

An Integer Programming Formulation For The Concept Selection Problem With An Axiomatic Perspective (Part I): Deterministic Formulation

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ABSTRACT

The concept selection problem is to select the 'best' conceptual design solution entity from a pool of feasible alternatives in the early concept design stage. The determination of a good selection criteria is a key for successful design release. In this paper, the concept selection problem is formulated as an integer programming problem. Complexity, value, cost and customer satisfaction are used to derive the objective function criterion. The mathematical form of the proposed criterion can be conveniently obtained by borrowing from the concepts of QFD, axiomatic design and value engineering. The criterion is then employed into our integer programming formulation which is expanded to include technical feasibility and assembly feasibility as constraints. The proposed formulation is sufficiently robust to adapt design situations with deterministic information (Part I) or fuzzy information (Part II).

Keywords: Axiomatic Design, Complexity, Integer Programming, QFD, Concept Selection

1 INTRODUCTION

The goal of engineering design is to create the design entities that satisfies the needs and delights of customers. The designer's creativity, experience and scientific knowledge are essential for developing good design entities. Usually, more than one conceptual entity will be conceived in a customer-based design assignment. The concept selection problem is to select the best design entity that not only satisfies the customer requirements but also outperforms the other alternative solutions based on a set of selection criteria. The selection problem involves the following three major steps: (1) identification of the selection criteria, (2) the ranking (scoring) of different design entities against the selection criteria, and (3) the identification of the 'best' (optimum) entity. The selection problem is a trivial problem when only one criterion is used. The best conceptual entity is the one that scores favorably in the ranking. However, the problem become more complex when multiple criteria are involved. In common industrial practice, the selection problem may become judgmental and exposed to bias as ranking will be driven to favor some pre-selected conceptual entity. The bias problem can be eliminated by

the systematic employment of a disciplined selection process. The process creditability and robustness are greatly enhanced when coupled with the state-of-the-art design theories.

In this paper, we propose to formulate the selection problem as an integer programming problem with, mainly, two selection criteria: customer satisfaction and design complexity. The choice of design complexity as a selection criterion is stemming from the information axiom (Suh 1990) of the axiomatic design (AD) approach. In addition, the proposed formulation is built around generic the conceptual framework of Quality Function Deployment (QFD).

This paper is developed as follows: Section 2 contains the needed background, Section 3 is the core section of Part I and is devoted for the deterministic formulation of the selection problem. Section 4 is the conclusion section.

2 BACKGROUND

2.1 Axiomatic Design

Motivated by the absence of scientific design principles, Suh (1990) proposed the use of axioms as the scientific foundations of design. Out of the twelve axioms first suggested, Suh introduced the following two basic axioms along with six corollaries that a design needs to satisfy :

Axiom 1: The Independence Axiom
Maintain the independence of the functional requirements

Axiom 2: The Information Axiom
Minimize the information content in a design

In axiomatic design approach, the engineering design process is described in Figure 1, in which the array of functional requirements (*FRs*) is the minimum set of independent requirements that completely characterizes the design objective based on customer attributes (*CAs*). Design is defined as the creation of synthesized solution to satisfy perceived needs through the mapping between the *FRs* in the functional domain

and the design parameters (DPs) in the physical domain and through the mapping between the DPs and the process variables (PVs) in the process domain.

The physical and process mappings can be expressed mathematically as

$$\{FR\}_{mx1} = [A]_{mxr} \{DP\}_{rx1} \quad (1)$$

$$\{DP\}_{rx1} = [B]_{rxn} \{PV\}_{nx1}$$

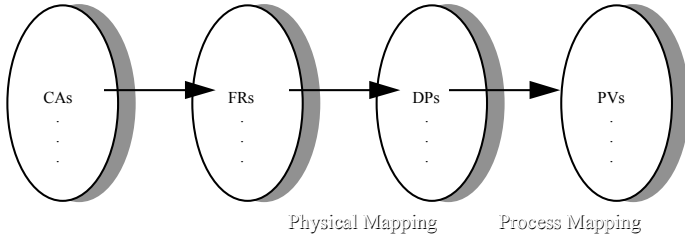


Figure 1. The design process mappings

where $\{FR\}_{mx1}$ is the vector of independent functional requirements with m components, $\{DP\}_{rx1}$ is the vector of design parameters with r components, $\{PV\}_{nx1}$ is the vector of process variables with n components, A is the physical design matrix, and B is the process design matrix. The mapping process can be mathematically abstracted as the following matrix equation: $\{FR\} = [A]\{DP\}$, where FR is the array of FRs , DP is the array of DPs , and A is the design matrix that contains the sensitivity coefficients of the FRs to the mapped-to DPs . The process mapping is described by: $\{DP\} = [B]\{PV\}$. The subsequent development uses the physical mapping for illustration purposes. Nevertheless, the results and conclusions are equally applicable to the process mapping as well.

