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COMPUTING THE INFORMATION CONTENT OF DECOUPLED DESIGNS

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ABSTRACT

The information content of uncoupled designs can be computed by summing the information content of each functional requirement. This paper proves that information cannot be summed for decoupled designs due to correlation among the functional requirements. To overcome this problem, this paper presents two algorithms for computing information content of decoupled designs. One algorithm is applicable to any joint density function for the design parameters. The second, more computationally efficient algorithm, applies only to uniformly distributed design parameters. The algorithm for uniform distributions is based on a recursive procedure for computing the volume of a convex polytope in *n* dimensional real space where *n* is the number of design parameters. An engineering application of the algorithms is presented. The example demonstrates that summing information content can significantly overestimate total information when compared to an algorithm that accounts for correlation. The example also demonstrates that decoupled designs can have lower information content than uncoupled systems with the same functional requirements and similar components.

1 MOTIVATION

The second axiom proposed in Suh's *The Principles of Design* [1] states that engineers should minimize the information content of their designs. In order to do this, it is essential that designers have means to calculate (or at least estimate) the information content of design alternatives. Theorem 13 [1] provides a simple means to compute information content of a system under certain conditions. The theorem states that information content for a system is the sum of the information content associated with each functional requirement *if the events are probabilistically independent*. This paper will show that these conditions are not met for decoupled designs even when the design parameters of the system are probabilistically independent.

When the functional requirements are not independent, as noted by Suh [1], the appropriate conditional probabilities must be considered. However, none of the literature provides specific guidance on how to account for these conditional probabilities. This paper helps to fill this gap by providing algorithms to compute information content for decoupled designs. These algorithms should be useful to any designer who must evaluate several alternatives, some of which are decoupled rather than uncoupled.

2 BRIEF REVIEW OF AXIOMATIC DESIGN

In the Axiomatic approach, design is modeled as a mapping process between a set of functional requirements (FRs) in the functional domain and a set of design parameters (DPs) in the physical domain. This mapping process is represented by the design equation.

$$\{\mathbf{FR}\} = [\mathbf{A}]\{\mathbf{DP}\} \tag{1}$$

where

$$\mathbf{A}_{i,j} = \frac{\partial \mathbf{FR}_i}{\partial \mathbf{DP}_j} \tag{2}$$

Suh defines an *uncoupled design* as a design whose **A** matrix can be arranged as a diagonal matrix by an appropriate ordering of the FRs and DPs. He defines a *decoupled design* as a design whose **A** matrix can be arranged as a triangular matrix by an appropriate

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ordering of the FRs and DPs. He defines a *coupled design* as a design whose **A** matrix **cannot** be arranged as a triangular or diagonal matrix by an appropriate ordering of the FRs and DPs.

$$\begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \quad \begin{bmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{bmatrix} \quad \begin{bmatrix} X & X & X \\ X & X & X \\ X & X & X \end{bmatrix}$$

uncoupled decoupled coupled

Figure 1. Categories of design based on the structure of the design matrix

In Axiomatic Design, the probability that a product can satisfy *all* of its FRs is called the probability of success (p_s). Based on the notion of probability of success, information content I is defined as

$$I = \log(1/p_s) \tag{3}$$

In the case that the probability density function over DP is uniformly distributed over the system range, and given that the tolerance on the functional requirements determines a tolerance range on the DP, then the information content can be expressed as

$$I = \log\left(\frac{\text{system range}}{\text{common range}}\right) \tag{4}$$

where the common range is the intersection of the system range and the design range (Figure 2).

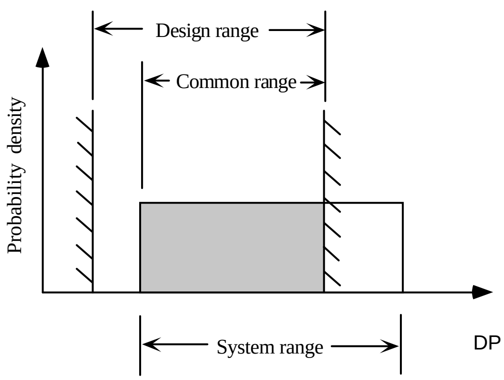

Figure 2. Information in the case of uniformly distributed variation

If a set of events are statistically independent, then the probability of the union of the events is the product of the probabilities of the individual events. From these facts follows a theorem [1].

