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AN AXIOMATIC DESIGN APPROACH TO ONE-DOF VEHICLE SUSPENSION SYSTEMS

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ABSTRACT

The design of a vehicle suspension system requires satisfaction of many design aspects including good vibration isolation to secure the occupants comfort, good road-vehicle holding ability and safety. Several vehicle suspension models with structural or actively controlled configurations have been reported in the open literature. The key design objectives are represented by the acceleration of the vehicle body and relative displacement between the vehicle and various suspension components. Other constraints such as the overall system robustness, power and cost requirements are also taken into account.

In this paper, two 1-DOF models with a Maxwell type suspension and a Maxwell type suspension with a parallel spring are considered. Mean square acceleration and relative displacement responses of these models are derived for a realistic random road input given in terms of its power spectrum density function.

The optimization problem of suspension systems is usually solved by defining a single performance index which is a weighted sum of mean square acceleration and the relative displacement. However, the results are dependent on the weighting coefficient which is arbitrary; and the solution can not tell us which design is better.

Independence Axiom of the Axiomatic Design can easily distinguish between different designs and can provide an index showing which is better. In this paper, the above derived suspension models are examined using this axiom, and semangularity and reangularity concepts. It has been shown that under certain conditions and in certain range of design parameters, it is possible to obtain decoupled designs. Axiomatic design approach and new relations derived during these investigations may provide a new look at the vehicle suspension system design.

Keywords: axiomatic design, vehicle suspensions

1 INTRODUCTION

Transportation is a very important service for the human society. Advanced research is undertaken for different engineering aspects of transportation such as reliability, safety, maintenance of vehicle structures, vibration and shock effects in transportation, to meet the needs for cost efficient, fast and comfortable travel.

A major influence on passenger comfort, and maintenance and repair cost of both vehicles and guideways comes from dynamic motions. A vehicle travelling on a given road surface is subjected at each wheel to a disturbance which is random function of time. It is also subjected to aerodynamic forces. To design a vehicle and its suspension so that the vehicle response to disturbances is at an acceptable level is one of the objectives of a vehicle design engineer. Among the design objectives are acceptable passenger comfort, small suspension deflections, and good road and track holding ability. A good vibration isolation is required for better ride comfort, whereas good road holding is important for safety. For suspension design, acceleration level of vehicle body, maximum allowable relative displacement between the vehicle and the various suspension components, including wheels, vehicle and other unsprung masses are taken into account. Additional constraints are imposed by the overall system robustness, reliability, power and cost requirements.

As suspension becomes softer, it tends to reduce the effects of road disturbances and also requires increased vehicle-road dynamic clearance. The effects of external forces tend to be reduced as suspension becomes harder and requires less vehicle-road dynamic clearance. The presence of different types of external disturbances and the passenger comfort results in a number of competing approaches to vehicle suspension design.

Modern approaches in design demand optimal performance. Considerable research devoted over last four decades to determine the optimum suspension design for a vehicle travelling over randomly profiled tracks realized that the optimum design is a compromise between several competing requirements.

Conventionally, road vehicle suspension systems are passive devices consisting of energy storage and dissipative elements tuned to one particular design point which is a compromise between sprung mass isolation, suspension travel, and tire-road contact forces [Sevin and Pilkey (1971), Hrovat and Hubbard (1981), Tseng and Hrovat (1990)].

Recent developments in active control means open a new era for the design of electronically controlled vehicle suspension systems, with potential in overall performance. Such advanced concepts include actively controlled systems that use sensing devices and servomechanisms [Young and Wormley (1973), Hedrick (1973), Karnopp(1989), Redfield and Karnopp (1989), Hrovat (1993)], externally-controlled passive systems called semi-active suspensions [Karnopp et al. (1974), Margolis (1983)]. Passive suspensions are found on most of the conventional vehicles. They do not include external power sources, whereas active suspensions require additional energy sources, such as compressors and pumps to achieve superior ride and/or road holding performance. The semi-active suspensions fill the gap between passive and active suspensions and they offer significant performance improvements while requiring relatively small external power sources.

Modern optimization techniques are heavily used in the design of suspension systems. Both single objective and multi-objective techniques are used for this purpose [Hedrick (1972), Young and Wormley (1973), Rao (1984)]. These techniques may indicate which design is optimal with respect to a given objective function, but they can not tell if and why that design is better.

