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AXIOMATIC DESIGN OF A CONCEPT OF CAR-PLATFORM FOR AN ELECTRICAL REAR-WHEEL DRIVE VEHICLE: A COMPARISON WITH A FUZZY APPROACH

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ABSTRACT

In this paper we analyze the concept design of an alternative propelled rear-wheel drive vehicle's chassis, of car-platform, focusing the interest on pavement and sub-pavement design, so that it will contain 18 accumulators to supply the electrical energy (for the electrical or hybrid engine) needed.

The spatial configuration of internal cross-bars (that are integrant part of vehicle chassis) and accumulators' spatial placing in available space requires a careful optimization research both in functional character and in structural behavior, since they involve vehicle dynamic, accessibility and overall dimensional problems in presence of dynamic stresses due to external factors.

Axiomatic Design can help us to make several strategic choices, in order to achieve the better solution for our problem; nevertheless, the presence of not precisely computable factors suggests fuzzy-logic application at optimization problems, in fact, in the plate optimization, not numerically quantifiable characteristics, such as accumulators' accessibility (FR_3), numerically determinable structural values, such as stresses and strains (FR_1) under particular load condition, analytically calculable properties, such as inertias (FR_2), come into play.

Three above-mentioned parameters drive the choices on prototype concept, and design parameters are the followings:

1. Accumulators placing coordinate
2. Car-platform cross-bars placing (integrant parts of chassis)
3. Internal cross-bars number in car-platform

We want to check if the Axiomatic approach allows achieving the same optimization results of Fuzzy Logic Approach, using, for requirements that have to be expressed by Linguistic variables, the Fuzzy Membership Functions.

Keywords: Axiomatic Design, Fuzzy Logic, Concept Design, Electrical Vehicle, Information Theory

1 INTRODUCTION

Vehicle manufacturers and designers, because of increasing air pollution, are now paying attention to alternative propulsion like electrical and hybrid one, to use according to the circumstances (i.e. electrical propulsion in city centre, air-terminal, bus terminal, seaport and railway station; fuel propulsion for other uses as on highway and outskirts roads).

It goes without saying that fuel propulsion and electrical or hybrid propulsion shows different designing problems such as travel comfort, car performances and aesthetic.

Low speed and short range does not allow the use of electrical propulsion alone; on the other hand hybrid propulsion involves size and weight problems that affect vehicle design.

Number, dimension and weight of accumulators involve volume and space distribution problems. In fact, the space assigned to double propulsion-system, to its interface and to the accumulators has obvious consequences on passengers comfort and on passengers' compartment.

There are other important factors in vehicle design, such as driver's side ergonomics, easy access to the vehicle, and easy access to components for ordinary and extra-ordinary maintenance and, above all, passengers and driver safety.

Considering the above statements, a rear wheel drive by rear transverse engine seems to be the best choice for a hybrid vehicle [Gillespie, 1992].

In order to maintain vehicle habitability, in the past, we've suggested to create a sandwich floor under passengers' feet for accumulators allocation [Naddeo, 1999]. This solution has to be combined with a right choice of modular front and rear seats easy to be dismantled. That case is the typical concept design problem, in which a multi-objective optimization is required before that the product is designed; that's the applying field for Axiomatic and Fuzzy design approach.

2 LOGICS AND AXIOMS IN DESIGN

Several methods were studied for helping design choices in concept design, and several mathematical instruments are useful for that topic. In University of Salerno a new approach, based on the use of Fuzzy Logic, was experimented for design problems.

2.1 FUZZY-ANALYSIS FUNDAMENTALS

Based on L. Zadeh theory [1965-1974], that kind of logic allows to express in mathematical terms several not precisely defined concepts; unlike of binary logic, that logic doesn't require that a proposition assumes a defined truthful value, true or false, but allows to assign a membership value (between 0 and 1) to truthfulness of it. Generally we can declare that an element satisfy a requirement [Klir, 1995], even if this requirements has a not clearly sense, giving to it a membership value in the range {0 – 1}.

In Design process, it's very important to underline the key role of mapping process between what we want to achieve and how we want to achieve it: using that definition, we can declare that Design problem formulation start from Functional requirements (FRs) and Design parameters (DPs) identification.

The Fuzzy logic approach help designers to identify the relationship between FRs and DPs, to formulate a judgment on several design hypothesis and to compare different concept design solutions each other, putting into account exact, not precise and not quantifiable requirements.