Axiom 1 states that the design parameters (DPs) and the functional requirements (FRs) are related such that a specific DP can be adjusted to satisfy its corresponding FR without affecting the other functional requirements, which will require that A should be either a diagonal matrix or triangular matrix.

After satisfying the Axiom 1, design simplicity is pursued by minimizing the information contents per Axiom 2, where the information content is defined as a measure of complexity. One popular measure of information content is *entropy* (Shannon 1948). An FR entropy is related to the probability of satisfying its specification in the physical mapping (the DP in the process mapping). Entropy H can be defined as

$$H = -\log_v p \quad (2)$$

Where $v = 2(e)$, H is measured in bits (nats), p can be defined as the probability of meeting design specifications, which is the area of intersection between the *design range* ' dr ', (design specifications) and the *system range* ' sr ', (process capability). (see Figure 2). The overlap between design range and system range is called the

common range ' cr '. The probability of success is defined as the area (probability) ratio of the common range to system range, i.e.

$$\frac{p_{cr}}{p_{sr}} \quad (\text{Suh 1995-1996}). \text{ Substituting in Eq. (2), we have:}$$

$$H = \log_v \frac{p_{sr}}{p_{cr}} \quad (3)$$

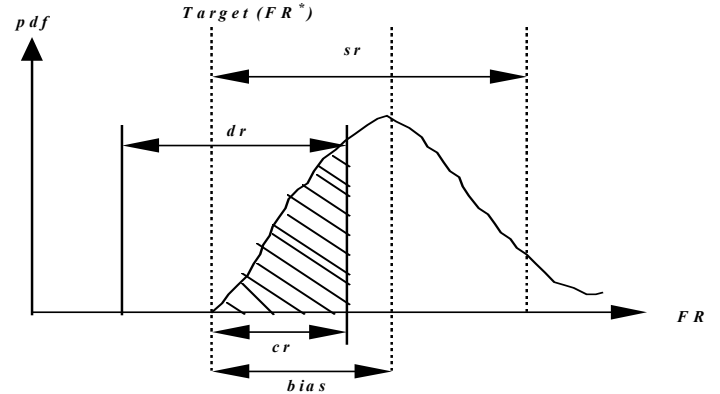


Figure 2: The probability of success definition

2.2 Design feasibility in concept selection problem

2.2.1 Modules

In practical design process, a product is made of several subsystems, or modules. Each module is designed to deliver an array of independent functional requirements (FRs). The physical entity of a module is a set of design parameters DPs grouped together in the form of a product.

Let a_i be the number of the independent FRs in the i^{th} module and V be the number of modules in the product, then total number of independent functional requirements, m , in the product should satisfy: $\sum_{v=1}^V a_v = m$. Assuming probabilistic

independence, and for any module, the information content of the module can be defined as:

$$\begin{aligned} H_{\text{module}_i} &= -\log_v p_{\text{module}_i} \\ &= -\log_v (p_1 p_2 \dots p_{a_i}) \\ &= -\sum_{j=1}^{a_i} \log_v p_j \end{aligned} \quad (4)$$

where P_j is the probability of success of the FR indexed $j, j=1, \dots, a_i$. H stands for entropy. In an independent design, each FR can be viewed as an stand-alone information source, or equivalently, a complexity source. Due to independence, the probability of success is multiplicative. Eq. (4) and it takes Eq. (5) as an average form

$$H = -\sum_{i=1}^m p_i \log p_i \quad (5)$$

can be generalized to quantify the information for the whole product where the summation is taken up to, m . The entropy described in equation (4) or (5) can be used as an index of complexity for evaluation of design alternatives. The smaller H indicates less complexity. It is obvious that H and hence overall design complexity can be reduced by maximizing the probability of success.