Theorem 13 (Information Content of the Total System) – If each FR is probabilistically independent of other FRs, the information content of the total system is the sum of information of all individual events associated with the set of FRs that must be satisfied.

This section outlined the basics of Axiomatic Design which are required to understand the developments of the subsequent sections of this paper. The following section will present related work on the subject of information content in Axiomatic Design.

3 RELATED WORK

Shannon [2] first proposed entropy as a measure of information in communications. The entropy of a discrete random variable X is

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log p(x) \tag{5}$$

where $p(x)$ is the probability mass function of the random variable X defined over its support set $\mathcal{X}$. Shannon also defined the joint information of two random variables X and Y as

$$H(X,Y) = -\sum_{x \in \mathcal{X}} \sum_{y \in Y} p(x,y) \log p(x,y) \tag{6}$$

and showed that the joint entropy of two random variables obeys the inequality

$$H(X,Y) \le H(X) + H(Y) \tag{7}$$

with equality only if the variables are probabilistically independent. So, since the beginnings of Information Theory, it has been recognized that information can only be summed under the condition of probabilistic independence of the relevant variables or events.

The information axiom in Axiomatic Design was first introduced in a paper by Suh et. al. [3] in a paper entitled "On an Axiomatic Approach to Manufacturing Systems." In this paper, the information axiom was simply stated as "minimize information content" where information content was defined as the instructions necessary to describe the parts of a product, the processes for making them, and the procedures for assembling them. Information content in Axiomatic design was later given a mathematical definition by Wilson [4] as the logarithm of the inverse of the probability of satisfying a tolerance. Wilson acknowledged that information defined in this way could be summed only if the relevant dimensions were independent, but his thesis simply left this as an area for future research. The text later published on Axiomatic Design [1] also acknowledged that summation of information requires probabilistic independence of the relevant variables. However, most of the subsequent applications of the independence axiom such as those published in Albano and Suh [5], Suh [1, 6] simply sum information content assuming that the relevant dimensions are independent.

In his most recent book, Suh presented a means for propagating tolerances in decoupled designs [6]. He showed that if the specified tolerances on a set of FRs are ±ΔFR1, ±ΔFR2, and ±ΔFR3, then the tolerances on the DPs may be expressed as

$$\Delta DP1 = \frac{\Delta FR1}{A11}$$
$$\Delta DP2 = \frac{\Delta FR2 - |A21 \cdot \Delta DP1|}{A22} \quad (8)$$
$$\Delta DP3 = \frac{\Delta FR3 - |A31 \cdot \Delta DP1| - |A32 \cdot \Delta DP2|}{A33}$$

This procedure is useful for defining "worst case" tolerances for decoupled designs. Those designs in which the values of the DPs can be guaranteed to lie entirely within the ranges as specified above will have zero information content. However, this procedure does not allow one to compute the information content for decoupled designs in which the tails of the probability distributions over the DPs extend beyond their specification widths as is most often the case in industrial practice. The algorithms in this paper will enable one to compute information content for these important cases.

El-Haik and Yang [7] addressed the issue of correlation in axiomatic design. They defined a measure of complexity which is composed of three aspects: variability, correlation and vulnerability. Vulnerability relates to the design's size, interdependency of design parameters (i.e. correlation), and the FR's sensitivity to changes in design parameters. This division of complexity in three parts gives, together with Boltzman's entropy measure, a formula for complexity. See equation 9.

$$h(\phi\{DP\}) = \sum_{l=1}^{p-1}\sum_{k=1+l}^{p} \ln\left(2\alpha e\sqrt{(1-p_{kl}^2)\sigma_l\sigma_k}\right) + \ln\left((2\pi e)^p \prod_{l=1}^{p}\sigma_l^2\right)^{\frac{1}{2}} + \ln|[A]| \quad (9)$$

El-Haik and Yang have therefore proposed a new measure of complexity which accounts for correlation among DPs. Equation 9 is not used in the presented research and is therefore not further defined. In contrast to equation 9, this paper presents a means to compute information content as defined by Suh [1] while accounting for correlation.

Although there has been substantial progress in Axiomatic Design, no one has yet devised a way to compute the information content of decoupled designs. One problem may be that the need has not been clearly articulated. The following section establishes this need with a proof that decoupled designs do not meet the criteria for summing information as listed in Theorem 13 in *The Principles of Design* [1].

