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LINKING AXIOMATIC DESIGN AND TAGUCHI METHODS VIA INFORMATION CONTENT IN DESIGN

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ABSTRACT

Designing high quality products and processes at low cost has become an economical and technological challenge to producers in order to survive in todays competitive market place. In turn, engineers are forced to find systematic and efficient ways to meet this challenge. Axiomatic Design method is developed to answer this challenge by providing design axioms to place the product and process design on a scientific base. Independence and Information Axioms have provided scientific ways of looking at the decision process in design. In another approach, Robust Design method uses Taguchi's Quality Loss Function and a new measure of quality, called S/N ratio, to predict the quality from customers' point of view.

A reasonable approach is suggested in Axiomatic Design to measure information content in design and manufacturing but it still need a scientific backing. In Robust Design S/N ratio is used as a plausible measure of quality because it is used often in engineering for similar purpose.

To provide a rigorous treatment of the Information Axiom, new approaches to the information content in design and manufacturing are developed in this paper using Shannon's communication channel and entropy definition from information theory, and Kullback's symmetric divergence measure from statistics. Using Information Axiom and Kullback's symmetric divergence measure for information, it is also shown that the information content in product and process design is directly proportional to the Taguchi's Quality Loss Function and inversely proportional to the signal-to-noise ratio (S/N). In this way a rigorous link between the information content in Axiomatic Design and Robust Design methods has been established which indicates that a product which has low information content also has low quality loss and high S/N ratio.

Keywords: axiomatic design, information, Taguchi methods

1 INTRODUCTION

The main task of an engineer is to apply his scientific knowledge to the solution of technical problems to satisfy the needs of the society. His solution may be a new design or modification of an existing design for the need. In any case, designing high quality products and processes at low cost is an economic and technological challenge to the engineer. This activity requires his creative ability, practical knowledge and experience in special fields as well as a sound background in mathematics, physics, chemistry, mechanics, thermodynamics, fluid mechanics, electrical engineering, production processes, materials technology and design methods.

Every field of knowledge has its subject matter and its methods, along with a style for handling them. Consequently, there have also been many attempts to draw up maps or models of the design process [(Pahl and Beitz, 1984), (French, 1971), (Hubka, 1982), (Suh, 1990), (Hollins and Pugh, 1990), (Levitt, 1962)]. Some of these models simply describe the sequences of activities that typically occur in designing; other models attempt to prescribe a better or more appropriate pattern of activities.

Although there are many variations in the traditional design methods, the following steps are common : the expression of the need, analysis of the problem and statement of the requirements, conceptual design using creative ideas, preliminary design where concepts are embodied to satisfy the need, analytical process for the selection of the optimum design among the preliminary concepts and detailing the selected design in the form of design drawings, information for manufacturing, etc.

The existing traditional design methods represent the best thinking about how to regularize the design process. However, they are essentially empirical methods without much scientific basis. Rather, they represent a distillation of the best ideas about what works to enhance the design practice.

There is a natural desire to improve upon this situation in order to upgrade the design process from an empirical art to a branch of science by developing a theory of design. This would extend intuition and experience by providing a framework for evaluating and extending design concepts. When the designer is faced with many similarly competitive designs, a design theory would make it possible to answer his questions such as [Suh, 1990]: Is this a good design ?, Why is one design better than the others ?, How many design parameters does one need to satisfy the functional requirements expected from design to satisfy the perceived need?, Should one abandon the existing idea or modify the concept?.

The creative process in design is complemented by the analytical process by which the design is analyzed. The analysis of the design implies making correct design decisions as well as evaluating the details of specific design features. In the absence of a criterion for selecting a good design, we can not make good design decisions.

Design axioms provide scientific principles that aid the creative process of design by enabling good designs to be identified from an infinite number of possible designs. There are two design axioms: the Independence Axiom, and the Information Axiom. They are stated in declarative form as follows [Suh, 1990]:

Axiom 1 The Independence Axiom
Maintain the independence of functional requirements (FR).

Axiom 2 The Information Axiom
Minimize the information content of the design.