Axiomatic design is developed with these questions in mind [Suh (1990)]. Design axioms provide principles that aid the creative process of design by enabling good designs to be distinguished among plausible designs. So far these axioms are successfully used by many researchers for design for producibility[Suh (1988)], information-based design for environmental problem solving [Wallace and Suh (1993)], design decision making support problems [Bras and Mistree (1993)], design of artificial skin [Gabela and Suh (1992)] as well as for other design related problems.

In this paper, two 1-DOF models with a Maxwell type suspension and a Maxwell type suspension with a parallel spring are investigated using independence axiom, and semangularity and reangularity concepts derived from this axiom. Mean square acceleration and relative displacement responses of these models are derived for a realistic random road input given in terms of its power spectrum density function.

2 MATHEMATICAL MODELS

In this section suspension and input models will be presented, mean square acceleration and relative displacement responses will be derived, and reangularity and semangularity relations based on these responses will be obtained.

2.1 SUSPENSION MODELS

Several suspension models were used in previous studies to study the optimal behavior of vehicle suspension systems. While a multidegree-of-freedom suspension model is required to describe a vehicle completely, considerable insight may be gained into the basic behavior of a suspension system by focusing attention upon models of small degree of freedom, such as the single degree of freedom models. The suspension models used in this paper are shown in Figure 1. These models consists of a vehicle mass m, and an equivalent suspension system involving springs and dashpot. Suspension system shown in Figure 1(a), called Maxwell-type suspension, proven to be the optimum suspension configuration for a vehicle subjected to random road inputs [Young and Wormley (1973), Hedrick (1973)]. Although it can not support static load, it is used to compare other suspensions with its performance. Maxwell-type suspension system with a parallel spring is shown in Figure 1(b) does support static loads.

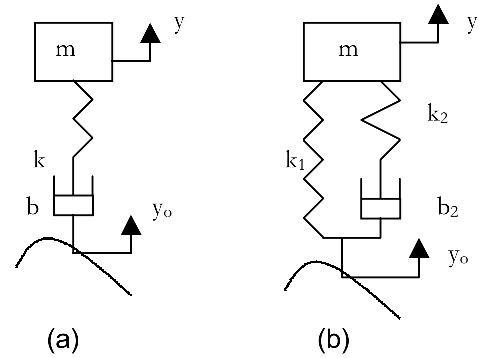

Figure 1. One-DOF suspension systems

Equations of motion between the acceleration of carbody a, relative displacement $d=y-y_o$ and road input y_o are given in Laplace domain by

$$a(s) = \frac{s^2 G(s)}{ms^2 + G(s)} y_o(s) \quad (1)$$

$$d(s) = \frac{-ms^2}{ms^2 + G(s)} y_o(s) \quad (2)$$

where s is the Laplace operator and G(s) is the transfer function of the suspension system .

Transfer function of the Maxwell-type suspension model is

$$G_1(s) = \frac{kbs}{k + bs} \quad (3)$$

and that of the Maxwell-type suspension with a parallel spring is

$$G_2(s) = k_1 + \frac{k_2 b_2 s}{k_2 + b_2 s} \quad (4)$$

2.2 ROAD DISTURBANCE INPUT

Real road force input is modelled using its power spectrum density function . A general form used for road power spectrum density function is given by

$$S_{yo}(s) = -\frac{AV}{s^2} \quad (5)$$

where A is roughness factor and V is forward vehicle velocity[Hedrick et al. (1974)]. Using the value of A for smooth highway from Table 1 [Young and Wormley (1973)] and V=100 km/h , $2\pi AV=2.10^{-4}$ m^2/s is obtained and used in this study.

2.3 PERFORMANCE MEASURES

Several criteria have been used in literature to describe the performance of a suspension system quantitatively. In this study, a comfort criterion in terms of RMS acceleration level and a sizing criterion in term of RMS relative suspension deflection are used separately. In literature, these two measures are combined with a weighting coefficient and optimized together. In this study these two will be used as the Functional Requirements (FRs).

