**Proceedings of ICAD2011**
**The Sixth International Conference on Axiomatic Design**
**Daejeon – March 30-31, 2011**
ICAD-2011-04

# SOLUTION FOR COUPLED DESIGN IN AXIOMATIC DESIGN USING THE LDU METHOD

**Jeonghun Ahn**
akandl@yonsei.ac.kr
Department of Mechanical Engineering
Yonsei University
262 Sengsanno Seodaemun-gu
120-749, Seoul, South Korea

**Young Ho Kim**
toga05@yonsei.ac.kr
Department of Mechanical Engineering
Yonsei University
262 Sengsanno Seodaemun-gu
120-749, Seoul, South Korea

**Sung Woon Cha**
swcha@yonsei.ac.kr
Department of Mechanical Engineering
Yonsei University
262 Sengsanno Seodaemun-gu
120-749, Seoul, South Korea

## ABSTRACT

When the design processes is executed, an existing axiomatic design becomes a coupled design when the Independence Axiom (Axiom 1) is unsatisfied. If needed, designers are supposed to redesign from the beginning, which can be very troublesome. The LDU method, presented in this paper, is an easy way to revise design problems and proceed with design processes. The LDU method adopts the LDU decomposition method from linear algebra in axiomatic design, which is applied to the design matrix. When the design matrix is determined to be a coupled design, the design matrix is decomposed into a lower triangular matrix, diagonal matrix, and upper triangular matrix. These decomposed matrices are converted to decoupled designs through Functional Requirements (FRs*) and Design Parameters (DPs*), which are obtained by the FRs and DPs of the coupled design. In order words, the LDU method can convert a coupled design into an uncoupled design through the FRs* and DPs*.

**Keywords**: axiom, LDU method, decomposition

## 1 INTRODUCTION

### 1.1 THE INDEPENDENCE AXIOM IN AXIOMATIC DESIGN [Suh,1990; Suh 2001]

An axiom is always considered to be true without any evidence of disproof or exceptions. In Axiomatic Design, there are two axioms that determine the design quality when designing products. The first axiom is the independence axiom, which maintains the independence of the FRs. The second axiom is the minimization of the design's information content. The first axiom is the one discussed in this paper. The independence axiom describes the independence of the functional requirements during the design processes. In other words, changing the DPs should maintain the independence of the FRs through concrete physical selections. Uncoupled designs, which allow for complete independence between the FRs and DPs, and path dependent decoupled designs, can solve design problems without altering the design plans. For the case of the FRs and DPs within a coupled design, there exist unique FRs, which lack flexibility and violate Axiom 1. Therefore, a coupled design is determined to be an unsatisfactory design and, therefore, needs to be revised.

### 1.2 LDU DECOMPOSITION [Anton and Busby, 2003; Thomas, 2006]

LDU decomposition is defined as a process to build a square matrix, "A", via the multiplication of three matrices, L (lower triangle matrix), D (diagonal matrix), and U (upper triangle matrix). Therefore, "A" can be expressed in terms of a 3X3 square matrix as shown in equation (1)

$$\overset{\text{A}}{\begin{Bmatrix} a_A & b_A & c_A \\ d_A & e_A & f_A \\ g_A & h_A & i_A \end{Bmatrix}} = \overset{\text{L}}{\begin{Bmatrix} 1 & 0 & 0 \\ d_L & 1 & 0 \\ g_L & h_L & 1 \end{Bmatrix}} \overset{\text{D}}{\begin{Bmatrix} a_D & 0 & 0 \\ 0 & e_D & 0 \\ 0 & 0 & i_D \end{Bmatrix}} \overset{\text{U}}{\begin{Bmatrix} 1 & b_U & c_U \\ 0 & 1 & f_U \\ 0 & 0 & 1 \end{Bmatrix}} \quad (1)$$

Using this method, the number of unknowns "n", can be solved using "n" linear equations in the form of Ax=b. If most matrices are composed of zero and nonzero components centered around a diagonal line, then the problem can be more easily solved using the Gauss-Jordan elimination method, which is the general solution for a linear system. Merits of the LDU decomposition method are used in axiomatic design when there are design problems due to the coupled design.