4 INFORMATION CANNOT BE SUMMED FOR DECOUPLED DESIGNS

Before presenting an algorithm to compute information content in decoupled designs, it seems appropriate to prove that the simpler procedure of summing information content will not be adequate. As noted in Theorem 13 [1], information can be summed if the functional requirements are probabilistically independent. The theorem below establishes that this condition fails to hold for decoupled designs under most conditions.

Proposition – If the design matrix **A** is decoupled and the DPs are probabilistically independent with non-zero variance and the on-diagonal elements of **A** are non-zero, then the FRs CANNOT be probabilistically independent and the information content cannot be summed.

Proof

Let the covariance among the DPs be represented by the covariance matrix $\mathbf{K_{DP}}$. If the DPs are probabilistically independent, then the covariance matrix $\mathbf{K_{DP}}$ must be diagonal. If the DPs are related to the FRs by the design matrix (Equation 1), then the covariance matrix of the functional requirements is

$$\mathbf{K_{FR}} = \mathbf{A}\mathbf{K_{DP}}\mathbf{A}^T \quad (10)$$

Without loss of generality, we may assume that **A** is lower triangular rather than upper triangular since an upper triangular matrix may be rearranged into a lower triangular matrix by inverting the order of the FRs and DPs. If **A** is lower triangular and $\mathbf{K_{DP}}$ is diagonal, any element of the covariance matrix of the functional requirements is

$$\mathbf{K_{FR}}_{ij} = \mathbf{K_{FR}}_{ji} = \sum_{p=1}^{i} \mathbf{A}_{jp}\mathbf{K_{DP}}_{pp}\mathbf{A}_{ip} \quad (11)$$

For the functional requirements to be probabilistically independent, it is a necessary condition that $\mathbf{K_{FR}}$ is diagonal. From Equation 11, we will now show that $\mathbf{K_{FR}}$ is diagonal if and only if **A** is uncoupled rather than decoupled.

If the matrix $\mathbf{K_{FR}}$ is diagonal, then all the off-diagonal elements in its first row must be zero. For the first row, this implies that $\mathbf{K_{FR}}_{1j} = \mathbf{A}_{j1}\mathbf{K_{DP}}_{11}\mathbf{A}_{11} = 0$ for all j>1. Since the proposition stipulates that the on-diagonal elements of **A** and $\mathbf{K_{DP}}$ are non-zero, we know that $\mathbf{K_{DP}}_{11} \neq 0$ and $\mathbf{A}_{11} \neq 0$. It follows therefore that all the off-diagonal elements in the first column of **A** must be zero (i.e. $\mathbf{A}_{j1} = 0$ for all j>1). So, probabilistic independence of the FRs demands that the off-diagonal elements of the first column of **A** are zero.

The argument in the paragraph above can be extended to the second row. If the matrix $\mathbf{K_{FR}}$ is diagonal, then all the off-diagonal elements in its second row must be zero. For the second row, this implies $\mathbf{K_{FR}}_{2j} = \mathbf{A}_{j1}\mathbf{K_{DP}}_{11}\mathbf{A}_{21} + \mathbf{A}_{j2}\mathbf{K_{DP}}_{22}\mathbf{A}_{22} = 0$. But we have established that the off-diagonal elements in the first row of **A** must be zero, so the expression simplifies to $\mathbf{K_{FR}}_{2j} = \mathbf{A}_{j2}\mathbf{K_{DP}}_{22}\mathbf{A}_{22} = 0$. Since the on-diagonal elements of **A** and $\mathbf{K_{DP}}$ are non-zero, it follows that all the off-diagonal elements in the second row of **A** must be zero. So, probabilistic independence of the FRs demands that the off-diagonal elements of the second row of **A** are zero.

The argument above extends naturally to all *n* rows of the design matrix establishing that $\mathbf{K_{FR}}$ is diagonal *if and only if* **A** is uncoupled rather than decoupled. Therefore, since **A** is not uncoupled, the matrix $\mathbf{K_{FR}}$ cannot be diagonal, the FRs cannot be probabilistically independent, and the information content of the design is NOT the sum of the information content of the FRs. □

In the proposition proven above, the requirement that the design matrix must have non-zero on-diagonal elements is not very restrictive. In Axiomatic design, it doesn't make sense to have diagonal elements equal to zero since that would imply that a DP provides no control over its corresponding FR. If any on-diagonal element is zero in an uncoupled or decoupled design, the design matrix will be rank deficient and it will generally not be possible to satisfy all the FRs.