2.4 DERIVATION OF EQUATIONS

Using equations (1-5), mean square values of carbody acceleration a_m^2 and relative suspension travel d_m^2 can be calculated for Maxwell type suspension as [Newton et al. (1967), Martin (1999)]

$$a_m^2 = C\frac{kb}{m^2} \tag{6}$$

$$d_m^2 = C\left(\frac{m}{b}+\frac{b}{k}\right) \tag{7}$$

where C=πAV. In order to simplify the analysis, equations (6-7) are written as

$$a_m^2 = CKB \tag{8}$$

$$d_m^2 = C\left(\frac{1}{B}+\frac{B}{K}\right) \tag{9}$$

where B=b/m, K=k/m.

From axiomatic design point of view, there are two FRs (a_m, d_m), and two DP's (K, B). Axiomatic design equation for Maxwell-type suspension may be written as [Suh (1990)]

$$\begin{bmatrix} a_m \\ d_m \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \cdot \begin{bmatrix} K \\ B \end{bmatrix} \tag{10}$$

Since functional requirements and design parameters are nonlinearly related, elements A_{ij} of the design matrix may be expressed as

$$A_{ij} = \partial FR_i / \partial DP_j \tag{11}$$

Using this definition, design matrix elements can be obtained as

$$A_{11} = \frac{1}{2}C^{1/2}\cdot B^{1/2}\cdot K^{-1/2}$$

$$A_{12} = \frac{1}{2}C^{1/2}\cdot B^{-1/2}\cdot K^{1/2}$$

$$A_{21} = \frac{-1}{2}C^{1/2}\cdot\left(\frac{1}{B}+\frac{B}{K}\right)^{-1/2}\cdot\left(\frac{B}{K^2}\right) \tag{12}$$

$$A_{22} = \frac{1}{2}C^{1/2}\cdot\left(\frac{1}{B}+\frac{B}{K}\right)^{-1/2}\cdot\left(\frac{1}{K}-\frac{1}{B^2}\right)$$

Reangularity in axiomatic design is an indication of the orthogonality between DP-axes which is a necessary condition for functional independence. Reangularity R can be obtained for this case, using design matrix elements, as [Suh (1990)]

$$R = \left[1-\frac{(A_{11}A_{12}+A_{21}A_{22})^2}{(A_{11}^2+A_{21}^2)\cdot(A_{12}^2+A_{22}^2)}\right]^{1/2} \tag{13}$$

Semangularity is a measure indicating the angular positioning of corresponding axes of DPs and FRs. Functional independence can not be characterized by R alone. Semangularity S can be written as

$$S = \frac{|A_{11}|}{(A_{11}^2+A_{21}^2)^{1/2}}\cdot\frac{|A_{22}|}{(A_{12}^2+A_{22}^2)^{1/2}} \tag{14}$$

For the Maxwell-type suspension with a parallel spring, mean square values of carbody acceleration a_m^2 and relative suspension travel d_m^2 relations become [Martin (1999)]

$$a_m^2 = C\left(\frac{B_2}{K_2^2}(K_1+K_2)^3+\frac{K_1^2}{B_2}\right) \tag{15}$$

$$d_m^2 = C\left(\frac{B_2}{K_2^2}(K_1+K_2)+\frac{1}{B_2}\right) \tag{16}$$

where $B_2=b_2/m$, $K_2=k_2/m$, $K_1=k_1/m$.

For this case there are three parameters, K_1, K_2, B_2 and design matrix will not be square. To prevent this, K_2 and B_2 are selected as design parameters DP1 and DP_2,, and K_1 is kept as a redundant parameter. For small values of K_1, design will approach the first model. With this setup, design matrix elements are obtained as

$$A_{11} = \frac{1}{2}C^{1/2}\cdot\left(\frac{B_2(K_1+K_2)^3}{K_2^2}+\frac{K_1^2}{B_2}\right)^{-1/2}\cdot\left(\frac{B_2(K_1+K_2)^2(K_2-2K_1)}{K_2^3}\right)$$

$$A_{12} = \frac{1}{2}C^{1/2}\cdot\left(\frac{B_2(K_1+K_2)^3}{K_2^2}+\frac{K_1^2}{B_2}\right)^{-1/2}\cdot\left(\frac{(K_1+K_2)^3}{K_2^2}-\frac{K_1^2}{B_2^2}\right) \tag{17}$$