The concept of membership function play a key role in that approach: FRs can be correlated, by membership function, to DPs that characterize the project, and FRs for a project's "element" (space-frame design and accumulators positioning in our paper) can be decomposed into simple ones (sub-requirements) directly depending from design parameters; that operation allows to decompose complex property, associated to a requirements, in simple ones, combined each other by fuzzy membership function composition laws [Scott-Antonsson, 1998].

Finally, the Design Goal is defined through all requirements opportunely weighted or composed by simple rules.

Those rules can be combined each other in order to create an Objective Function (OF) that provides all design aspects.

The concept design process finish with the formulation of several design hypotheses.

Each hypothesis is evaluated and makes a score defined by the final composition rule. The score expresses the membership value to the chosen objective; the best design solution will be naturally chosen among ones which have the best score [Antonsson, 1992].

After the complete characterization of problem, by identification of FRs and DPs, the second, and most important step of fuzzy formulation, is the Fuzzyfication of the problem, and so the definition of the membership functions (mf) and the evaluation rules.

There are a lot of papers in literature dealing with membership function definition [2, 11, 21], their construction and method of composition; for our application we will use several simple mf such as triangular, trapeziform and simple mathematical function, for evaluating quantifiable parameters, while, for evaluating several not quantifiable requirements, we will use the "one expert direct method"; for the last one we need to give to readers a brief explanation:

"One expert direct method" allows to assign a membership value directly for each of examined alternatives, in comparison with other methods that indirectly (by membership function) make this operation [Naddeo, 2001]. The first step for that method is the interview with an expert that gives a judgment for each design solution; after that, his evaluation is expressed in terms of adjective (that modifies the truthfulness value of a proposition) or by collocating the alternative in a predefined list, in which several kind of judgment are provided. Finally, for each alternative, the judgment is transformed in membership value by using a table of predefined correspondence judgment<–>value.

Once the membership functions are defined, they have to be combined by composition rules; some of these are: minimum rule, maximum rule, arithmetical average rule, geometrical average rule. The first of those is applied in evaluating requirements that have to be necessarily satisfied, and assigns, to requirements, minimum of obtained scores among all; the second is applied especially when at least one of the requirements has to be satisfied, and assigns to element the maximum among scores; arithmetical average is applied when requirements interact each other compensating themselves, and assigns to the element a score calculated as weighted average of single requirements scores; geometrical average is applied when every judgment on design's requirement makes worse the final one.

That rules are used to define the Objective Function for evaluating the Designs' hypotheses.

Finally the Design Problem requires a Defuzzyfication, in order to extract the physical values of DPs from Fuzzy formulation.

2.2 AXIOMATIC DESIGN

Motivated by the absence of scientific design principles, Suh (1990 - 2001) proposed the use of axioms as the scientific foundations of design. Out of the twelve axioms first suggested, Suh introduced the following two basic axioms along with six corollaries that a design needs to satisfy:

Axiom 1: The Independence Axiom

> Maintain the independence of the functional requirements

Axiom 2: The Information Axiom

> Minimize the information content in a design

In axiomatic design approach, the engineering design process is described in Figure 1, in which the array of functional requirements *(FRs)* is the minimum set of independent requirements that completely characterizes the design objective based on customer attributes (*CAs*). Design is defined as the creation of synthesized solution to satisfy perceived needs through the mapping between the *FRs* in the functional domain and the design parameters (*DPs*) in the physical domain and through the mapping between the *DPs* and the process variables (*PVs*) in the process domain.

The physical and process mappings can be expressed mathematically as

$$\{FR\}_{m\times1} = [A]_{m\times r} \{DP\}_{r\times1}$$
$$\{DP\}_{r\times1} = [B]_{r\times n} \{PV\}_{n\times1}$$

where $\{FR\}_{mx1}$ is the vector of independent functional requirements with m components, $\{DP\}_{rx1}$ is the vector of design parameters with r components, $\{PV\}_{nx1}$ is the vector of process variables with *n* components, ***A*** is the physical design matrix, and ***B*** is the process design matrix.

For our purposes, the mapping process can be mathematically abstracted as the following matrix equation: $\{FR\}=[A]\{DP\}$, where ***FR*** is the array of *FRs*, ***DP*** is the array of *DPs*, and ***A*** is the design matrix that contains the sensitivity coefficients of the *FRs* to the mapped-to *DPs*. The process mapping is described by: $\{DP\} = [B]\ \{PV\}$ but doesn't affect our problem formulation.

Axiom 1 states that the design parameters (*DPs)* and the functional requirements (*FRs)* are related such that a specific *DP* can be adjusted to satisfy its corresponding *FR* without affecting the other functional requirements, which will require that ***A*** should be either a diagonal matrix or triangular matrix.

