

## DESIGN DECOUPLING METHOD BASED ON PARA-COMPLETE LOGICS

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### ABSTRACT

A primary “must” of axiomatic design theory is the first axiom, stating that independence of functional requirements should be maintained throughout the design process.

Para-complete logics, such as Fuzzy logic, give us a powerful instrument to express “mathematical/functional” interaction between FRs and DPs, especially when this interaction cannot be expressed by a precise “mathematical function (i.e. the case in which we want to express several data from VOC (Voice of Customer) investigation, building an FR for a defined design performance), and so can be codified only using “Linguistic variables”.

Para-complete logics, among which the Fuzzy logic is, for us, the most powerful, violate the principle of the excluded third party, so that the effects of DPs’ changes on the same FR can be considered partially independent each other.

Our paper wants to investigate changes in Decoupled Design’s concept when para-complete logics are applied in FRs-DPs link definition.

We want to evaluate the impact of decoupling capability of designer using composition rules on FRs, in order to make the design matrix diagonal or lower triangular by decoupling effects of several DPs on different FRs using Fuzzy formulation.

Para-complete logics, mathematically codified, have also the instruments for Information Measurement; so that we can evaluate different designs using the Axiom of Information (minimize the information content).

This formulation can extend the concept of decoupled design in Axiomatic design matter.

**Keywords:** Axiomatic Design, Para-complete Logics, Decoupling Methods

### 1 INTRODUCTION

With increasing demand for shorter development time and higher quality, design effectiveness has received growing attention from both academia and industry. In industry, unsatisfactory design results in a great number of process iterations, so improving the effectiveness of design is crucial in order to shorten product development time and lower costs. The goal of

effective engineering design is to minimize unnecessary process iterations. To reduce the probability of design failures, systematic approaches have become the trend to efficiently realize designs in recent decades.

Since 1990 the research Group of University of Salerno, headed by Prof. Antonio Donnarumma and Prof. Michele Pappalardo has introduced the Systematic approach to Design in Mechanical Engineering, especially using Fuzzy logic approach and Entropy based approach. The Axiomatic Design (AD) method proposed by Suh (1990) represents, for us, a powerful approach that provides a systematic guideline for evaluating the acceptability of designs, so we have imagined to use that approach in Concept Design phase, and support it, in Independence and Information evaluation, using Fuzzy logic.

That approach would be very useful when designer has to work with large and complex systems, in which several kinds of couplings are still considered acceptable in practice. This is due to the fact that some couplings are weak and have little influence on the design outcome so they can be ignored, in particular conditions, in order to proceed with fewer interactions, thus expediting the design process.

In light of this situation, our research intends to develop methods for measuring functional dependency and, if possible to develop decoupling methods based on the use of para-complete logics, such as Fuzzy logic, in engineering design in order to improve the design process.

The objectives of this research are twofold:

- (1) To investigate changes in Decoupled Design’s concept when para-complete logics are applied in FRs-DPs link definition.
- (2) To evaluate the impact of decoupling capability of designer using membership function formulation, in order to make the design matrix diagonal or lower triangular using Fuzzy formulation, and also using  $\alpha$ -cut methodology.

### 2 LOGICS AND AXIOMS IN DESIGN

Several methods were studied for helping design choices in concept design, and several mathematical instruments are useful for that topic. In University of Salerno a new approach, based on

the use of Fuzzy Logic, was experimented for design problems as explained in following paragraphs.

## 2.1 FUZZY-ANALYSIS FUNDAMENTALS

Linguistic inexactness (imprecision) is the most common feature of many real life situations. Dutta (1985) classifies imprecision according to its source: measurement, stochastic, ambiguous definitions, incomplete knowledge, etc. In decision making, for example, the usefulness of mathematical algorithms is in having clearly defined objective criteria and constraints for evaluate the Information content.

Certainty formulations require structure with precise parameters. However, most real life situations are characterized linguistically with degrees of imprecision. Precision implies no ambiguity by assuming that variables, parameters, and structure represent deterministic situations. The imprecision issue is further complicated in the classification of concept design.