Fundamental to this theory of design is the idea of functional requirements and design parameters. These concepts have also been dealt in the traditional design processes with some differences in their interpretations. However, in axiomatic design they are fundamental part of the design process. Engineering design process is viewed as a constant interplay between what we want to achieve and how we want to achieve it. What the designer wants to achieve are stated as objectives of the design and defined in terms of specific requirements in the functional domain, which are called *functional requirements* (FRs). How he wants to achieve it is defined and created physically in the physical domain in terms of *design parameters* (DPs). The design process consists of mapping the FRs of the functional domain to the DPs of the physical domain to create a product, process, system or organization that satisfies the perceived social need [4]. However this mapping process is not unique. Therefore, more than one design may result from the generation of the DPs that satisfy FRs. Thus, the final outcome still depends on the creativity of the designer. However, the design axioms provide the principles that the mapping techniques must satisfy to produce a good design, and they offer a basis for comparing and selecting designs.

One of the attributes of a good designer is the ability to satisfy the needs with a minimal set of independent FRs. As the number of FRs increases, the solution becomes more complex. In addition to FRs, designers often have to specify constraints.

The problem of designing a product that can be manufactured is a very important industrial concern. Then the question is how to design a product and its components so that they can be manufactured in the most efficient manner, regardless of the specific nature of the product to be made. There is a relationship between the DPs of the product in the physical domain and the Process Variables (PV) in the process domain. During the product design phase the FRs specified in the functional domain must be satisfied by choosing a proper set of DPs, whereas during the process design phase, the DPs in the physical domain must be satisfied by selecting an optimum set of PVs, as shown in Figure 1. Therefore the independence axiom must also be satisfied by the process design.

Design Axiom 1 tells us that a design is a good design if it is an uncoupled design. However if we have obtained more than one uncoupled design, Axiom 2 tells us that the one with the minimum information content while fulfilling the functional requirements is the best design.

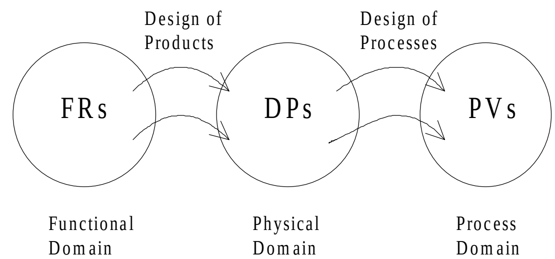

Figure 1. Three domains of the design/manufacturing world

Another systematic and efficient way to meet the requirements of designing high quality products and processes is Taguchi's Quality Loss Function and the Robust Design [(Taguchi, 1989), (Phadke, 1989)]. This method uses a mathematical tool called orthogonal arrays to study a large number of decision variables with a small number of experiments. It also uses a measure, called signal-to-noise (S/N) ratio, to predict the quality from the customer's side. This method tries to accomplish the most economical product and process design from both manufacturing and customers' viewpoints at the smallest and affordable development cost.

Although it seems that there are close similarities between the Axiomatic Design and the Robust Design methods, so far no analytical connections have been established between them. In this paper, an attempt is made to establish a connection between them using the concept of information measure in product and process design.

2 INFORMATION CONTENT USING SHANNON'S INFORMATION MEASURE

Similar to the communication channel in information theory [Shannon and Weaver, 1964], let us assume that the manufacturing process is a communication channel where a design parameter D is sent from a source with a probability distribution and a range. It will be received at the other end after manufacturing as a modified design parameter S with another probability distribution and a different range, as shown in Figure 2. If the channel were perfect, one could ask a 20±0.010 mm diameter shaft to be machined and would receive a machined shaft with 20±0.010 mm diameter. There would be no need for additional information to satisfy the tolerance range set up during the design process. However, due to limitations on the machine, it may or may not be manufactured within the requested range, one may need additional information to bring the result within the design range.

According to the definition of mutual information in information theory [(Shannon and Weaver, 1964), (Beckmann, 1967)], for the manufacturing channel we can write

$$I(D,S) = H(D) - H(D|S) \quad (1)$$

that is, the mutual information between the design variable D before manufacturing and design variable S after manufacturing (which we will call system variable after (Suh, 1990)) is the reduction in the uncertainty of D due to the knowledge of S. This is the information supplied by D to S through the communication channel.