After satisfying the Axiom 1, design simplicity is pursued by minimizing the information contents per Axiom 2, where the information content is defined as a measure of complexity. One popular measure of information content is *entropy* (Shannon 1948). FR entropy is related to the probability of satisfying its specification in the physical mapping (the *DP* in the process mapping).

Entropy and Information content can be mathematically expressed in different ways; the more useful measures are those that evaluate the probability of meeting design specifications, which is the area of intersection between the *design range* '*dr*' , (design specifications) and the *system range* '*sr*' , (process capability). The overlap between design range and system range is called the common range '*cr*'. The probability of success is defined as the area (probability) ratio of the common range to system range. The common measures are based on the logarithmic function: in probability the information related to an event of probability p is $I = \log_2 (1/p)$; on that concept we will base our Information content evaluation [Donnarumma, 1997].

When we formulate the Information Content for the Fuzzy Design approach we can declare that its measure is based not only on the "process capability", but also on the "agreement index" that expresses how much a DPs value has the capability to achieve a desired FRs value.

We also need to remember that in fuzzy logic formulation many membership functions contain irregular mathematical functions (i.e. Min and Max) that can uncouple an FR apparently dependent from more DPs.

The total information content is calculated composing the information content of each FR by rules that will be explained in the following paragraphs.

3 OUR APPLICATION: SANDWICH FLOOR PAN STRUCTURAL DESIGN

Our application is based on a study made in cooperation with FIAT Automotive Industry, for designing a new kind of floor-pan for Hybrid propulsion vehicle, in order to allow the containment of the energy accumulators; we've thought to design a "sandwich floor pan", formed as a space-frame, obtained combining cross-bars (with closed section) that form an internal frame and that are closed up and down by two plates and sideways by two backstays. In this way we obtain a volume for containing energy storage elements, like electrical accumulators, in respect of good habitability and static and dynamic constraints such as common engineering rules suggest.

That application was developed in Dept. of Mechanical Engineering of University of Salerno, by using a Fuzzy evaluation method; results of that work were presented in two papers in 2000 and in 2001[Naddeo].

3.1 SHORT DESCRIPTION OF SPACE-FRAME

The space-frame is provided with an access to accumulators in the upper side, so the first imposed constraint is the presence of some opening space in the upper part of the same, in order to allow maintenance; this constraint involves several transformation of initial sandwich space-frame idea (first hypothesis) into a hive composed by a reinforced-by-ashlars base (whose study doesn't concern this paper), over hanged by a grating frame with longitudinal development (second hypothesis: continuous cross-bars in the best impact adsorption's direction, in case of frontal impact) with some transversal cross-bars in order to stiffen the space frame on plane. The space-frame is been conceived as composed by "U" formed steel sheet, upper-closed by a grating covering structure in order to form a closed-section, as shown in fig.1.

The so composed space-frame is a hive divided into "cells" each of that can contain a number variable from four to six accumulators (third hypothesis). For our application, and in particular considering available space, that on average, on the pavement of a "B" segment vehicle is of 1500X1500 millimeters (fourth hypothesis), we can individuate four possible configurations that allow positioning a maximum of 24 accumulators, and we can choose between four possible dispositions of the accumulators into, for each of them.

The size of accumulators, useful to supply the propulsion system, is shown in fig.2.

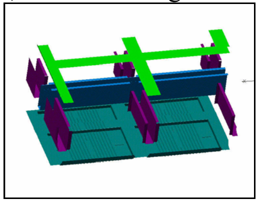


Fig.1: Assembled hive space-frame

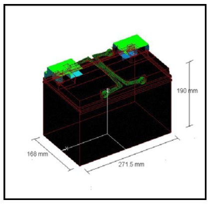


Fig.2: Accumulator

When we've designed the prototype of our space-frame, we have formulated several hypotheses in order to allow the good availability of supply system and the access to accumulators for ordinary and extra-ordinary maintenance.

The space-frame conceptual model, so assembled, has to be placed under the habitability-scheme, typical of a "B" segment vehicle; we can make some change to this scheme because it has to be adapted to a servo-controlled rear-wheel drive vehicle; by overlapping this scheme to the space-frame we can deduce the accessibility to accumulators, that is a parameters related to configuration of elements that makes the space-frame and to disposition of accumulators into the frame.