In the early phases, a design is a collection of scattered conceptual thoughts and rough drawings. The difficulty in design problem formulation often has in establishing precise objectives, constraints as functional requirements which are uncertain, do not fall between what we consider as definite and precise.

All Design matter is not deterministic, but has to be used to make deterministic assertion and to take deterministic decisions.

The first used approach is the use of probability theory to handle randomness.

In customer oriented design, customers have wants and needs that are hard to interpret. They are expressed, linguistically, using terms which have no precise definition. A statement is not always right or wrong; in such cases solution can be found using the logics that violate the principle of the excluded third party, like Fuzzy logic.

The dichotomous property is the basis of classical set theory but we cannot use it because, for complex systems, a property may be viewed as a continuous measure of some possibility distribution.

The Fuzzy logic, based on L. Zadeh theory [1965-1974], allows to express in mathematical terms several not precisely defined concepts; unlike of binary logic, that logic does not require that a proposition assumes a defined truthful value, true or false, but allows to assign a membership value (between 0 and 1) to truthfulness of it. Generally we can declare that an element satisfy a requirement [Klir, 1995] even if this requirements has a not clearly sense, giving to it a membership value in the range  $\{0 - 1\}$ .

An example that may be used to facilitate the fuzzy concepts is as follows. Assume that there are 3 design proposals (solution entities); say the crisp set S (S1, S2 and S3).

We would like to select a solution entity at random from S. The probability distribution in this case is:

$$p(\{S1\}) = p(\{S2\}) = p(\{S3\}) = 1/3.$$

If we were asked to select randomly a successful creative design, we can't use the probability distribution above because of the fuzziness in the word 'Successful'. The answer is in defining 'design solution', say Y, as a variable that takes in values in the set S, according to a probability distribution constructed around the proposition "Y is successful".

A fuzzy set accepts objects with certain degree, the so called membership function (Zadeh 1965). The fuzzy set A is

represented as:  $A = \{(FR, \mu_a(FR)) / FR \in FRs\}$  with  $m_f(FR)$ , understood to represent a mapping of membership of

$$FR, m_f / FRs \rightarrow [0,1], FR \rightarrow m_f(FR) \quad (1)$$

It is understood that in the crisp case,  $\forall FR \in A, \mu_a(FR) = 1$  and zero otherwise. Every mapping of this nature with some conceptual realization (in alignment with intuitive semantics of imprecise description of FR) is a fuzzy set. For example, FRs can be the universe of fuzzy functional requirements, such as stylish, cheap, convenient, etc.

In Design process, it's very important to underline the key role of mapping process between what we want to achieve and how we want to achieve it: using that definition we can declare that Design problem formulation start from Functional requirements (FRs) and Design parameters (DPs) identification.

The Fuzzy logic approach helps designers to identify the relationship between FRs and DPs, to formulate a judgment on several design hypothesis and compare different concept design solutions each other, putting into account exact, not precise and not quantifiable requirements, thanks to the formulation explained in (1).

The concept of membership function plays a key role in that approach: FRs can be correlated, by membership function, to DPs that characterize the project while FRs for a project's "element" can be decomposed into simple ones (sub-requirements) directly depending from design parameters; this operation allows to decompose complex property, associated to a requirements, in simple ones, and to combine each other by fuzzy membership function composition laws [Scott-Antonsson, 1998].

In our approach, defined for Mechanical Design, but extendible to all Design problems, we've to define the Design Goal through all requirements opportunely weighted or composed by simple rules. Those rules can be combined each other in order to create an Objective Function (OF) that provides all design aspects. The design process finishes with the formulation of several design hypotheses.

Each hypothesis is evaluated and makes a score defined by the final composition rule. The score expresses the membership value to the chosen objective; the best design solution will be naturally chosen among ones which have the best score [Naddeo, 1999 - Antonsson, 1992].