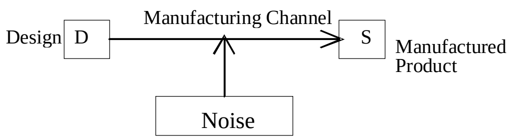

Figure 2. Manufacturing process as a communication channel

By symmetry, it follows that [(Shannon and Weaver, 1964), (Beckmann, 1967)]

$$I(D,S) = H(S) - H(S|D) \tag{2}$$

Thus D says as much about S as S says about D. Our interest is in this form of mutual information.

When we obtain the actual range of design variable after manufacturing, and it has certain amount of uncertainty associated with it. We can not derive its information content or the amount of information required to bring it in conformance with the design variable requested by the functional requirement. However we can look through the manufacturing channel, that is we can see what D says about S as given in mutual information relation in Eq. (2) which indicates that the initial uncertainty H(S) is reduced by looking at variable D. For a perfect manufacturing system the mutual information must be maximized. Assuming a stable process where H(S) is not changing, this, in turn, requires the minimization of H(S|D) which indicates the uncertainty in S given D and it is an indication of the quality of manufacturing process (i.e., the quality of communication channel). In order to improve the quality of the manufacturing process, the loss in the manufacturing process, that is H(S|D), must be minimized.

For a given design parameter ranges before and after the manufacturing, shown in Figure 3, we can calculate H(S|D) from [Lipschutz,1974]as

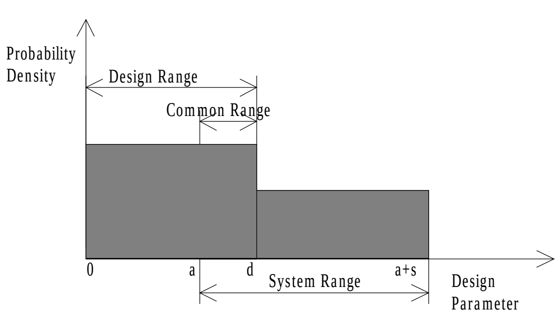

Figure 3. Probability density functions of design and system ranges

$$P(S \mid D) = \frac{P(S \cap D)}{P(D)} = \frac{(d-a)/s}{1} = \frac{(d-a)}{s} \tag{3}$$

using Shannon's entropy formula

$$H(S \mid D) = -\log \frac{(d-a)}{s} = \log \frac{s}{(d-a)} \tag{4}$$

This is the uncertainty or information content to be minimized to bring the design parameter range after manufacturing in line with the design parameter range before manufacturing in order to successfully satisfy the functional requirements on this design parameter.

Eq. (4) is the one defined in the axiomatic design as the information content of the system range with respect to the design range, and given by the log of the system range divided by the common range. It has been shown here that it has a theoretical background to support its usage.

3 INFORMATION CONTENT USING KULLBACK'S INFORMATION MEASURE

[Kullback, 1959] provides a measure of the mean information for discrimination in favor of one probability distribution against another. Given a set of n possible values x_i with probability $p_i = p(x_i)$ and a second set of probabilities $q_i=q(x_i)$, the mean information the second distribution provides about the first, called Kullback's measure, may be computed as

$$I(p \,\|\, q) = \sum_{i=1}^{n} p_i \log(\frac{p_i}{q_i}) \tag{5}$$

This information measure is zero when the second distribution is the same as the first distribution. In information theory the above measure is called the relative entropy or Kullback's distance between two probability distributions. It is always nonnegative; however it is not symmetric. Furthermore, it is not a true distance between distributions since it does not satisfy the triangle inequality [Fraser, 1957].

Kullback also defines a measure of divergence between the distributions p_i and q_i, based on the mean information between them, called the symmetric divergence and given by

$$J(p \,\|\, q) = I(p \,\|\, q) + I(q \,\|\, p) \tag{6}$$

is a measure of difficulty of discriminating between them. This may be regarded as a measure of the distance between two distributions. This divergence measure has all the properties of a distance (or metric) as defined in topology except the triangle inequality property and therefore is not termed a distance [Fraser, 1957].