3.2 LOADS ON SPACE-FRAME

Vehicle floor pan, object of our study, is subjected to the common torsional (1000 Nm applied) and flexional (1000 Nm applied) test, in order to define, as concentrated parameters, the torsional and flexional stiffness as normally happens for the space-frame of new-marketed vehicles (and not for obtaining an accurate map of physical strength on the same space-frame).

Our space-frame is also subjected to another test of strength and deformability under critical dynamic load condition of 3g (three times gravity acceleration) applied on accumulators (standard condition suggested by homologation normative, in case of hole, pebble or hump on the road); for this test the space-frame is clamped on front and rear side, with an applied load by four rigid plate for simulating accumulators. For space frame base-plate we can calculate Von Mises equivalent stress, strain and maximum deformation under described load condition.

3.3 FINITE ELEMENTS MODELING AND LOAD SIMULATION

In modeling phase only a half of space-frame is analyzed for obvious symmetry reason, Cad modeling is made by parametric/variational CAD system Think Design (Think3), thanks to which we are able to plan, in parametric way, the generic generating profiles of model surfaces, in order to obtain, by extrusion, the longitudinal and transversal CAD surfaces, making the trim operation if needed.

This solution makes possible to draw in Think Design and to export, in IGES format, mathematics of surfaces versus the FEM pre-processor Hypermesh™ (Altair Inc.) for FEM generation; also the FEM model was parametric.

All Fem simulations were made in static condition using ANSYS™ FEM solver.

4 PROBLEM FORMULATION

4.1 DESIGN PARAMETERS

The spatial configuration of internal cross-bars (that are integrant part of vehicle chassis) and accumulators' spatial placing in available space requires a careful optimization research both in functional character and in structural behavior since they involve vehicle dynamic, accessibility and overall dimensions problems in presence of dynamic stresses due to external factors.

Chassis CAD and FEM models are parameterized with three design variables:

1. Accumulators placing coordinate
2. Car-platform cross-bars placing position
3. Internal cross-bars number in car-platform

Several hypotheses are formulated about each of these respecting good engineering rules, and finally the better 16 configuration were individuated; those were tested and evaluated in order to find the best one. Detailed description of structural hypotheses is in the paper [Naddeo, 2001].

4.2 FUNCTIONAL REQUIREMENTS

Final Design Goal is assessed by three requirement measures:

FR1: Structural requirement: Containment of maximum Von Mises stress under fatigue limit for chosen material (FeP04, Fe355), granting a good safety margin too, for stress and strain under dynamic load; good values of torsional and flexional stiffness, equal or greater than common vehicle of the same segment values.

FR2: Dynamic requirement: Minimization of space-frame weight in order to not increase yet heavy vehicle; equal-distribution of weights depending on accumulators' positioning. Hoping to reach the value of 50%-50% as weights distribution between front and rear axles; concentration of heavy elements around the centre of gravity, in order to diminish inertia values to improve comfort and driving performance.

FR3: Accessibility to accumulators requirement: this performance can be defined through four characteristics that ideally compose it: number of parts that have to be dismantled, time and costs of dismantling operation to have an access to accumulators, dismantling or not wiring into central tunnel and possibilities to have an easy visual access to accumulators in order to control their working status.

In the plate optimization not numerically quantifiable characteristics such as part's accessibility and numerically determinable (by FEM simulation) structural values such as stresses and strains come into play; presence of not precisely computable factors has suggested fuzzy-logic application at optimization problems.

Since several years, in fact, fuzzy-logic is applied, obtaining good results, to mechanical design; using this logic we can evaluate different design alternatives putting into account not precisely computable factors involved as ergonomic, aesthetic and functional ones.

Membership function values for FRs have the same meaning of the quantifying of the common range as overlap of design and system ranges: when a DP value changes we can imagine that a variation of the probability distribution of design range, with unaltered system range probability distribution, happens [El-Haik, 2000].

The FR value associated to a DP domain value, by membership function, wants to represent the agreement value (also called agreement index) and so the quantification, in Fuzzy domain, of the overlapping between design range and system range.

When we make that formulation, we can quantify the Information content of a design solution using the membership values as the quantified value of common range between probability distributions.