In practical design process, standard DPs (modules) are often used and those have higher probability of success. In addition, there are several advantages of using standard DPs . First, designers do not have to reinvent what have already exist so the design efforts can be saved. Secondly, the use of standard DPs will improve the quality and reliability levels.

2.2.2 Design feasibility

Clearly, a system or product is made of a number of modules. Each module can be made of a set of DPs . So the key decision in design is to select the groups of DPs that are design feasible. The design feasibility here has two aspects; first, the selected group of design parameters is able to deliver the FRs required by the module and it is called technical feasibility, and secondly, the selected group of DPs should be assembly and manufacturing feasible, which means that they can be grouped together with current manufacturing and assembly processes. The design parameters in this context are not limited to the intuitive assumption of hardware components, but rather as a generic physical instances that can be materialize by hardware, software, or fields. Also, the matrices that are used for feasibility assessment (see example) are mathematical representation of the mappings at the same level of decomposition.

Mathematical Formulation of Technical Feasibility

Let i be the index of FRs , $i = 1, 2, \dots, m$; k be the index of DPs , $k = 1, 2, \dots, K$; F_i be the set of potential (alternative) DPs of the functional requirement FR_i with cardinality N_i , and F be the union set of the overall unique potential DPs .

Example

A given design problem has the following arrays of design functional requirements where the symbol ' \rightarrow ' denotes the possible mapping between the FRs domain and the DPs domain.

$$\begin{aligned} FR &= \{FR1, FR2, FR3\} \\ FR1 &\rightarrow F_1 = \{DP1, DP2\} && \text{with } N_1=2 \\ FR2 &\rightarrow F_2 = \{DP1, DP3\} && \text{with } N_2=2 \\ FR3 &\rightarrow F_3 = \{DP1, DP4, DP5\} && \text{with } N_3=3 \end{aligned}$$

For example, $FR1 \rightarrow F_1 = \{DP1, DP2\}$ means that $FR1$ can be performed by either $DP1$ or $DP2$. It is also assumed that there is no duplication of identical DPs in each module. For example, $DP1$ can be used to deliver all $FR1, FR2$ and $FR3$ in above example. When we select this option, then the module will have

only one $DP1$. In general, to furnish each module, the union set $F = \bigcup_i F_i$, the set of unique potential DPs will be selected by dropping overlaps and its cardinality $K \leq N_1 \times N_2 \times \dots \times N_m$.

When a single DP serves more than one FR , and in general, $F_c \cap F_d \neq \emptyset$ will hold for some F_i pair, where c and d are two arbitrary functional requirements in the product, the coupling vulnerability may be created as the result of Axiom 1 violation. In other words, a potential coupling-free (independent) design solution can be achieved when $F_c \cap F_d = \emptyset$ for $c=1, 2, m-1; d=2, 3, \dots, m$, or for a sufficient subset of F that covers the FRs .

In last example, the set $F = \{DP1, DP2, DP3, DP4, DP5\}$ and $K=5$ ($\leq 2 \bullet 2 \bullet 3 = 12$). In addition, the mapping process can be coded mathematically via the variable

$$T_{ik} = \begin{cases} 1 & \text{if } FR_i \rightarrow DP_k \\ 0 & \text{otherwise} \end{cases}. \text{ Thus, the technology matrix } T_{m \times K} \text{ is}$$

defined as

$$T = \begin{matrix} & DP1 & DP2 & DP3 & DP4 & DP5 \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} & FR1 \\ & FR2 \\ & FR3 \end{matrix}$$

There are 12 solution combinations in this example and not all of them satisfies the independence condition. This condition is only satisfied by two overall solutions(S1, S2), each of which is a subset of DPs which can deliver all FRs and also satisfy the independence axiom.

$$T_{S1} = \begin{matrix} DP2 & DP3 & DP4 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & FR1 \\ & FR2 \\ & FR3 \end{matrix} \quad T_{S2} = \begin{matrix} DP2 & DP3 & DP5 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & FR1 \\ & FR2 \\ & FR3 \end{matrix}$$

Mathematical Formulation of Manufacturing & Assembly Feasibility

The assembly feasibility should be tested in the physical domain among the DPs themselves subsequent to the technology determination. The binary

$$\text{characterization } Z_{kl} = \begin{cases} 1 & \text{if } DP_k \rightarrow DP_l \\ 0 & \text{otherwise} \end{cases} \text{ denotes the}$$

assembly feasibility between pairs of the DPs . $Z_{kj} = 1$ indicates that DP_k and DP_j can be assembled together. Hence, a 0-1 assembly matrix $Z_{K \times K}$ can be constructed as follows. Clearly, the Z matrix is symmetrical, i.e. $z_{kl} = z_{lk}$.