For the case when both probability densities are Gaussian with $p(x) = N(m_p, \sigma_p^2)$ and $q(x) = N(m_q, \sigma_q^2)$, the Kullback's measure of information I(p||q) becomes [(Kullback, 1959), (Pinsker, 1964)]

$$I(p\|q)=\frac{1}{2}\left(\frac{\sigma_p^2+(m_p-m_q)^2}{\sigma_q^2}-1-\log\frac{\sigma_q^2}{\sigma_p^2}\right) \qquad (7)$$

Then the symmetric divergence J(p||q) can be obtained as

$$J(p\|q)=\frac{1}{2}\frac{(\sigma_p^2-\sigma_q^2)^2+(m_p-m_q)^2(\sigma_p^2+\sigma_q^2)}{\sigma_p^2\sigma_q^2} \qquad (8)$$

If we take the design range as the input to the manufacturing channel and the system range as the output of the channel, we can utilize Kullback's divergence in Eq. (5) as a measure of information content to satisfy the requirements of the designer by the manufacturing system. However, this measure has two shortcomings. First, this measure is not symmetric. Second, it requires that both probability definitions be nonzero in the same domain. Therefore it can not be used with uniform probabilities of varying domains. We will use continuous normal probability distributions to overcome this problem. However, to overcome the symmetry problem we will use the symmetric divergence measure defined in Eq. (6).

Let us assume that the design and system ranges have normally distributed probability densities with means m_d and m_s, and standard deviations σ_d and σ_s, respectively, as shown in Figure 4.

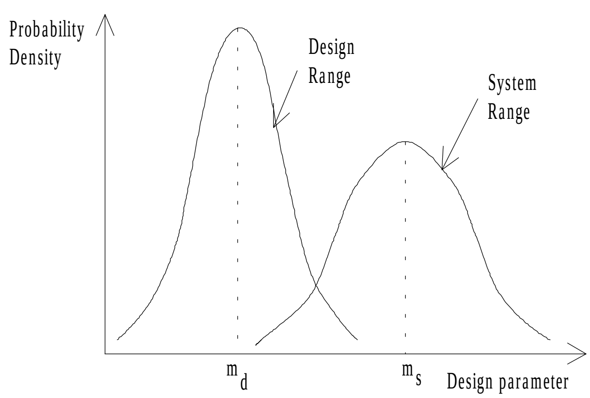

Figure 4. Probability density functions for design range and system range

Then the information content of the system range with respect to the design range can be calculated by using Eq. (8) as

$$J=\frac{1}{2}\frac{(\sigma_s^2-\sigma_d^2)^2+(m_s-m_d)^2(\sigma_s^2+\sigma_d^2)}{\sigma_s^2\sigma_d^2} \qquad (9)$$

This symmetric divergence function is symmetric and positive semi-definite[Pinsker, 1964].

Figure 3 and Eq.(9) shows a similarity between K-L information measure and Suh's information content. When both the system range and design range overlap completely, both measures are zero, and information measures increase as overlap decreases.

Let us now investigate this information measure for different design and system range combinations.

(a). Assume that standard deviations of the design and system ranges are equal, that is

$\sigma_s = \sigma_d$; but average values are different, that is $m_s \neq m_d$. The information function J in Eq. (9) becomes

$$J=\frac{(m_s-m_d)^2}{\sigma_d^2} \qquad (10)$$

It is a parabolic function as shown in Figure 5. It is zero when $m_s=m_d$, that is when the system range completely overlaps the design range.

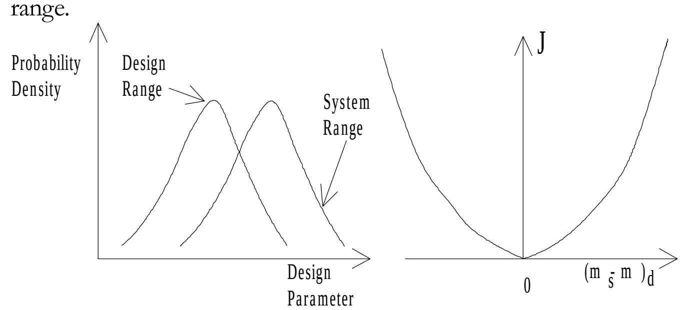

Figure 5. Probability density functions and the information measure for $\sigma_s=\sigma_d$.

(b). In design and manufacturing, generally the mean values of design and system ranges are different and the standard deviation of the system range is greater than the required design range. In this case, $m_s \neq m_d$, and $\sigma_s > \sigma_d$. Then we obtain from Eq. (9)

$$J\cong\frac{1}{2}\frac{\sigma_s^2+(m_s-m_d)^2}{\sigma_d^2} \qquad (11)$$

It is again parabolic and minimum value is at $m_s = m_d$. Probability density functions of the design and system ranges and the information function J are shown in Figure 6.