5 FUZZY CHARACTERISATION

5.1 MEMBERSHIP FUNCTION CONSTRUCTION

Membership functions corresponding to requirements are defined in following way:

Requirement *"structural performance"* for our application torsional (mf11) and flexional (mf12) stiffness of floor pan is very important parameters for finding the best solution among the hypotheses; we have to make the choice by evaluating the hypotheses each other, because we've not reference parameters into bibliography; however we're sure that (cfr. Thesis of Alessandro Naddeo) our space-frame stiffness is double in comparison with classical ones; dealing with dynamic stress parameters coming from acceleration on accumulators (mf13), we have to not override fatigue limit stress for our material (steel FeP04 with $\sigma_{fatigue} = 0.8\ \sigma_{yielding} = = 0.8*173.5\ N/mm^2 =$ $=138.8\ N/mm^2$). We prefer to adopt a good safety factor, and we have to consider, as optimum solution, that whose factor is about 1.4. Equivalent Von Mises stress is the evaluated one and is calculated as interpolation of nodal average stress in FEM model; as regards strain and maximum displacement under dynamic loads, experience suggests to keep displacement of vehicle chassis' parts, due to external dynamic stresses, in the same order of magnitude compared with sheet-steel thickness; that membership function (mf14) is shown in fig.4. All those membership functions are built by the formula:

$$mf = 1 - \frac{bestvalue - value}{bestvalue} \qquad (1)$$

that is the best for the comparison of values each other.

Requirement "*dynamic performance*": these performances are evaluated through three parameters:

- weights distribution (mf21), that has to be, as possible, near the value of 50%-50% between front and rear axle, that is considered the best solution, in order to improve comfort and handling capabilities.
- inertia values (mf22) calculated around the centre of gravity, have to be as small as possible; for this parameter, for that kind of vehicle, we have not other values for making a comparison with, so we have to consider the best solution (smaller) as the reference parameter, and then calculate the value of membership by the following formula (2)
- weight of space-frame (mf23) has to be as small as possible in order to reduce the vehicle weight; best solution is the classical space-frame's one, and the worst is his double value because of not industrial suitability.

For all above mentioned parameters we have defined the membership function by the formula:

$$mf = 1 - \frac{bestvalue - value}{bestvalue} \qquad (2)$$

Requirement "*accessibility*": this parameter can be defined through three sub-parameters:

- the first one (mf31) takes into account the number of parts and the difficulties of dismantling the internal parts of vehicle in order to have an access to accumulators for ordinary and extraordinary maintenance: membership function value is defined in hundredth, assigned by a score in function of following scheme:

Table 1. Dismantling difficulty score

Front seats, guides and lining	30
Rear seats, guides and lining	40
Central supply tunnel	20
Translation of accumulators for their dismantling	4
Width of trapdoor to be dismantled for extracting accumulators	6

Scores of this kind of job are assigned according to official tables of ANIA (Associazione Nazionale fra le Imprese Assicuratrici) [2000] and with the help of a team of three experts (two coachbuilder and one mechanic), superimposing the cad profiles of space-frames on habitability scheme for a "B" segment vehicle as shown in the Fig.3. Results are shown in the paper [Naddeo, 2001].

- the second one (mf32), takes into account the space to allow the extraction of accumulators and that is expressed as the percentage of covering of accumulators' surface by the trapdoor in the upper part of floor pan; the best solution is the one in which this covering reaches exactly the value of 100%
- the third one (mf33) takes into account the direct visual accessibility to accumulators for their status control (ordinary maintenance) without extract them; the best solution is the one in which all the accumulators are visible without translation operations.

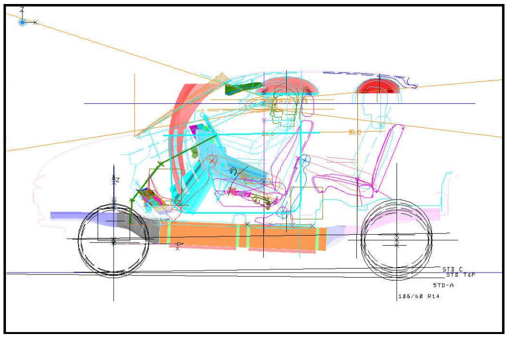
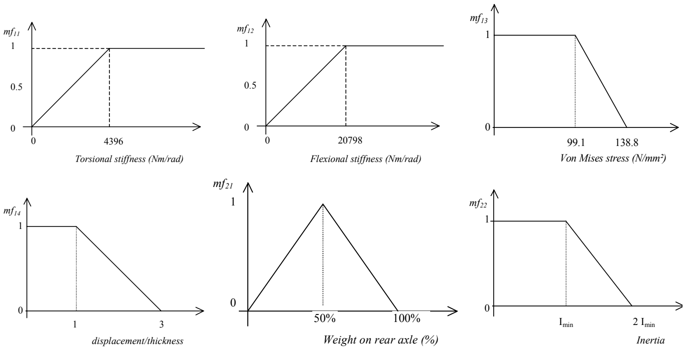


Fig.3: Overlapping of habitability scheme on our space-frame

Those three membership functions, now defined, are expressed by the formula

$$mf = \frac{value}{bestvalue} \qquad (3)$$

All the membership function graphs are shown in the fig.4.