## 2 SOULUTION FOR COUPLED DESIGN

### 2.1 EXISTING PROCESS IN AXIOMATIC DESIGN

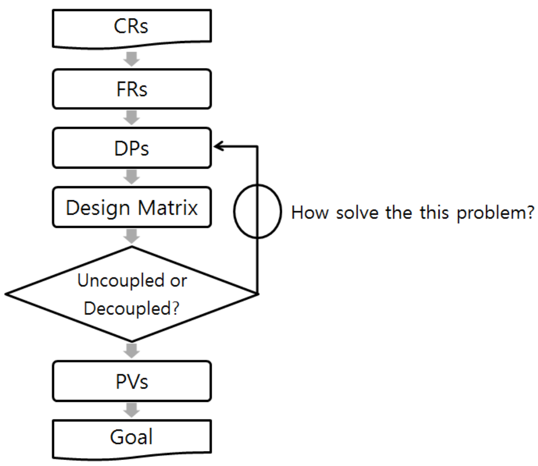


**Figure 1. Flow chart for the existing process in axiomatic design.**

The above flow chart demonstrates the existing design processes by employing Axiom 1. When Customer Requirements (CRs) occur, designers then set the FRs and the DPs, followed by the mapping process for both the FRs and DPs. Then, the design matrix is made and tested in order to determine whether it complies with Axiom 1. Upon compliance with Axiom 1, it follows a designated process. When the design matrix does not comply with Axiom 1, it selects new DPs followed by the mapping processes of DPs and FRs. Then, a revision is made to the uncoupled design or decoupled design so that the mapping results satisfy Axiom 1. This method demonstrates the designer's goal but does not provide a clear path for solving the given problem and is, therefore, considered to be this method's limitation.

## 2.2 LDU METHOD [Anton and Busby, 2003; Thomas, 2006]

The LDU method is a way to solve design problems through a design matrix obtained from the Mapping Results of the FRs and DPs within the Axiomatic Design and LDU-Decomposition procedure. Design problems can be approached with a 2X2 Design Matrix, as shown in Figure 2.

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ c/a & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & (ad-bc)/a \end{bmatrix} \boxed{\begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix}}$$

$$= \begin{bmatrix} 1 & 0 \\ c/a & 1 \end{bmatrix} \boxed{\begin{bmatrix} a & 0 \\ 0 & (ad-bc)/a \end{bmatrix} \boxed{\begin{Bmatrix} FR_1^* \\ FR_2^* \end{Bmatrix}}}$$

$$= \begin{bmatrix} 1 & 0 \\ c/a & 1 \end{bmatrix} \boxed{\begin{Bmatrix} DP_1^* \\ DP_2^* \end{Bmatrix}}$$

**Figure** 2. **LDU Method (in a 2*2 Matrix).**

When the mapping of the FRs and DPs is complete, it can be expressed mathematically via matrices as follows: FRs=A·DPs, A=L·D·U. and FRs=L·D·U·DPs. Designers using this mathematical formula set the FRs* and DPs*, and all design problems can then be converted to a decoupled design.

## 2.3 LDU METHOD –APPLICATION [Kim *et al.,* 2001; Hwang *et al.,* 2002]

When designers solve design problems using axiomatic design in compliance with Axiom 1 and get an uncoupled or decoupled design, they do not need to revise their designs and can instead move forward with their design processes. For the case of a coupled design, designers can easily revise their problem using the LDU method by setting the FRs*, satisfying FRs*=U·DPs and DPs* and satisfying FRs*=D-1·DPs*. Then, the designers get FRs=L·DPs* with newly-set DPs* and the design problem can be revised to a decoupled design. This is illustrated in the flow chart shown in Figure 3. Designers can provide solutions using the method shown in Figure 3 with coupled design problems.

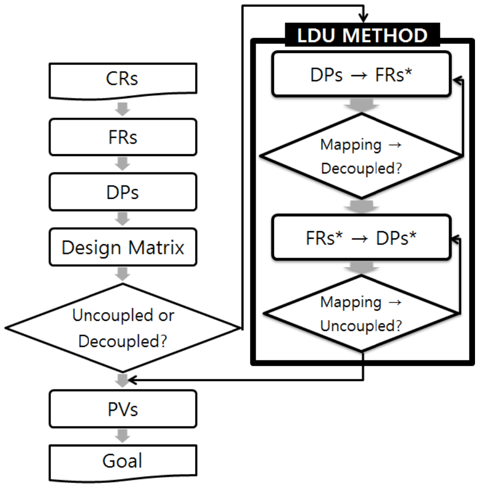


**Figure 3. Flow chart for the LDU method process.**

## 2.4 SCHEMATIC EXPRESSION

Figure 4 Shows a pictorial expression of the design issues given in Figure 2. Solution processes utilizing the LDU method are shown from Figure 5 to Figure 7. Figure 5 shows the pictorial presentation of the FRs* setting using DPs and the upper triangular matrix obtained through the LDU decomposition within the design matrix. Figure 6 depicts the DP's* setting using the previously determined FRs* and the diagonal matrix. Figure 7 shows the pictorial expression on how the final FRs, obtained through the lower triangular matrix, and DPs* were determined in the previous process, and can be solved for using the decoupled design.