After the complete characterization of the design problem, by identification of FRs and DPs, the second, and most important step of fuzzy formulation, is the Fuzzyfication of the problem, and so the definition of the membership functions ( $m_f$ ) and of the evaluation rules.

There are a lot of papers in literature dealing with membership function definition [2, 11, 21], their construction and methods of composition; for our application we will use several simple  $m_f$  such as triangular, trapeziform and simple mathematical function, for evaluating quantifiable parameters, while, for evaluating several not quantifiable requirements, we will use the "one expert direct method"; for the last one we need to give to readers a brief explanation:

"One expert direct method" allows to directly assigning a membership value for each of examined alternatives, in comparison with other methods that indirectly (by membership

$$\begin{aligned} \{FR\}_{mx1} &= [A]_{m \times r} \{DP\}_{rx1} \\ \{DP\}_{rx1} &= [B]_{rxn} \{PV\}_{nx1} \end{aligned}$$

function) make this operation [Naddeo, 2001]. The first step for that method is the interview with an expert that gives a judgment for each design solution; after that, his evaluation is expressed in terms of adjectives (that modify the truthfulness value of a proposition) or by collocating the alternative in a predefined list, in which several kind of judgment are provided. Finally, for each alternative, the judgment is transformed in membership value by using a table of predefined correspondence judgment $\leftrightarrow$ value.

Once the membership functions are defined, they have to be combined by composition rules; some of these are: minimum rule, maximum rule, arithmetical average rule, geometrical average rule. The first of those is applied in evaluating requirements that have to be necessarily satisfied, and assigns, to requirements, minimum of obtained scores among all; the second is applied especially when at least one of the requirements has to be satisfied, and assigns to element the maximum among scores; arithmetical average is applied when requirements interact each other compensating themselves, and assigns to the element a score calculated as weighted average of single requirements scores; geometrical average is applied when every judgment on design's requirement makes worse the final one.

That rules are used to define the Objective Function for evaluating the Designs' hypotheses.

Finally the Design Problem requires a Defuzzyfication, in order to extract the physical values of DPs from Fuzzy formulation.

## 2.2 AXIOMATIC DESIGN

Motivated by the absence of scientific design principles, Suh (1990 - 2001) proposed the use of axioms as the scientific foundations of design. Out of the twelve axioms first suggested, Suh introduced the following two basic axioms along with six corollaries that a design needs to satisfy:

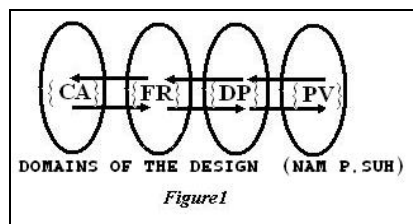
Axiom 1: The Independence Axiom

*Maintain the independence of the functional requirements*

Axiom 2: The Information Axiom

*Minimize the information content in a design*

In axiomatic design approach, the engineering design process is described in Figure 1, in which the array of functional requirements ( $FR_s$ ) is the minimum set of independent requirements that completely characterizes the design objective based on customer attributes ( $CA_s$ ). Design is defined as the creation of synthesized solution to satisfy perceived needs through the mapping between the  $FR_s$  in the functional domain and the design parameters ( $DP_s$ ) in the physical domain and through the mapping between the  $DP_s$  and the process variables ( $PV_s$ ) in the process domain.



The physical and process mappings can be expressed mathematically as

where  $\{FR\}_{mx1}$  is the vector of independent functional requirements with  $m$  components,  $\{DP\}_{rx1}$  is the vector of design parameters with  $r$  components,  $\{PV\}_{nx1}$  is the vector of process variables with  $n$  components,  $A$  is the physical design matrix, and  $B$  is the process design matrix.

For our purposes, the mapping process can be mathematically abstracted as the following matrix equation:  $\{FR\} = [A]\{DP\}$ , where  $FR$  is the array of  $FR_s$ ,  $DP$  is the array of  $DP_s$ , and  $A$  is the design matrix that contains  $FR_s$ - $DP_s$  relationships. The process mapping is described by:  $\{DP\} = [B]\{PV\}$  but doesn't affect our problem formulation.