$$A_{21} = \frac{-1}{2}C^{1/2}\cdot\left(\frac{B_2(K_1+K_2)}{K_2^2}+\frac{1}{B_2}\right)^{-1/2}\cdot\left(\frac{B_2(2K_1+K_2)}{K_2^3}\right)$$

$$A_{22} = \frac{1}{2} C^{1/2} \cdot \left(\frac{B_2 (K_1 + K_2)}{K_2^2} + \frac{1}{B_2} \right)^{-1/2} \cdot \left(\frac{(K_1 + K_2)}{K_2^2} - \frac{1}{B_2^2} \right)$$

Reangularity and semangularity can be calculated as before.

3 PERFORMANCE EVALUATIONS

Performance of suspension systems are evaluated based on the functional requirements derived in Section 2. Firstly a_m and d_m equations are examined. Then contours of constant rms acceleration and relative displacement are plotted against DPs. Reangularity and semangularity contours are also plotted and examined in conjunction with a_m and d_m.

3.1 MAXWELL-TYPE SUSPENSIONS

To investigate the limits on B and K, based on given levels of rms acceleration a_a and relative displacement d_m, solving K from Equation (9) we get

$$K = \frac{C \cdot B^2}{B \cdot d_m^2 - C} \tag{18}$$

which means that for a positive K, we must have

$$d_m^2 > C / B \tag{19}$$

Solving Equation (8) for K and substituting into Equation (9), we obtain

$$B = (K/2)^{1/2} \tag{20}$$

is obtained for the optimum case where

$$a_m^2 \cdot d_m^6 = (27/4) C^4 \tag{21}$$

But the limiting performance is

$$a_m^2 \cdot d_m^6 \geq (27/4) C^4 \tag{22}$$

otherwise there is no feasible solution [Martin (1999)]. This is similar to the findings of earlier researchers reached through a single objective optimization [Hedrick (1973)].

Plot in Figure 2 shows the constant rms acceleration and relative displacement contours on K and B-axes. This figure clearly shows that it is a coupled design, because isograms for rms acceleration and relative displacements are not orthogonal to each other, and they are not parallel to DP-axes. However, for low values of B and high values of K or low values of K and high values of B, it approaches to uncoupled design.

To investigate this premise further, reangularity and semangularity contours are plotted in Figures 3 and 4. In these figures we can clearly verify above mentioned premise. For high values of B, there is a value of K around 1 that can make the suspension decoupled. If we examine the design matrix in Equation (10) in light of Equation(12) approximately we get