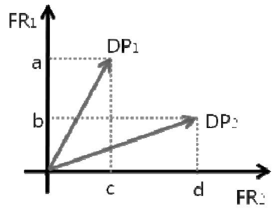


**Figure 4. Schematic diagram of FRs and DPs for a coupled design.**

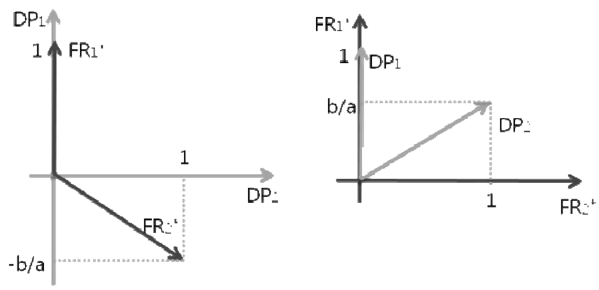


**Figure 5. Schematic diagram of FRs* from DPs under the triangular matrix. The diagram on the right is drawn with the coordinates transformed from the diagram on the left.**

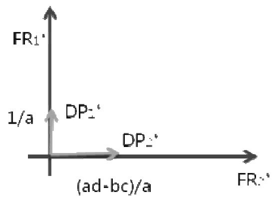


**Figure 6. Schematic diagram of DPs* from FRs* and a diagonal matrix.**

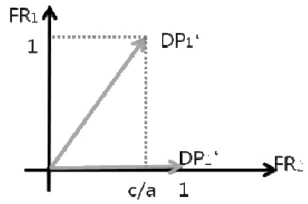


**Figure 7. Schematic diagram of FRs and DPs* for a decoupled design.**

## 2.5 MULTI-SOLUTION VIEW OF THE LDU METHOD

The LDU method shows a variety of solutions to designers who are having compliance problems with Axiom 1, since they are able to find all solutions in the LDU method concerning redesign problems. We are going to present three solution directions using the LDU method on design problems.

First, the LDU method suggests the direction of revisions for the DPs. This is the basic application of the LDU method. The first step of the design matrix, A, is expressed as three multiplied matrices via LDU decomposition. Then the FRs and DPs are converted to FRs* and DPs* and solving problems of FRs via DPs* can convert problems into decoupled design problems.

Second, the LDU method shows the FR revisions. Instead of using the whole LDU method process, it stops in the middle of the process and solves any issues. In other words, designers set new FRs* by using an upper triangular matrix and the DPs and get a new expression, FRs*=U·DPs. Then, problems are converted to decoupled design problems. Designers should confirm whether the changed FRs* can be set in the original CRs. Without this confirmation, designers get solutions which do not match the problems.

Third, design problems can be solved with a two-step process by setting the Goal* in solving coupled design problems. The goal is defined as the ultimate destination of the design which designers set in their designs.

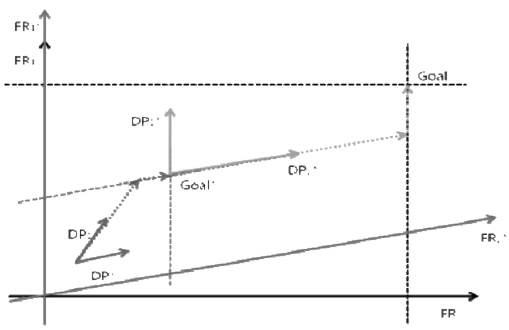


**Figure 8. Solution of the coupled design problem using a multi-goal.**

In the existing design problems, designers find their goal by finding the FRs in the given CRs, whereas the LDU method reaches its original goal via the FRs* and Goal* as determined by the upper triangular matrix and the DPs and not conjectured from the CRs. This is the method for solving the problems concerning coupled designs, and solutions are pictorially shown in Figure 8. Designers use the initial DPs to reach the Goal* of FRs*, which is set through the upper matrix and DPs, and then the diagonal matrix and FRs* are utilized to define the DPs*. The design problems are solved by reaching the initial goal.

## 3 APPLICATION – HYBRID CAR

Now we are going to apply the LDU method to a hybrid car. The car is equipped with a combustion engine, motor, and batteries, and, for the sake of its environmentally-friendly power source, it is termed a hybrid car. In designing a hybrid car, designers used the existing CRs, even though this car will be manufactured with two constraints as follows: environmentally friendly and low emission standard. These constraints are not CRs, and designers can design it like an ordinary car.