Axiom 1 states that the design parameters ( $DP_s$ ) and the functional requirements ( $FR_s$ ) have to be related such that a specific  $DP$  can be adjusted to satisfy its corresponding  $FR$  without affecting the other functional requirements, which will require that  $A$  should be either a diagonal matrix or triangular matrix.

After satisfying the Axiom 1, design simplicity is pursued by minimizing the information contents per Axiom 2, where the information content is defined as a measure of complexity. One popular measure of information content is *entropy* (Shannon 1948).  $FR$  entropy is related to the probability of satisfying its specification in the physical mapping (the  $DP$  in the process mapping).

Entropy and Information content can be mathematically expressed in different ways; the more useful measures are those that evaluate the probability of meeting design specifications, which is the area of intersection between the *design range* ' $dr$ ', (design specifications) and the *system range* ' $sr$ ', (process capability). The overlap between design range and system range is called the common range ' $cr$ '. The probability of success is defined as the area (probability) ratio of the common range to system range, i.e. the common measures are based on the logarithmic function: in probability the information related to an event of probability  $p$  is  $I = \log_2 (1/p)$ ; on that concept we will base our Information content evaluation [Donnarumma, 1997].

When we formulate the Information Content for the Fuzzy Design approach we can declare that its measure is based not only the "process capability", but also on the "agreement index" that express how much a  $DP_s$  value has the capability to achieve a desired  $FR_s$  value.

We also need to remember that in fuzzy logic formulation many membership functions contain irregular mathematical functions (i.e. Min and Max) that can uncouple an  $FR$  apparently dependent from more  $DP_s$ .

## 3 METHODS OF MEASURING THE COUPLING STRENGTH

In order to overcome the shortcoming of the binary design matrix in AD wherein the dependencies between  $FR_s$  and  $DP_s$  are shown as binary measures, an effective method for extracting the quantifiable measure of couplings is needed.

In this section, we review three methods that are capable of transforming qualitative information into quantitative measures.

They are: fuzzy techniques, utility theory, and analytic hierarchy process.

Introduced by Zadeh (1965), fuzzy theory has been applied to many areas such as control, decision making, etc. Fuzzy theory explores the degrees of membership in extension to the binary properties of membership or non-membership in traditional set theory models. With the percentage of confidence, fuzzy technique can define crispy information in terms of some meaningful membership functions. This concept provides further decomposition of binary or crispy systems. Several researchers have used Fuzzy Decision Analysis in order to take decision based on non quantitative factors: (Liang and Wang [1991] for site selection and personnel selection, Ghotb and Warren [1995] for Hospital information system, Naddeo and Cappetti [1999] for mechanical topology optimization, Antonsson [1998] for automotive structural optimization.

Such applications for decision-making have shown the capability of Fuzzy approach to handle multi-criteria decision problems with qualitative information. The method could be also appropriately used to quantify the binary measures. However, the membership functions for FRs and DPs are case dependent and need justifications accordingly. The membership functions for FRs and DPs are not easy to obtain because many iterative evaluations will be involved in validating the selected membership functions. Further, a systematic procedure for generating the membership functions is not present in fuzzy theory. That problem is overridden using, for such analysis, the One Expert Method (before explained).

Utility theory, which has been applied for decades, has its strength in multi-criteria decision problems. It is a method for assessing the worth of a particular alternative that a decision maker attaches during the decision process. Von Neumann and Morgenstern (1947) and Savage (1954) introduced the axiomatic foundations for utility theory. The lottery method developed by Keeney and Raiffa (1976) enables the utility model to determine the individual attribute utility functions and weighting/scaling factors. This makes the utility model a more precise approach than fuzzy technique in determining the attribute/membership functions. In brief, the FRs and DPs can be seen as attributes or criteria and the coupling strength serves as weight.

This advantage suits the utility model to measure the binary coupling strengths. Despite its outward appearance of mathematical precision, the lottery type utility function requires a measurable unit for the attributes or criteria. However, FRs and DPs do not always have explicit measurement units.