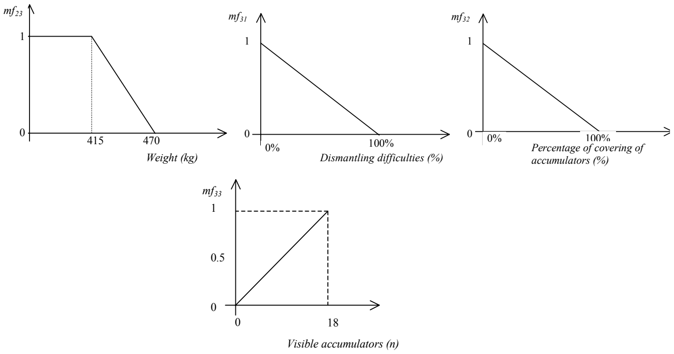


Fig.4: Membership functions

When all membership function are defined, we have to combine them by composition rules. In our research we suppose that the best "Objective function" is represented by the following composition rule, involving all the above-mentioned consideration and expert ones (4).

$$
\begin{aligned}
\mathrm{MEAN}\{0.40\,\mathrm{MEAN}[\;& 0.40\,\mathrm{MIN}(mf_{11}, mf_{12}),\\
& 0.60\,\mathrm{MIN}(mf_{131}, mf_{132}\}\;],\\
0.35\mathrm{MEAN}[\;& 0.80\,\mathrm{MIN}[mf_{21},\mathrm{GMEAN}(mf_{22a},mf_{22b}, mf_{22c})],\\
& 0.20\,mf_{23}\;],\\
0.25\,\mathrm{MEAN}[\;& 0.60\,mf_{31},\\
& 0.40\,\mathrm{GMEAN}(mf_{32}, mf_{33c})],\quad \} \qquad (4)
\end{aligned}
$$

Using that formula we can make a comparative evaluation simply maximizing the Objective function and we can make the first selection of the best "Fuzzy" result for our Concept design choice [Antonsson, 1992 – Cappetti, 1998]. In the following table 2 we can find the first classification and identify the best solution: the B1 solution.

6 FUZZY LOGIC IN AXIOMATIC DESIGN

Fuzzy logic was recently applied as a measure and a choice method in design; the design development requires, as shown, maximization of its objective function. Each requirement is evaluated by DP dependence (*mf*) or by direct expert's judgment.

Difficulties in application of fuzzy logic are especially in identifying requirements and requirement's dependence on DPs, and in defining the objective function, such as in Axiomatic Design.

Table 2. Fuzzy evaluation scores

	torsion	bending	stress accum.	displac.. accu.	centre gravity	Ixx	Iyy	Izz	weight	access.	Trap-door	visual control	
	mf_{11}	mf_{12}	mf_{131}	mf_{132}	mf_{21}	$mf_{22\,a}$	$mf_{22\,b}$	$mf_{22\,c}$	mf_{23}	mf_{31}	mf_{32}	mf_{33}	score
A3	0.317	0.332	1.000	0.569	0.894	0.775	0.638	0.476	0.919	0.000	0.750	0.667	0.495
A2	0.317	0.332	1.000	0.569	0.927	0.775	0.638	0.476	0.919	0.060	0.750	0.667	0.504
C2	0.280	0.271	1.000	0.531	0.948	0.644	0.767	0.886	0.929	0.260	0.830	0.556	0.555
C4	0.280	0.271	1.000	0.531	0.892	0.644	0.767	0.886	0.929	0.200	0.830	0.778	0.559
A4	0.317	0.332	1.000	0.569	0.869	0.775	0.889	0.985	0.919	0.040	0.750	1.000	0.587
C3	0.280	0.271	1.000	0.531	0.892	1.000	0.965	0.886	0.929	0.200	0.830	0.778	0.596
C1	0.280	0.271	1.000	0.531	0.948	1.000	0.965	0.886	0.929	0.260	0.830	0.556	0.608
A1	0.317	0.332	1.000	0.569	0.952	0.775	0.889	0.985	0.919	0.460	0.750	0.667	0.637
B3	1.000	0.968	1.000	0.556	0.893	0.963	0.749	0.490	0.930	0.200	0.750	0.667	0.652
D4	0.998	1.000	1.000	0.568	0.875	0.629	0.652	0.669	0.916	0.200	0.750	0.889	0.654
D2	0.998	1.000	1.000	0.568	0.933	0.629	0.652	0.669	0.916	0.260	0.750	0.778	0.657
B2	1.000	0.968	1.000	0.556	0.927	0.963	0.749	0.490	0.930	0.260	0.750	0.667	0.661
D3	0.998	1.000	1.000	0.568	0.875	0.984	0.850	0.669	0.916	0.200	0.750	0.889	0.702
D1	0.998	1.000	1.000	0.568	0.933	0.984	0.850	0.669	0.916	0.260	0.750	0.778	0.706
B4	1.000	0.968	1.000	0.556	0.868	0.963	1.000	1.000	0.930	0.240	0.750	1.000	0.719
B1	1.000	0.968	1.000	0.556	0.951	0.963	1.000	1.000	0.930	0.660	0.750	0.667	0.789