This section proved that information content cannot simply be summed for decoupled designs. This motivates the need for the more sophisticated algorithms presented in the following sections.

5 COMPUTING THE INFORMATION CONTENT OF DECOUPLED DESIGNS

This section will present a means for computing information content of a decoupled design without assuming any specific form of the distribution of the design parameters. In the most general case, the probability of success of a design is the integral of the joint density function of a vector of design parameters $f(\mathbf{DP})$ over the design range

$$p_s = \int_{\text{design range}} f(\mathbf{DP})\,d\mathbf{DP} \qquad (12)$$

The design range is the set of points in design parameter space that satisfy *all* the tolerances on the functional requirements. Let the bi-lateral tolerance on the j^{th} FR be represented as $\delta\mathbf{FR}_j$ and the center of the tolerance range (or target value) of the j^{th} FR be represented as $\tau\mathbf{FR}_j$. If the design is nearly linear within the system range, then the design equation (Equation 1) can be modeled as a linear mapping between design parameters and functional requirements. Under these conditions, the design range is a set defined by a system of linear inequality constraints.

$$\text{design range} = \left\{ \mathbf{DP} \left| \begin{bmatrix} \mathbf{A} \\ -\mathbf{A} \end{bmatrix} \cdot \mathbf{DP} \le \begin{Bmatrix} \tau\mathbf{FR} + \delta\mathbf{FR} \\ -\tau\mathbf{FR} + \delta\mathbf{FR} \end{Bmatrix} \right. \right\} \qquad (13)$$

The matrix on the left hand side of the inequality in Equation 13 is formed by stacking the design matrix and the negation of the design matrix. Similarly, the vector on the right hand side of the inequality in Equation 13 is formed by stacking a vector of upper limits on the FRs and the negation of a vector of lower limits on the FRs.

If the design is decoupled and its design matrix is reordered into lower triangular form and the on-diagonal entries are all positive, it is possible to rearrange Equation 13 into a more useful form in which the linear inequality constraints are applied directly and separately to the DPs

$$\text{design range} = \left\{ \mathbf{DP} \left| \begin{Bmatrix} \mathbf{DP}_1 \\ \mathbf{DP}_2 \\ \vdots \\ \mathbf{DP}_n \\ -\mathbf{DP}_1 \\ -\mathbf{DP}_2 \\ \vdots \\ -\mathbf{DP}_n \end{Bmatrix} \le \begin{Bmatrix} \dfrac{\tau\mathbf{FR}_1 + \delta\mathbf{FR}_1}{\mathbf{A}_{1,1}} \\ \dfrac{\tau\mathbf{FR}_2 + \delta\mathbf{FR}_2 - \mathbf{A}_{2,1}\mathbf{DP}_1}{\mathbf{A}_{2,2}} \\ \vdots \\ \dfrac{\tau\mathbf{FR}_n + \delta\mathbf{FR}_n - \sum_{i=1}^{n-1}\mathbf{A}_{n,i}\mathbf{DP}_i}{\mathbf{A}_{n,n}} \\ \dfrac{-\tau\mathbf{FR}_1 + \delta\mathbf{FR}_1}{\mathbf{A}_{1,1}} \\ \dfrac{-\tau\mathbf{FR}_2 + \delta\mathbf{FR}_2 + \mathbf{A}_{2,1}\mathbf{DP}_1}{\mathbf{A}_{2,2}} \\ \vdots \\ \dfrac{-\tau\mathbf{FR}_n + \delta\mathbf{FR}_n + \sum_{i=1}^{n-1}\mathbf{A}_{n,i}\mathbf{DP}_i}{\mathbf{A}_{n,n}} \end{Bmatrix} \right. \right\} \qquad (1$$

Equation 14 can be adapted to designs with negative on-diagonal elements by switching the constraint for the two corresponding rows of the matrix from "less than or equal to" to "greater than or equal to."

In order to better understand the design range as expressed by Equation 14, it is useful to plot the design range for the case of a design with 2 FRs and 2DPs. In Figure 3, the space of the design parameters is represented as a plane. For a lower triangular design matrix, the tolerances on $\mathbf{FR}_1$ will plot as lines perpendicular to the $\mathbf{DP}_1$ axis. By contrast, the tolerances on $\mathbf{FR}_2$ will be parallel to neither the $\mathbf{DP}_1$ nor the $\mathbf{DP}_2$ axis. The design range (over which the probability density must be integrated) is the set of points satisying all four linear inequality constraints. This set of points will be a parrallelpiped in $\Re 2$ if each FR has both upper and lower bounds. In the more general case that there are *n* FRs and *n* DPs, the design range will be a convex polyhedron in $\Re n$. See Figure 3.

