the solution of a redundant decoupled design equation. In fact, the tolerance equation is represented by Eq. 13 that can be also expressed by Eq. 26, which triangular matrix confirms the decoupled nature of the problem. As a result, the process of synthesis always involves some assumptions are needed to establish the values of the tolerance chains. Thus, tolerance synthesis concerns should be integrated in the design structure, instead of being done in the final steps of the design.

At last, one can stress that Axiomatic Design easily allows systematizing the synthesis of tolerances.

In the future, the authors are planning to work on a holistic roadmap applicable to the process of tolerances synthesis under an AD viewpoint.

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