To overcome the effect of conflicting information from different disciplines on the design, the Analytic Hierarchy Process (AHP) method developed by Saaty (1980, 1990)) is capable of prioritizing qualitative information using a pair wise comparison technique. It has been demonstrated to be a suitable method for the selection of the functionally most appropriate components of technical systems. AHP has been applied to various areas of multi-criteria decision and conflict solving problems showing the power of the method.

These merits qualify AHP as a solid approach to assessing the binary coupling strengths. AHP uses the pair wise comparison technique to obtain the quantifiable measures for competing elements. It enables the evaluation for comparison consistency and it does not require the explicit unit for the attributes or

criteria. AHP method for evaluating coupling strength and for implementing decoupling method was utilized by Su, Chen, Lin (2003) with good results; we want to make the same using Fuzzy Logic.

#### 4 DECOUPLING IDEA BASED ON MEMBERSHIP FUNCTION ANALISYS

Our research starts from a basic hypothesis: relationship between FRs and DPs has to be considered as flexible. In fact concepts about coupled, decoupled and uncoupled design are often explained without considering (during problem definition phase at least) the real influence of the design parameters on requirements.

When we have to choice among several different design solution, it's very useful and interesting to measure the accordance of a design solution to the FRs using a membership value (typical of Fuzzy approach); that's approach is suggested especially when we consider FRs that suffer the user's or customer's subjectivity ("a car has to be capable to move itself" is a proposition that express an objective FR while "a car-seat has to be comfortable" express a subjective FR).

This hypothesis allows to investigate several optimization methods also for coupled design; in order to explain our approach we will formulate a simple example: *We have to design a new car bonnet.*

Problem analysis takes us to define the following FRs:

FR<sub>1</sub> = Style

FR<sub>2</sub> = Accessibility to the engine compartment

FR<sub>3</sub> = Pedestrian safety

A stylistically pleasant shape for our bonnet is a must for the front of the car because it affects the aesthetic pleasure, the driver visibility and the aerodynamic property (the last affects also the petrol use and the vehicle silence).

Our bonnet has to be designed in order to allow the access to the under-bonnet compartment, for ordinary and extraordinary maintenance for a front wheel drive vehicle (that has the engine under the bonnet) and for loading and unloading operation for a rear wheel drive (rear engine) vehicle.

Pedestrian safety is also to be taken into account by vehicle designer because since 2005 all vehicles will be homologated (in Europe) only if will pass the Pedestrian tests for evaluating the aggressiveness of the front of the vehicle towards pedestrians. The vehicle-aggressiveness strongly depends from the shape and the configuration of the bonnet over which the pedestrian could impact.

For all the above statements, we can define the Design Parameters for the bonnet:

DP<sub>1</sub> = Stiffness

DP<sub>2</sub> = Shape

DP<sub>3</sub> = Opening system (opening compound levers and safety opening hook mechanism)

As we can see in the following Fig.2, a car-bonnet is constituted by an external skin (a), a reinforcement frame (b) and an opening system (compound levers and safety opening hook mechanism) (c). External skin outlines the bonnet shape; the reinforcement skin gives to the bonnet the required stiffness; compound levers and hook mechanism allow to open the bonnet, when needed, and to lock it during the drive.

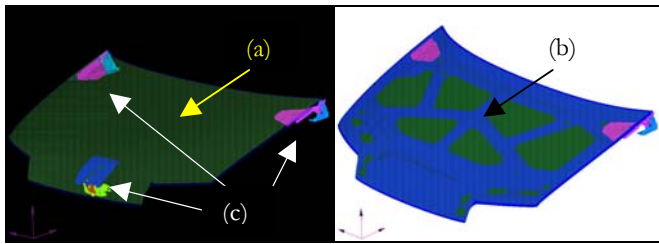


Fig.2 Car-bonnet

We can now to define the relationships between FRs and DPs:

The style is obviously influenced by the bonnet shape; the opening system is always positioned under the bonnet but shrewdness is needed in order to hide the compound levers and the hook; we have to pay attention also to clamp points with the car-frame because of the interference with other sheet metal parts.