From the expression for the design range given in Equation 14, one can define, in closed form, the upper and lower bounds of integration required to evaluate Equation 12.

Equation 15 below applies only if all the on-diagonal elements of the design matrix are positive. For any negative on-diagonal elements, the upper and lower limits of integration for the corresponding DP must be switched.

$$p_s = \int_{\frac{-\tau\mathbf{FR}_1+\delta\mathbf{FR}_1}{\mathbf{A}_{1,1}}}^{\frac{\tau\mathbf{FR}_1+\delta\mathbf{FR}_1}{\mathbf{A}_{1,1}}} \int_{\frac{-\tau\mathbf{FR}_2+\delta\mathbf{FR}_2+\mathbf{A}_{2,1}\mathbf{DP}_1}{\mathbf{A}_{2,2}}}^{\frac{\tau\mathbf{FR}_2+\delta\mathbf{FR}_2-\mathbf{A}_{2,1}\mathbf{DP}_1}{\mathbf{A}_{2,2}}} \cdots \int_{\frac{-\tau\mathbf{FR}_n+\delta\mathbf{FR}_n+\sum_{i=1}^{n-1}\mathbf{A}_{n,i}\mathbf{DP}i}{\mathbf{A}_{n,n}}}^{\frac{\tau\mathbf{FR}_n+\delta\mathbf{FR}_n-\sum_{i=1}^{n-1}\mathbf{A}_{n,i}\mathbf{DP}i}{\mathbf{A}_{n,n}}} f(\mathbf{DP}_1,\mathbf{DP}_2,\ldots,\mathbf{DP}_n)\,d\mathbf{DP}_n \ldots d\mathbf{DP}_2\,d\mathbf{DP}_1 \quad (15$$

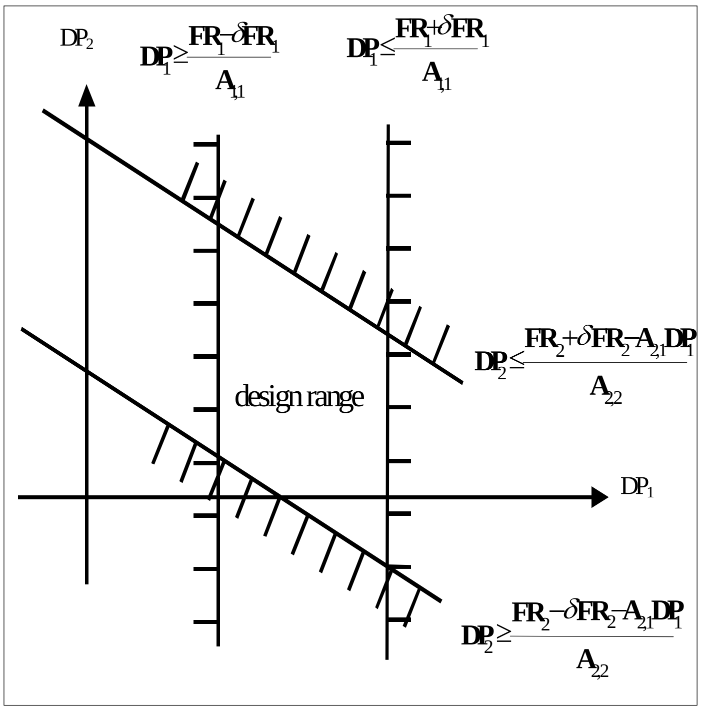

Figure 3.The representation of the design range as a convex polyhedron in ℜ2.

To evaluate the integral in Equation 15 numerically, any of the commonly used numerical algorithms may be employed including Simpson's rule or Gaussian quadrature. Note that the order of integration is essential. For example, $\mathbf{DP}_1$ must be the outermost integral since it is the only integral whose limits are a function of known quantities -- the target value $\tau\mathbf{FR}_1$ and the tolerance $\delta\mathbf{FR}_1$ and the first element of the design matrix $\mathbf{A}_{1,1}$. Each of integrals nested within are functions of all the DP values in the outer integrals.