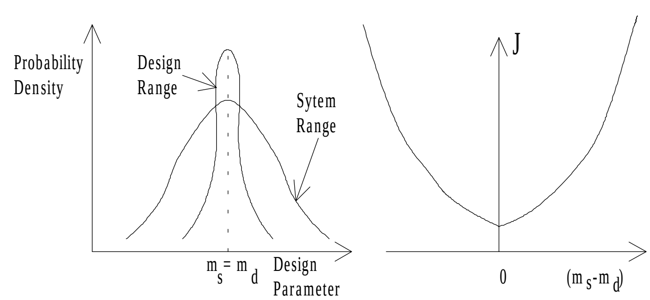

Figure 6. Probability density functions for design range and system range $\sigma_s >> \sigma_d$

3 INFORMATION CONNECTION BETWEEN THE AXIOMATIC DESIGN AND TAGUCHI METHODS

Taguchi measures the quality of a product in terms of the total loss to a society due to functional variation and harmful side effects. He assumes that the cost of making products that deviate from the target value (m_d in this case) increases as the square of the deviation. He then defines the quality loss function of a product with mean y and a target mean m as

$$Q = k(y-m)^2 \qquad (12)$$

where k is a constant called quality loss coefficient [Taguchi, 1989]. The expected value of the quality loss function, for a product with mean value of y, variance of σ^2 and target value of m , is given by Hunter as [Hunter, 1985]

$$E[Q] = k[(y-m)^2 + \sigma^2] \qquad (13)$$

The numerator in Eq. (11) is proportional to the expected value of this quality loss function where the observed mean of the system is m_s, the observed variance is σ_s^2 and the required target mean is the design mean m_d. Information measure in Eq. (11) indicates that the quality loss is proportional to the information content of the product after it is produced as compared to the designed product. Furthermore, Eq. (11) shows that for the same quality loss function, the required information is inversely proportional to the variance of the design set by the designer. As the design variance gets smaller more information is required to provide the same level of quality.

This relation proves a direct link between the quality loss function of the Taguchi and the information content of the design. For a multiparameter design, the information content obtained in Eq. (11) can be computed for every design parameter and added, provided that they are statistically independent [Pinsker, 1964]. This requires that the Independence Axiom be satisfied before information content is calculated.

Furthermore, using Eqs. (11) and (12), we can obtain a relationship between the quality loss function and the S/N ratio of the Taguchi. If the system mean m_s is changed by (m_d/m_s) it could be brought to the target mean m_d. Then the standard deviation of the system after this adjustment becomes $(m_d/m_s)\sigma_s$, assuming linearity.. The expected quality loss function becomes

$$E[Q\} = k\left(\frac{m_d^2}{m_s^2}\sigma_s^2\right) = k\, m_d^2 \cdot \left[\frac{\sigma_s^2}{m_s^2}\right] \qquad (14)$$

For a given product design, k and m_d are constant, $(m_s/\sigma_s)^2$ can be called the S/N ratio because σ_s^2 is the effect of noise factors and m_s is the desired mean [Phadke, 1989].

Then from Eq.(11) we obtain

$$J = \frac{1}{2}\frac{m_d^2\sigma_s^2}{m_s^2\sigma_d^2} = \frac{1}{2}\frac{m_d^2}{\sigma_d^2}\frac{\sigma_s^2}{m_s^2} = K\left(\frac{\sigma_s^2}{m_s^2}\right) = K\frac{N}{S} \qquad (15)$$

where $K = (m_d^2 / 2\sigma_d^2)$ is a constant for a given design. This equation indicates a direct relation between the information content of the design and the Taguchi S/N ratio. In fact, the term in parenthesis in Eq. (15) is the N/S ratio of the Taguchi. Thus Eq. (15) indicates that minimizing the information content in the design/manufacturing means maximizing the S/N ratio as described by Taguchi. This is another result linking information content directly to quality.