Bonnet stiffness is due to reinforcement-frame shape and doesn't affect bonnet aesthetic.

Accessibility to the engine compartment is affected by position of compound levers and also by shape and dimension of the bonnet; the bonnet has to be so stiff that it can support itself, independently from the way by which it has to be mounted on the car; the stiffness doesn't affect the accessibility.

Pedestrian safety is obviously affected by all DPs considered because the impact of the head of pedestrian may happen in several different parts of the bonnet, and injuries to pedestrian are seriously dependent by the local bonnet stiffness (the compound lever zone and the hook zone are the hardest for pedestrian impact).

Based on the last statements the Design matrix can be expressed as follows and the Design is evidently coupled:

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} 0 & X & X \\ 0 & X & X \\ X & X & X \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix} \quad (2)$$

Now we've to make a more deepened consideration on FRs and on DPs in order to understand the "importance level" of the impact of a variation of Design Parameters on different FRs and understand how our coupled Design can be decoupled.

The accessibility, which certainly is improved by particular bonnet shapes, is strongly affected by the position of the bonnet during the opening operation, and by compound levers position. For example we can think to design the opening system in such way to be able to remove the bonnet by appropriate joints; in that way we can consider the accessibility as independent from bonnet shape.

At the same time if the compound levers and the hook are positioned completely under the bonnet, they don't affect the style.

It's evident that the concept of "coupled design" has to be deepened. We are saying that a procedure that allows to compare how much  $DP_i$  affects  $FR_i$  is needed; it's also important to investigate if we can select a particular sub-domain (in DP

functional domain) in which we can choose a DP value without affecting the FR value in order to consider a design as decoupled or uncoupled. The coupled design problem can become a good-constrained, decoupled design.

It's necessary to establish a coupling measure that allows to evaluate not numerically quantifiable parameters (such as a "good shape" that affects the style; it's an aesthetic problem not quantifiable). That aspect is treated in a paper from the same author Naddeo (2004).

Necessity to evaluate not quantifiable parameters makes indispensable to use a methodology based on a logic system that, using the linguistic or the comparative approach, allows to do that.

In this paper we are illustrating how we can use the Fuzzy logic for designing a car-bonnet (Designing will mean the individuation and the characterization of macro-values of chosen DP). Expressed considerations can be applied at the same time to every decisional processes in which the existent link between FRs and DPs makes a design coupled.

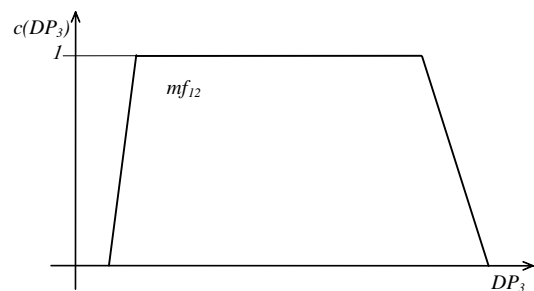
The first step, starting with Fuzzy approach, is the detailed analysis of the relationship between FRs and DPs for evaluating the "satisfaction" value of the proposition "the bonnet is  $FR_i$ " for each FR, on the DPs domain. The satisfaction value will be expressed by the value of membership, whose significance was before explained, to the Fuzzy set individuated for the evaluated proposition.

For example when we ask ourselves "how much the bonnet is safe for pedestrians" when the stiffness of the reinforcement frame varies or "how much the engine compartment is accessible" when the compound levers position varies, we can build the correspondent membership function, defined on the physical domain of DPs.