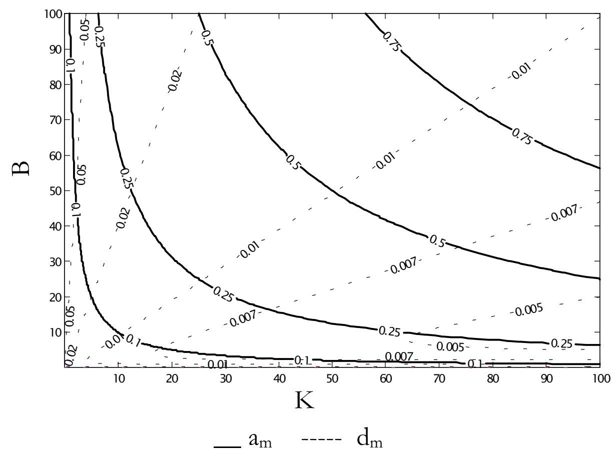

Figure 2. Contours of constant rms acceleration and rms relative displacement

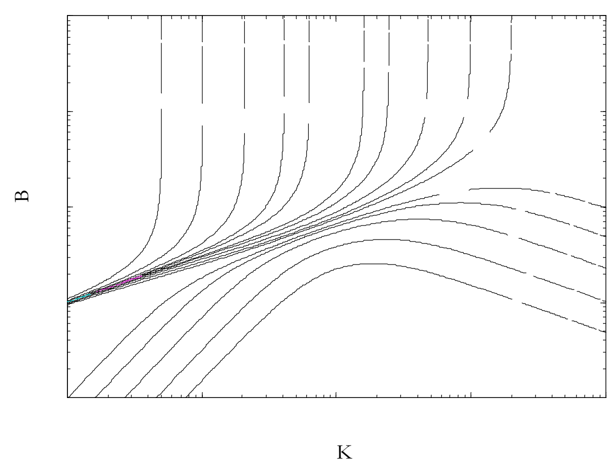

Figure 3. Contours of reangularity

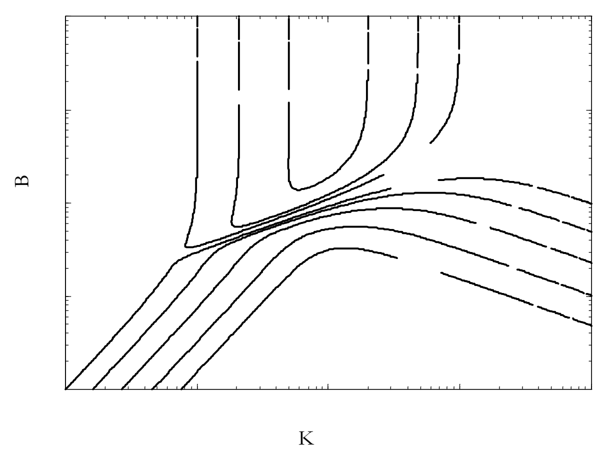

Figure 4. Contours of semangularity

$$A = \begin{bmatrix} X & 0 \\ X & 0 \end{bmatrix} \tag{23}$$

which shows the decoupling nature of the design under these conditions where Xs are nonzero terms as compared to almost zero terms. Again for low values of B and high values of K Equation (10) approximately becomes

$$A = \begin{bmatrix} 0 & X \\ 0 & X \end{bmatrix} \tag{24}$$

which again clearly indicates a decoupled system.

3.2 MAXWELL-TYPE SUSPENSIONS WITH A PARALLEL SPRING

For small values of the parallel spring, we obtain the behavior of the Maxwell-type suspensions.

For medium values of K_1, behavior of a_m and d_m begins to change. However , it can be seen from Figure 5 that it is still a coupled design, but for small values of B_2 and large values of K_2, isograms of a_m and d_m are parallel to K_2 axes. For low values of K_2 and large values of B_2, isograms of a_m and d_m becomes approximately parallel to B_2 axes. It seems it is becoming decoupled design in these regions.

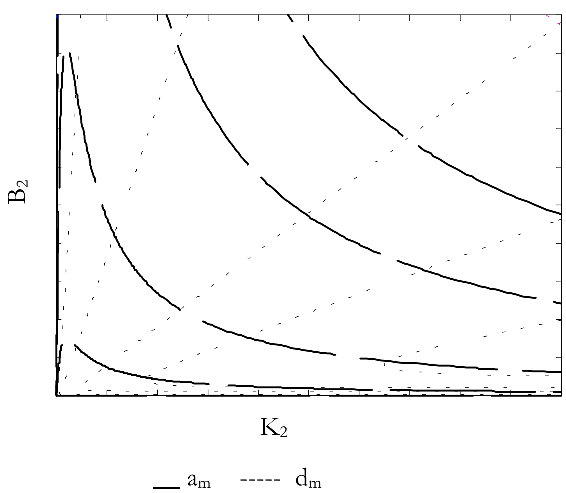

Figure 5. Contours of constant rms acceleration and rms relative displacement for K_1=1

This coupling behavior can be seen clearly from reangularity and semangularity plots in Figures 6 and 7. However if we examine design matrix elements for large values of K_2 and small values of B_2 using Equation (17), approximately we get

$$A = \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix} \tag{25}$$

which again indicates decoupled behavior in a certain region. Although the reangularity plot in Figure 6 shows orthogonal behavior among DPs, semangularity plot in Figure 7 does not indicate a similar relationship among pairs of DPs and FRs. This is the effect of K_1. Although K_1 provided load carrying capability under static load, it reduced the ranges where independence of FRs can be satisfied.

As K_1 is increased, the behavior of the system did not change appreciably, except the region where decoupled behavior is observed before became more restrictive, as it can be seen in reangularity and semangularity plots in Figures 8 and 9 for K_1 = 5.