Equation 15 allows one to numerically compute the probability of success for any joint distribution function over the DPs. The equation will automatically account for any correlation among the DPs. Once probability of success has been properly calculated, it is simple to compute the information content of the design using the log transform (Equation 8).

The computational complexity of deterministic numerical integration of Equation 15 grows non-polynomially with the number of DPs. If information content of a design with many DPs is to be computed, it is often more computationally efficient to use a non-deterministic integration technique such as the Monte Carlo method.

Unfortunately, Equation 15 does not admit closed form solutions even for the simplest cases such as independent, normally distributed DPs. However, there does exist a more efficient calculation procedure than Equation 15 for the special case of uniformly distributed DPs. This algorithm is discussed in the next section.

6 UNIFORMLY DISTRIBUTED DESIGN PARAMETERS

The information content of a design whose DPs are uniformly distributed can be expressed as the log of the ratio of the volumes of two *n* dimensional polytopes where *n* is the number of DPs

$$I = \log\left(\frac{V(\text{system range})}{V(\text{common range})}\right) \quad (16)$$

where $V(\bullet)$ denotes the volume of a set in *n* space. This expression can be viewed an extension into *n* dimensional space of the one dimensional Equation 4 as given by Suh [1].

To explain the meaning and use of Equation 16, let us consider a case in which there are just two DPs which are probabilistically independent and uniformly distributed within their specifications. In this case, the joint probability density function is uniformly distributed within the system range. If the bi-lateral specification on the i^{th} DP is represented as $\Delta\mathbf{DP}_i$, the system range is a rectangle with sides of length $2\Delta\mathbf{DP}_i$, (see Figure 4).

If the bi-lateral tolerance on the j^{th} FR is represented as $\delta\mathbf{FR}_j$ then each tolerance can be represented by two linear inequality constraints. These inequality constraints together define the design range which will be an *n* dimensional polyhedron – it may or may not have finite volume. The intersection of the system range and the design range is the common range which will be an *n* dimensional polytope -- it will have a finite volume less than or equal to that of the system range.

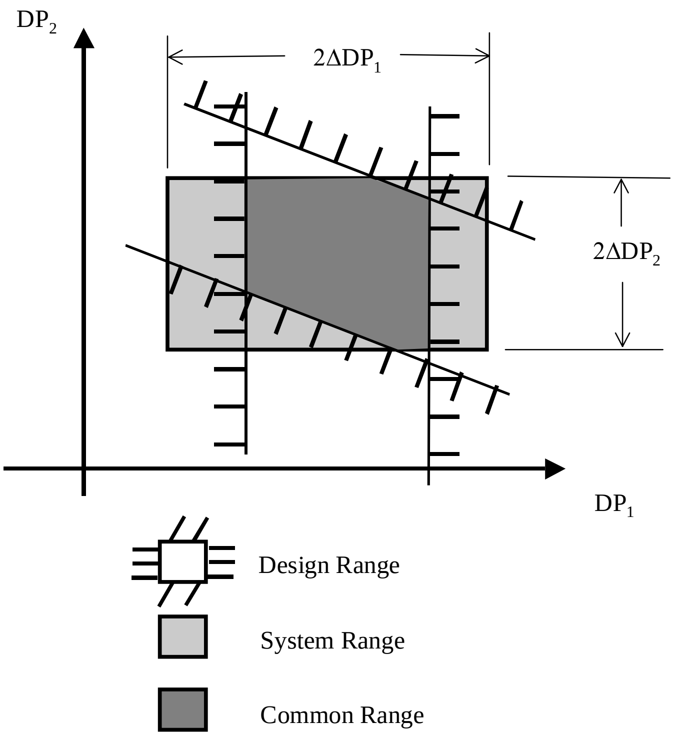

Figure 4. The representation of the system range and common range as convex polytopes in two dimensional space.