Our coupled design matrix, expressed by membership function, becomes the following:

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1-mf_{12} & 1-mf_{13} \\ 0 & 1-mf_{22} & 1-mf_{23} \\ 1-mf_{31} & 1-mf_{32} & 1-mf_{33} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix} = \underline{MF} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix} \quad (3)$$

If a weak-dependence between a FR and a DP exists, it's expressed by a  $mf_{ij}$  like a trapezoid one, for which the "satisfaction range" is wider when the dependence between FR and DP is lower. We've underlined that if, for example, the safety opening hook mechanism is mounted under the bonnet, the aesthetic satisfaction doesn't change when the joint position and the hook type varies, while the accessibility to the hook varies: that membership function is expressed by the following fig. 3



**Fig.3 -  $mf_{12}$  almost all solutions with under-bonnet hook have high aesthetic satisfaction**

It's now evident that since  $mf_{ij}$  are defined, we have quantified the dependence between DPs and FRs, but we can encounter three kinds of possibilities:

- when a  $mf_{ij}$  has a value identically equal to zero (0) then the  $FR_i$  cannot be never satisfied, so we've to redefine the DPs values
- If a DPs range for which the membership function is equal to one (1) exists, then the correspondent member of the design matrix became zero: that value means that we can choose, in that range, what value we want for  $DP_i$  without affecting the  $FR_{k \neq i}$  eventually  $DP_i$  dependent.
- If  $mf_{ij}$  has a value too different from 0 or 1 we come back to the original coupled design matrix.

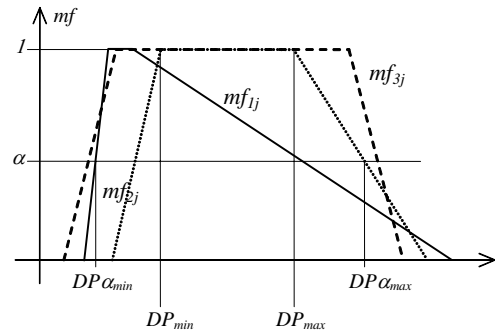
As we've formulated the elements of design matrix, we can usefully evaluate the impact of DPs variation on FRs: we can analyze the Design matrix searching for the "validity range" of DPs, expressed in Fuzzy way as satisfaction index, and evaluating the dependence level.

Basing our reasoning on what we've considered, we can generally follow those steps:

- 1) Individuation of FRs
- 2) Individuation and choice of DPs
- 3) Design matrix building
- 4)  $mf_{ij}$  definition for each couple  $FR_i - DP_j$
- 5) Elaboration of MF matrix
- 6) Individuation of decoupling DPs range (i.e. DPs range in which for one  $DP_i$  all the  $mf_{ij}$ , with  $i \neq j$  has value equal to 1)
- 7) Definition of Fuzzy constraints on DPs domain
- 8) Fuzzy decoupling of the problem and re-organization of Design Matrix
- 9) Defuzzification of the problem and transformation of DPs domain constraint in physical constraints
- 10) Compatibility verification of Physical constraints with Design goal and development.

Another powerful method we can use, accepting a weak approximation (that is a natural way to operate in Fuzzy set definition), is the widening of "fuzzy independence ranges" by  $\alpha$ -cut operation: the  $\alpha$ -cut allows to however consider satisfactory the solutions for which all the  $mf_{ij}$ , with  $i \neq j$  have values greater than a value  $\alpha$ , lower than 1, chosen by designer.

For example, if you see the Fig. 4, for  $mf_{ij}$ , with  $i \neq j$  independence range increase itself form  $[DP_{min} - DP_{max}]$  to  $[DP_{\alpha_{min}} - DP_{\alpha_{max}}]$ , simply using the  $\alpha$ -cut operation.



**Fig.4 - membership functions e  $\alpha$ -level cut**

That kind of operation can allow to define a right sequence of optimization of FRs: if we consider a different value of  $\alpha_i$  for each  $DP_i$  we can reduce each  $\alpha_i$  in order to encounter a situation in which every  $DP$  is important for only one  $FR$ . Repeating that procedure for each  $FR$  we can obtain the ideal optimization sequence and so a decoupled design.

Let's make an example to better explain the whole procedure; this example will deal with our Bonnet design.