$$DP1 \quad DP2 \quad DP3 \quad DP4 \quad DP5$$

$$Z = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} DP1 \\ DP2 \\ DP3 \\ DP4 \\ DP5 \end{matrix}$$

From Z , we can construct the following assembly matrices for the S1 and S2 solutions

$$Z_{S1} = \begin{matrix} DP2 & DP3 & DP4 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{matrix} \text{ and } Z_{S2} = \begin{matrix} DP2 & DP3 & DP5 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

It is obvious that S1 is only assembly feasible in two DP s: $DP3$ and $DP4$, while S2 is not assembly feasible at all. S1 is a technology feasible coupling free. It is some times inevitable to trade independence with feasibility as is the case with Solutions S3 and S4 below.

$$T_{S3} = \begin{matrix} DP1 & DP2 & DP3 \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} FR1 \\ FR2 \\ FR3 \end{matrix} \quad T_{S4} = \begin{matrix} DP1 & DP2 & DP3 \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$T_{S4} = \begin{matrix} DP1 & DP3 & DP4 \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix} \begin{matrix} FR1 \\ FR2 \\ FR3 \end{matrix} \quad T_{S4} = \begin{matrix} DP1 & DP3 & DP4 \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{matrix}$$

Note that S3 is assembly feasible because $DP3$ and $DP2$ are connected via $DP1$. There is no assembly related restrictions on S4. Once the feasibility criteria are satisfied, then the design's degree of coupling can be quantified using *Semangularity* and *Reangularity*, the axiomatic measures (Suh 1990).

3. THE CONCEPT SELECTION PROBLEM: CRISP FORMULATION

As the physical mapping process (product design) is performed, it is possible that a function may be mapped to many alternative physical entities (DP s) with each having its own customer perception, manufacturing processes, material variability, geometrical tolerance, and other physical attributes. Therefore, a DP is a complexity or an information source. In the mapping of interest, we would like to select the 'best' DP s (PV s) that satisfy the FR s (DP s) with the maximum customer satisfaction and minimal design vulnerabilities. To achieve this objective, the concept selection problem is formulated by using the framework of QFD and the axiomatic design principles.

In the QFD planning matrix (Figure 3), the product of customer *Attribute Value* (AV), targeted *Improvement Ratio* (IR) for a customer attribute (the row), and the *Sales Point* (SP) provides a weighted measure of the relative importance of this customer

feature, where SP is a measure of how the raw feature affects sales. The product is denoted AW (*Attribute Weight*). The other relative measure is the subjective cause-effect weight that a function (a column) may play in satisfying a customer attribute. For example, W_{ij} gives a measure of how much FR_i is related to CA_j . The summation of W_{ij} in each column is denoted here as FW (function weight), which gives a measure about how much this function is related to the overall customer attributes. The product of function weight (FW) of each function by the raw weight (AW) and sum over all the rows (customer attributes) on the right of House of Quality provides a measure of the relative importance of that function to the overall value for the customer.

For example, $\sum_{j=1}^J W_{ij} AW_j$ is a measure of customer perceived value for FR_i . Besides the customer perceived value, other design criteria should be also considered in design evaluation and selection. In this paper, we are interested in merging the complexity measure (Axiom 2) in the objective function of our integer program. The inclusion of complexity as an optimization criterion is justified because it relates many design criteria such as tolerance control effort, assembly and manufacturing cost, coupling among different DP s, etc.

For each functional requirement, FR_i , its complexity (entropy) is also related to which DP is selected to deliver it. If there are k DP s which can deliver FR_i , then there are k design instances. The entropy of FR_i at instance k ($FR_i \rightarrow DP_k$); $DP_k \in F_i$; $k = 1, 2, \dots, N_i$, can be denoted as H_{ik} . In addition, let j be the index of customer attributes; $j = 1, 2, \dots, J$, then the following value to information index:

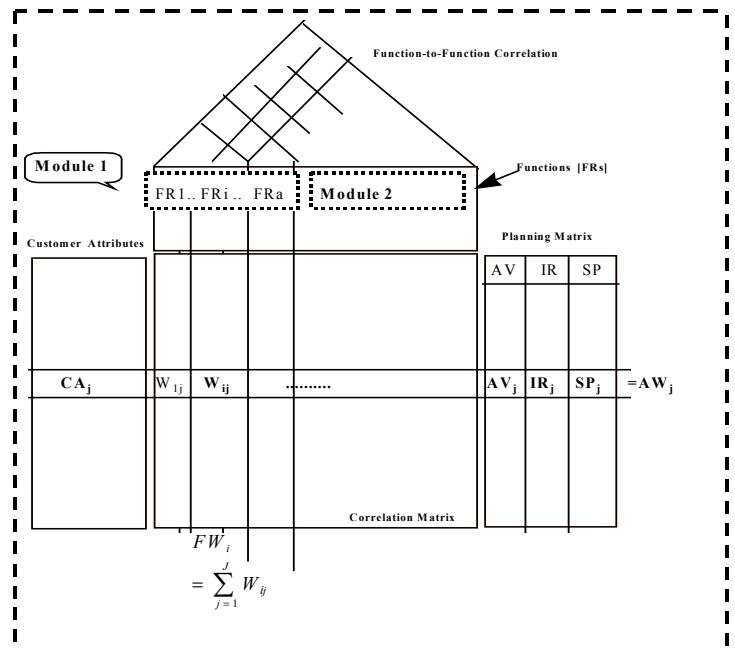


Figure 3. The QFD House of Quality at some design instance

$$VI_{ik} = \frac{\sum_{j=1}^J W_i AW_j}{H_{ik}}$$

can be used to evaluate the FR_i at the k th design instance, where H_{ik} can be computed by $H_{ik} = -p_{ik} \log p_{ik}$, and p_{ik} is the success probability of FR_i when DP_k is used. The larger the value to information ratio, the better the design is.

The weights in QFD matrix, that is, W and FW are more sensitive to particular DPs at different design instances. For example, electrical solution entities are usually highly rated in the 'convenience of operation' and 'ease of maintenance' attributes as compared to mechanical entities. Therefore, W_{ik} and FW_{ik} should be assessed with much attention.

Now, we can formulate the design concept selection problem as maximizing value to information ratio subject to technology feasibility and assembly feasibility constraints. Specifically, the concept selection problem can be formulated as the following integer programming problem:

$$Max. \frac{\sum_{i=1}^m \sum_{k=1}^K \sum_{j=1}^J W_{ik} Y_{ik} AW_j}{\sum_{i=1}^m \sum_{k=1}^K H_{ik} Y_{ik}} \quad (6)$$

Subject To:

$$\sum_{k=1}^K Y_{ik} T_{ik} = 1 \quad \forall i, i = 1, 2, \dots, m, DP_k \in F \quad (7)$$

$$\sum_{i=1}^m \sum_{k=1}^K \sum_{j=1}^J FW_{ik} Y_{ik} AW_j > \left(\sum_{i=1}^{m_d} \sum_{j=1}^J FW_{ik} Y_{ik} AW_j \right)_{datum} \quad (8)$$

$$\sum_{i=1}^m \sum_{k=1}^K H_{ik} Y_{ik} < \left(\sum_{i=1}^{m_d} H_i \right)_{datum} \quad (9)$$

$$Y_{ik} = 0 \text{ or } 1 \quad (10)$$

where m_d is the number of functions in the datum design and T_{ik} are the entries of matrix T . In this technology feasibility formulation, the decision variables are the binary variables Y_{ik} , where $Y_{ik} = 1$ indicates that DP_k is selected to deliver FR_i . The objective function is clearly the value to information ratio for the whole product, in which the numerator is the customer satisfaction index while the denominator is the design complexity level. This objective maximizes customer satisfaction while minimizing design complexity. Constraint (7) forces the selection of one solution entity per a given function. Constraints (8) and (9) translate the word 'best technology' into its mathematical definition. The 'best' selected design is therefore the design that outperforms the datum design from the perspectives of customer satisfaction and design simplicity.