6.1 INDEPENDENCE AXIOM

If the independence axiom is respected, the fuzzy system is well formed, but frequently fuzzy requirements depend on many DPs; if the design problem is rebuild with independent FRs, those FRs are very difficult to evaluate. So we need to assert that the proposition "FRs are independent" must to be true in the fuzzy sense of the word to consider the approximation of a fuzzy system; this independence is named *weak independence*.

In our problem we've made a Fuzzy formulation of the problem in order to deploy the original FRs in several sub-FRs expressing the dependence between FRs and DPs by membership function. First design matrix is the following:

Table 3. First hypothesized design matrix

	DP1	DP2	DP3
FR1		X	X
FR2	X	X	X
FR3	X	X	

The Fuzzyfication of the problem allows us to create a new design matrix as follows:

Table 4. Design matrix for fuzzyfied problem

	Torsional-stiff	Flexional-stiff	Von Mises stress	Displ/Thick	Rear weight	Inertia	Weight	Difficulty	Covering	N° accumulator
FR-mf11 - torsional	X									
FR-mf12 - flexional		X								
FR-mf13 - fatigue			X							
FR-mf14 - max displac.				X						
FR-mf21 - weight distrib.					X					
FR-mf22 - inertia value						X				
FR-mf23 - weights							X			
FR-mf31 - dismantling								X		
FR-mf32 - extraction									X	
FR-mf33 - visual access										X

We can note that, because of the particular formulation of the problem, initial FRs are not independent each other because each parameter affects more than one FRs.

The FRs decomposition in more sub-requirements, also formulated by membership functions on DPs domain, generates several sub-FRs that are completely independent in Fuzzy domain.

That kind of approach allows to respect automatically the Independence Axiom.

It's very important, but it's no matter of this paper, to investigate what is the real physic sense of fuzzyfication operation on the independence concept. We can also formulate a new design matrix that allows to express the defuzzyfication operation and to link the Fuzzy DPs domain to the phisics' one (number and position of longitudinal and transverse cross bars, accumulators X and Y positions, accumulators orientation, width and number of trapdoors, grouping of accumulators). That operation, made, for our problem, by an expert engineering team when the 16 hypotheses were formulated, have to be evaluated by independence axiom, after the defuzzyfication problem; it will be the next step of our research and represents the future development.

6.2 INFORMATION AXIOM

We've now to formulate any hypotheses to measure the information content of fuzzy design; in the past we've proposed to compare the membership value with the probability value [Pappalardo, 1998]:

- Information in probability $\log_2 (1/p)$

- Fuzzy information $\log_2 [1/mf(x)]$ (we've used the natural logarithm, obtaining information measure in nats)

When a decision is made in the traditional design process, a straight information is provided; when a decision is made in the fuzzy design process, the information provided is not completely significant because of the imprecision inherent the fuzzy evaluation. So it is not correct to add information supplied by DPs in order to obtain the total information value; moreover every information contribution has different weight in final evaluation.

Now we want to calculate the global information content provided by each alternative by a composition law similar to the fuzzy composition one in which the membership value is replaced by the logarithm of his reciprocal.

It is easy to verify that if $mf(DP_i) = 1$ for each i (i.e. when we are sure of design quality) then $I_{tot} = 0$.