For that design steps 1), 2) and 3) of procedure were been yet explained; now we've to define the  $mf_{ij}$  that express the relationship between DPs and FRs.

For doing that we will use the one-expert method, asking to experts something about our problem; methods for constructing Fuzzy  $mf_{ij}$  are explained in scientific literature (Klir -1995, Scott-Antonsson – 1998).

Example function will be created in order to depend only from one physical parameter for better explain the method, even if in reality the relationship are often more complex;

In fact for our bonnet we will consider that the bonnet shape depends only from width and that the opening system is defined only by the distance between compound levers and hook mechanism. The stiffness is calculated only using reinforcement frame information, not depending from other parameters.

In the Fig. 5 are  $mf_{12}$ :

- a bonnet is aesthetically satisfactory if it's not too small or too big ( $mf_{12}$ )
- if its width is at least such as the engine one, it easily allows the access to engine compartment, but if the width is too big, the engine will be positioned too far from the bonnet frontal edge, so becoming difficult to reach ( $mf_{22}$ )
- Pedestrian safety increases when the width is greater ( $mf_{32}$ ).

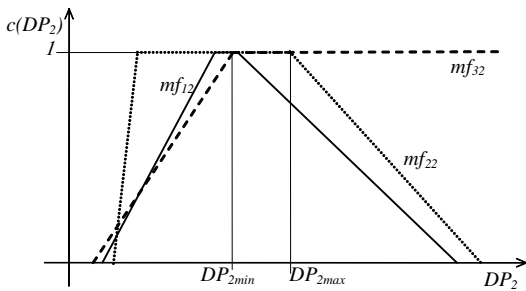


Fig.5 *m.f.* FRs-DPs

Step 6, by analysis of the  $mf_{ij}$ , allows the definition of eventual ranges in which DPs can vary without changing the value of more than one FR.

As we can see in the Fig.4, we've described dependencies of several FRs in function of  $DP_2$ .

If we choose a  $DP_2$  value in the range  $[DP_{2min}, DP_{2max}]$ , chosen by individuating the max overlap between  $mf$ , we can optimize the  $FR_1$  without taking into account what happens to  $FR_2$  and  $FR_3$ .

The same process can be made for  $DP_3$ , for which a range  $[DP_{3min}, DP_{3max}]$ , in which it can vary without affecting the other FRs, could be defined.

After the seventh step (Fuzzy constraints definitions) the problem becomes uncoupled with the following design matrix:

$$\begin{Bmatrix} FR_3 \\ FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix} \quad (4)$$

With  $DP_1$  defined without constraints and  
 $DP_2 \in [DP_{2min}, DP_{2max}]$   
 $DP_3 \in [DP_{3min}, DP_{3max}]$

## 5 CONCLUSIONS

Membership function values for FRs have the same meaning of the quantifying of the common range calculated as overlap of design and system ranges: when a DP value changes we can imagine that a variation of the probability distribution of design range change, with unaltered system range probability distribution, happens [El-Haik, 2000].

The FR value associated to a DP domain value, by membership function, wants to represent the agreement value (also called agreement index) and so the quantification, in Fuzzy domain, of the overlap between design range and system range.

The application of dependence concept, evaluated by Fuzzy logic, allows to operate with a rigorous method, if possible, in order to optimize coupled design for which is impossible to define an uncoupled or a decoupled version. The method explained allows to improve the design objective simply evaluating good constraints for Design parameters.

The powerful of  $\alpha$ -level cut has to be investigated because it can play a fundamental role in design development and optimization; it will be explained in the future works.

We just want to remember that when we make a Fuzzy formulation, we can quantify the Information content of a design solution using the membership values as the quantification of common range between probability distributions, so evaluating the project also by second axiom.

## 6 ACKNOWLEDGMENTS

This paper come on the follow of several papers published in last seven years by the Design methods research Group of University of Salerno that shows the possibility to use fuzzy logic as a design method.

The use of axiomatic approach and fuzzy problem formulation together allows to select a good design, especially in concept design phase, in which most of the choices are still to be made.

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