$FR_1$: Drive
$FR_2$: Start
$DP_1$: Combustion Engine
$DP_2$: Motor

However, two car designs are shown in Figure 9 as two matrices.

| Gas Car | Hybrid Car |
|---|---|
| $\begin{bmatrix} X & X \\ 0 & X \end{bmatrix}$ | $\begin{bmatrix} X & X \\ X & X \end{bmatrix}$ |

**Figure 9. Design matrix of automobile**

As can be seen in the design matrices, a motor in an ordinary car is a decoupled design and takes the function of starting a car, thus, the design processes can be executed with no revisions necessary to the original design. On the other hand, for the case of a hybrid car design, a motor has two functions, which are starting the car and employing driving mechanisms, and designers have problems due to the coupled design along with redesign issues.

By employing the LDU method presented in this paper, designers can revise their designs without any problems of axioms and can then execute their design processes.

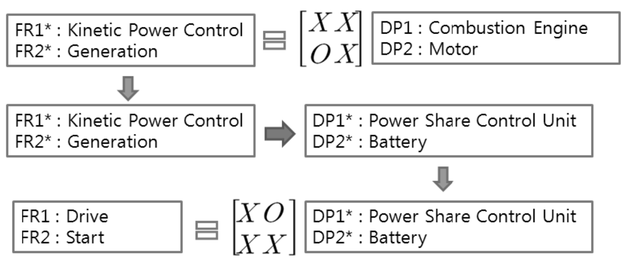


**Figure 10. Solution for a coupled design in hybridcar using LDU method**

Figure 10 presents how a coupled design can be transformed into a decoupled design by using a power sharing control unit and a battery, which are revised DPs* and determined by way of an upper triangular matrix, diagonal matrix, and lower triangular matrix within the hybrid car design.

## 4 APPLICATION – ONE TIME USE CAMERA

[Su *et al.* 2003]

The design problems of a one-time-use camera are adapted from Su *et al.* [2003]. Among the 6X6 design problems in the above mentioned journal, we will demonstrate how to solve coupled design problems with a 3X3 matrix by employing our LDU method. FRs, DPs, and related matrices are given below.

$FR_1$: Able to advance film

$FR_2$: Able to show the amount of film left or pictures taken

$FR_3$: Able to take picture

$DP_1$: Rolling mechanism

$DP_2$: Wheel counter mechanism

$DP_3$: Shutter mechanism

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} X\,X\,X \\ X\,X\,0 \\ X\,X\,X \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix}$$

**Figure 11. Design matrix of one time use camera**

As can be seen in the design matrix, the $DP_3$ item does not influence $FR_2$, which shows a typical coupled design form of the design matrix. The design problems in the original paper do not give concrete parameters for the DPs. Even with this design case, designers have to revise their designs in order to execute their design processes. The LDU method can solve this kind of design problem via revising designs and complying with axioms in the design processes, as shown in figure 12.

## 5 CONCLUSIONS

When executing the design processes using the existing axiomatic method, designers are supposed to redesign if their designs do not comply with Axiom 1. By utilizing the LDU method presented in this paper, designers are given the tools to discover ways to revise their designs without having compliance problems with Axiom 1. Also, this method will give designers solutions for coupled design problems when in the middle of the design processes.

However, this method is still is not ideal because when one of the FRs affects other DPs in the same way as when using the LDU method, its design matrix cannot use LDU decomposition like a coupled design case. The solution to this type of problem must be studied and developed in more detail.


## 6 ACKNOWLEDGEMENT

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0011337)


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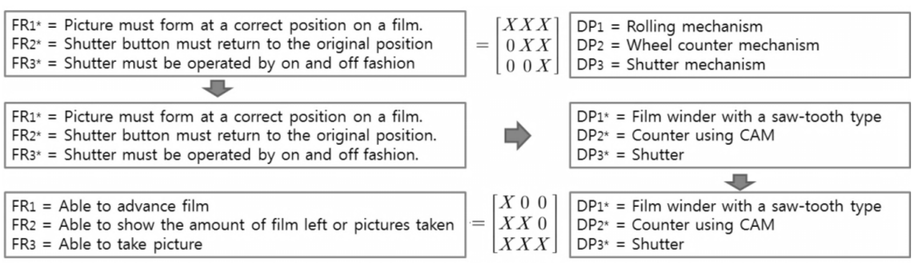


**Figure 12. Solution for coupled design in a one-time-use camera using the LDU method.**