For the composition operator we suggest to follow those rules:

- when the composition operator is "MIN" it means that the information for the worst mf is higher and so in the information composition rule we have to adopt the "MAX" operator;
- when the composition operator is "MAX" it means that the information for the best mf is lower and so in the information composition rule we have to adopt the "MIN" operator;
- when the composition operator is "MEAN" it means that the information for the composed solution is weighted on two or more mf and so in the information composition rule we have to adopt the "MEAN" operator too;
- when the composition operator is "GMEAN" it means that the membership function value is made worse by each sub-mf; in that case we cannot use a rule for the information because of the single domains of the sub-mf maybe different or cannot be super-imposed; for our work we've calculated the information content for the composed mf obtained using Gmean operator, by the following formula:

$$\text{Information} = I = \log [1/\mathrm{GMEAN}(mf_i)] \qquad (5)$$

New Information composition rule became the following:

$$\begin{aligned}
\mathrm{MEAN}\{0.40\ \mathrm{MEAN}\{ & \ 0.40\ \mathrm{MAX}[\ln(1/mf_{11}), \ln(1/mf_{12})], \\
& \ 0.60\ \mathrm{MAX}([\ln(1/mf_{131}), \ln(1/mf_{132})] \quad \}, \\
0.35\ \mathrm{MEAN}\{ & \ 0.80\ \mathrm{MAX}\{\ln(1/mf_{21}), \\
& \qquad \ln[1/\mathrm{GMEAN}(mf_{22a}, mf_{22b}, mf_{22c})]\}, \\
& \ 0.20 \ln(1/mf_{23})\}, \\
0.25\ \mathrm{MEAN}\{ & \ 0.60 \ln(1/mf_{31}), \\
& \ 0.40 \ln [1/\mathrm{GMEAN}(mf_{32}, mf_{33c})]\} \ \} \qquad (6)
\end{aligned}$$

Using that formula we can evaluate the information content of each design solution and select the best one (whose information content is lower).

Obtained results are shown in the following table:

Table 5. Axiomatic evaluation scores

4,985086	A3
0,917056	A2
0,861883	A4
0,706049	C4
0,683518	C2
0,660914	C3
0,621461	C1
0,524859	D4
0,524392	B3
0,512699	A1
0,492181	D2
0,485037	B2
0,458336	D3
0,425658	D1
0,419251	B4
0,262192	B1

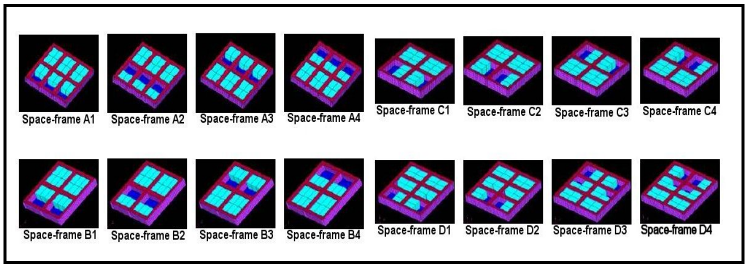


Fig.5: Examined spatial configuration

7 EVALUATION AND CONCLUSION

From Tab.1 results that B1, the best fuzzy alternative, has very low information content and it is the best hypothesis in the axiomatic context too.

The global classification of the hypotheses is lightly modified, but we can note that the three best solutions remain the same and remain in the same order too.

This indication means that our design is well-proposed and the choice is very robust both with fuzzy and with axiomatic formulation. Also the problem formulation seems to be very robust.

Shown procedure allows also to weigh impact of different design parameters on final scores (objective function); this characteristic becomes very important during vehicle prototyping and design planning stage, because allows to make planning choice on "new problem" dividing it in simple known sub-problems; we have also the opportunity to choose the best value or values range in which parameter can change, involving consideration on not calculable and not quantifiable parameter, constrained to technical and practical difficulties in realization, and to production and maintenance difficulties and costs.

The power of that method is that the fuzzy formulation for relation between FRs and DP makes the FRs automatically independent, while the content of information can be evaluated such as for the classical axiomatic approach. Another powerful characteristic is that we can make how many decomposition we want for the FRs: if we continue to use the fuzzy formulation for linking FRs to DPs we obtain always a set of independent FRs.

We also can introduce several FRs that normally are not quantifiable because Fuzzy approach allows to express them also with membership function, using the same domain used for quantifiable requirements.

Obviously we have to choose the best fuzzy formulation in order to not violate the physic meaning in the Fuzzyfication operations.

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The use of axiomatic approach and fuzzy problem formulation together, allows to select a good design, especially in concept design phase, in which most of the choices are still to be made.

Future work will be concentrated on deepening the meaning of independence when axiomatic formulation is adapted to fuzzy design problem. We've shown that at higher definition level, in Fuzzy domain, the relationship between FRs and DPs can be considered independent each other, but at lower level, after the defuzzyfication of the problem, independence may be not so immediately considered if an engineering team does not support the hypothesis selection.

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