Unfortunately, the above program (Eq.s: 6-10) does not eliminate the possibility of obtaining an overall assembly infeasible

solutions. An assembly infeasible solution can be achieved in one of two forms. First, all the DPs are assembly infeasible to each other or, second, some of the DPs are assembly feasible only at subsystem level and overall solution can not be synthesized. The assembly feasibility can be viewed as a *tour* between the selected design DPs where each is visited once starting from a DP of reference. Therefore, an overall assembly feasible solution is the one that has only one tour (loop) such that all sub-tours are eliminated. This reasoning is adopted from Traveling Salesman Problem (TPS) (Salkin & Mathur 1989). The program in Eq.s: 6-10 can be rectified to account for assembly feasibility when

augmented by $\sum_{i=1}^{m-1} \sum_{u=1+i}^m Y_{ik} Y_{ul} Z_{kl} \leq m-1, i \neq u$ where the binary

characterization Z_{kl} 's are the entries of matrix Z . An assembly-feasible design with m selected components is the one that has at most $m-1$ non-zero Z_{kl} 's. That is, in order to synthesize a solution, we need to satisfy simultaneously the technology requirement between a pair of functional requirements through the viable selection of DP_k for FR_i and DP_l for FR_u and the assembly requirement between DP_k and DP_l i.e. $Z_{kl} = 1$. This feasibility assurance process is expanded to all possible pairs of functional requirements. The use of this constraint prevents the selection of islands of physical entities that only assembly feasible at subsystem level.

This formulation can further be enhanced to include the value to cost performance index, PI . The formulation can be written as:

$$Max. \frac{\sum_{i=1}^m \sum_{k=1}^K \sum_{j=1}^J PI_{ik} W_{ik} Y_{ik} AW_j}{\sum_{i=1}^m \sum_{k=1}^K H_{ik} Y_{ik}} \quad (11)$$

Subject To:

$$\sum_{k=1}^K Y_{ik} T_{ik} = 1 \quad \forall i, i = 1, 2, \dots, m, DP_k \in F \quad (12)$$

$$\sum_{i=1}^m \sum_{k=1}^K \sum_{j=1}^J PI_{ik} FW_{ik} Y_{ik} AW_j > \left(\sum_{i=1}^{m_d} \sum_{j=1}^J PI_{ik} FW_{ik} Y_{ik} AW_j \right)_{datum} \quad (13)$$

$$\sum_{i=1}^m \sum_{k=1}^K H_{ik} Y_{ik} < \left(\sum_{i=1}^{m_d} H_i \right)_{datum} \quad (14)$$

$$\sum_{i=1}^{m-1} \sum_{u=1+i}^m Y_{ik} Y_{ul} Z_{kl} \leq m-1, i \neq u; i = 1, 2, \dots, m-1; \quad (15)$$

$$Y_{ik} = 0 \text{ or } 1 \quad u = 2, 3, \dots, m \quad (16)$$

Where PI_{ik} is the value-to-cost performance index of the function i at instance k .

The selected solution entity of the proposed framework will achieve higher performance of design requirements from a multi-disciplinary perspective. For example, from the perspective of value engineering, the selected optimum should possess a higher

total value than a datum design. Higher value performance is fostered by the process of eliminating unnecessary functions and delivering functional requirements with value-optimized physical solutions while customer satisfaction drives the selection process (Eq.s: 6 and 11).

The elimination or reduction of design coupling may result in added complexity. The use of additional DP_s to eliminate or reduce coupling may increase the overall design complexity because the cardinality, m , will increase. As formulated here, the entity's overall complexity takes m as an argument and is a function of the underlying probability distributions of the design parameters and/or process variables. The use of probability distributions indicate the case of the incremental design classification, i.e. experienced design situations with precise data that allow the calculation of H_{ik} . Incremental design is a design that is within a *slight* variation of the current design. In many design situations, especially those classified as creative design solutions, we do not have this luxury of information and H_{ik} can not be calculated. The type of information in the creative situation is qualitative and fuzzy in the form of engineering judgment. Another approach to assess complexity and other arguments in the integer program presented here is needed and should be based on fuzzy set theory. The fuzzy formulation is presented in Part II.

4. CONCLUSIONS

The concept selection problem can be solved using the integer programming formulation proposed here. The selection criteria include the complexity, customer satisfaction, and design value. Design complexity is measured by information content using Shannon entropy which in turn takes the probability of success as arguments. In incremental design situations, these probabilities can be quantified and to be used in the deterministic integer programming [Eq.s: (11)-(16)]. The formulation presented here produces an optimum, i.e. best selected entity that maximizes customer satisfaction, value, and simplicity within design and manufacturing feasible configurations.

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