4 CONCLUSIONS

In this paper an axiomatic design approach is presented to investigation of one-DOF vehicle suspension systems. Two

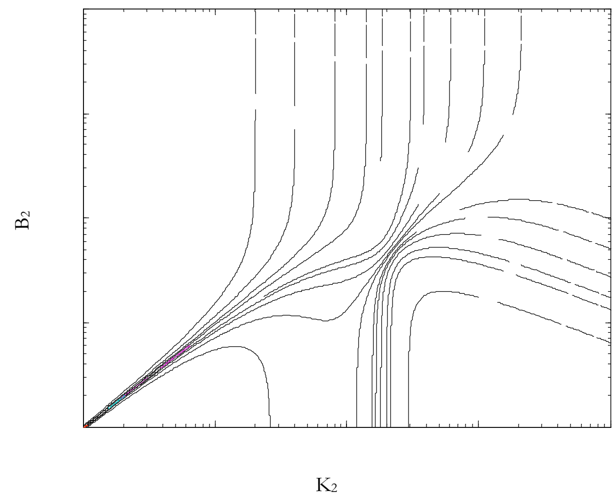

Figure 6. Contours of reangularity for K_1=1

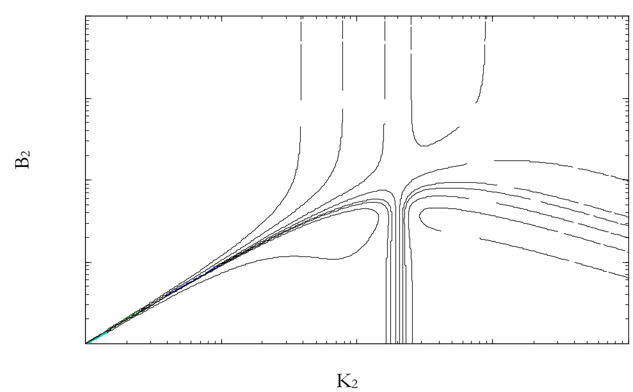

Figure 7. Contours of semangularity for K_1=1

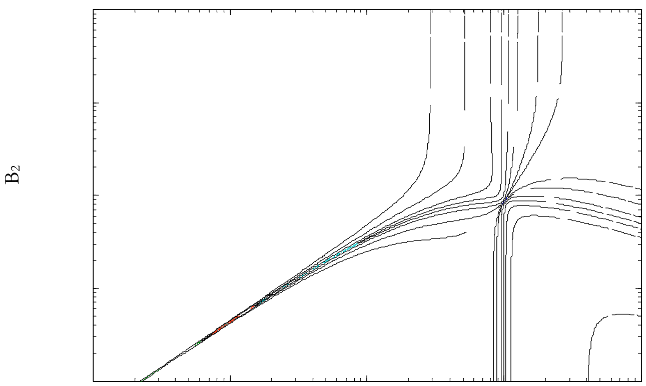

Figure 8. Contours of reangularity for K_1=5

systems were investigated using rms acceleration and rms relative displacement of vehicle body as functional requirements. It is shown that investigating the behavior of suspensions using reangularity and semangularity concepts from axiomatic design provide better answers clearly. this approach also presents another approach to the design of vehicle suspension systems.

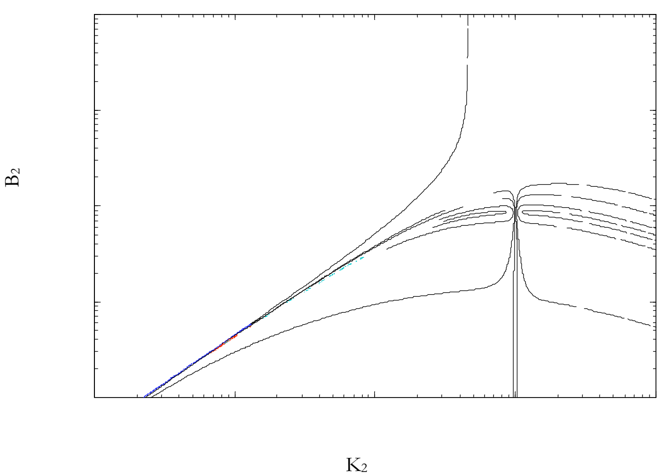

Figure 9. Contours of semangularity for K_1=5

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