In Robust Design three types of S/N ratios are defined: Nominal-the -best, larger-the-better and smaller-the-better. For a given design (given m_d and σ_d), the information content in Eq. (11) can be written as

$$J_N = \frac{1}{2}\frac{\sigma_s^2 + (m_s - m_d)^2}{\sigma_d^2} = C\cdot[\sigma_s^2 + (m_s - m_d)^2] \qquad (16)$$

where C is a constant for the requested design.

Eq. (16) can be used as an objective function for nominal-the-best problems instead of a S/N ratio. For a constant σ_s ,, this function is symmetric around m_s=m_d. To maximize S/N ratio means to minimize J_N in Eq. (16) which requires reducing the variance of the system range and equating means of the system and design ranges.

For smaller-the-better type problems, the most desired value m_d=0. Then, Eq. (12) becomes

$$J_S = C\cdot[\sigma_s^2 + m_s^2] \qquad (17)$$

In order to minimize J_S, m_s and σ_s must be minimized, as shown in Figure 7. Result is the same as maximizing S/N ratio for this case in Robust Design method.

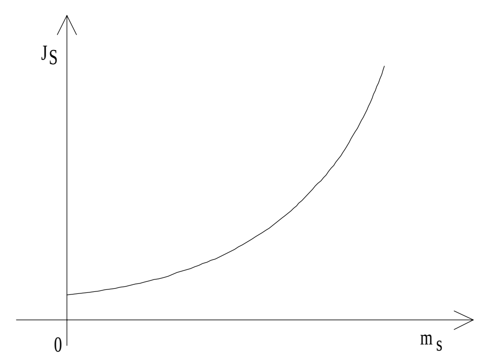

Figure 7. Smaller-the-better behavior of information content function

For larger-the-better type problems, the desired design value m_d is large. Then

$$J_L = C\cdot[\sigma_s^2 + (m_s - m_d)^2] \qquad (18)$$

can be used as it is. To minimize J_L (that is to maximize S/N ratio), σ_s must be minimized and m_s must be brought to closer to m_d which is desired thing to do [Phadke, 1989], as shown in Figure 8.

4 CONCLUSIONS

Traditional design methods provide some means of starting the design process, bringing about new designs or modifying the existing ones, but they are still ad-hoc in nature and do not provide any scientific basis for selecting the optimum design. The Axiomatic Design method provides two scientific axioms not only to select the best design among the candidates but to accelerate the process of design in the right direction without much trial and error.

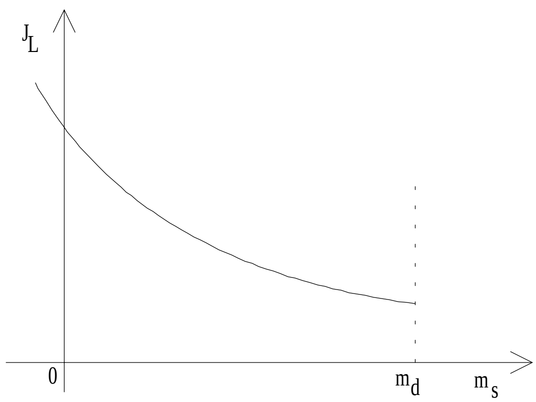

Figure 8. Larger-the-better behavior of information content function

Information content is a commonly used term to indicate the amount of information one gains after some processes and it has been used in thermodynamics, statistics, communication theory, pattern recognition, economics, etc. since the beginning of the century. Axiomatic Design utilizes a relationship from information definition in thermodynamics and information theory to determine the information content in design and manufacturing..

Manufacturing process has been modeled as a communication channel where the designed product as the source input and the manufactured product as the received output. It has been determined that the information content needed to manufacture the designed product directly related to the loss of the channel. By formulating this idea using the concepts from the information theory it was possible to derive the information content relation used in Axiomatic Design.

In another approach, the symmetric divergence measure of Kullback is used as the information content measure. Taking the probability density function of a design parameter of the designed product before manufacturing and that of the same parameter after manufacturing, it was possible to obtained a new measure in terms of averages and variances. Furhermore it has been shown that this measure is proportional to the quality loss function of the Taguchi. It was also possible to obtain a direct relationship between this information measure and Taguchi's S/N ratio. It has been concluded here that maximizing Taguchi's S/N ratio is directly related to the minimization of information content in design/manufacturing. This result provides a direct link between the Axiomatic Design and Robust Design and clearly indicates the power of the design axioms, especially the information axiom.

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