To evaluate Equation 16, one must compute the volume of both the system range and the common range. The volume of the system range can be seen by inspection to be

$$V(\text{system range}) = \prod_{i=1}^{n} 2\Delta \mathbf{DP}_i \tag{17}$$

To automatically compute the volume of the common range, one may use the following theorem by Lasserre [8]. Given a convex polyhedron defined by the set of linear inequalities

$$\mathbf{Ax} \le \mathbf{b} \tag{18}$$

the volume of the convex polyhedron satisfying those inequalities is

$$V(n,\mathbf{A},\mathbf{b}) = \frac{1}{n}\sum_{p=1}^{m} \frac{\mathbf{b}_p}{\left|\mathbf{A}_{p,q}\right|} \cdot V(n-1,\tilde{\mathbf{A}},\tilde{\mathbf{b}}) \tag{19}$$

where $\tilde{\mathbf{A}}\mathbf{x} \le \tilde{\mathbf{b}}$ is the system resulting from removing $\mathbf{x}_q$ from the system $\mathbf{Ax} \le \mathbf{b}$ by casting the p^th^ inequality as an equality.

To use Equation 19 to compute the volume of the common range, it is necessary to formulate a mathematical representation of the common range as a set of inequality constraints. This can be accomplished by adding the constraints that define the system range to Equation 14 which defines the design range

$$\text{common range} = \left\{ \mathbf{DP} \;\middle|\; \begin{bmatrix} \mathbf{A} \\ -\mathbf{A} \\ \mathbf{I} \\ -\mathbf{I} \end{bmatrix} \cdot \mathbf{DP} \le \begin{Bmatrix} \tau\mathbf{FR} + \delta\mathbf{FR} \\ -\tau\mathbf{FR} + \delta\mathbf{FR} \\ \mu\mathbf{DP} + \Delta\mathbf{DP} \\ -\mu\mathbf{DP} + \Delta\mathbf{DP} \end{Bmatrix} \right\} \tag{20}$$

where **I** is the n by n identity matrix and μ**DP** is a vector of the mean values of the DPs.

This section provided a conceptual overview of an algorithm for computing information content for decoupled systems with uniformly distributed design parameters. Equations 16-20 are sufficient in principle to enable the reader to carry out the necessary calculations. In practice, there are many implementation details required for correct numerical computation of information content of decoupled designs with uniformly distributed DPs. These details are provided in the Appendix. The next section presents a case study of the use of the two algorithms.

6.1 EXAMPLE APPLICATION – PASSIVE FILTER DESIGN

This case study is an adaptation of Example 4.2 from Suh [1] concerning the design of an electrical passive filter. The example was also discussed by Bras and Mistree [9] who noted some errors in the originally published formulae. We have adopted the corrected formulae for this paper. The two proposed circuit designs are given in Figure 5 as Network a and Network b. The variable values that define the model of the displacement transducer / demodulator and galvanometer are in Table 1. We chose to analyze the design options that employ (as a transducer) the strain gauge bridge rather than the LVDT (Linear-Variable Differential Transformer). The expressions for D and ω_c in terms of the design parameters and the transducer and galvanometer characteristics are from Suh [1] and are presented in Table 2.

The functional requirements of the system have been specified as:

FR1: ω_c = Design a low-pass filter with a filter pole at 6.84 Hz or 42.98 rad/sec.

FR2: D = Obtain D.C. gain such that the full-scale deflection results in ±3 in. light beam deflection.

The two design parameters are:

DP1: C = capacitance.

DP2: R = resistance. The design parameter for network a is R2. The design parameter for network b is R3.

One may solve for the nominal DP values that place the FRs precisely on their target values. Using the formulae in Table 2 and values in Table 1, the DP values that satisfy the FRs are given in Table 3. Taking the appropriate partial derivatives of the equations in Table 2 about the target values of the DPs in Table 3 yields the design matrices in Table 4.

It is clear from inspection of Table 4 that both designs are decoupled. However, it is also true that network A is much more nearly uncoupled than network B. This can be demonstrated by computing the Reangularity of the two designs (Table 4). The Reangularity of network A is nearly unity which is characteristic of a completely uncoupled design. The lower values of Reangularity of network B indicate a higher degree of coupling. Please note that the Reangularity of a matrix depends on the scaling of the rows. To compute the figures in Table 4, which are the same as those published in Suh [1], one must first normalize the rows of the matrix by the nominal values of the FRs.

Now, let us consider the information content of the two designs. To do this, we must specify the tolerances on the FRs and the distribution of the DPs. Let us assume that the tolerances on the FRs are ±5% of their nominal values. Let us further assume that the specification for the resistors is ±10% of their nominal values and that the specification for the capacitor is ±15μF. Let us further assume that the DPs are probabilistically independent. For the distribution shape of the DPs, let us consider two cases – normal and uniform. For normally distributed DPs, let us assume that the tolerance represents ±3σ of the distribution and that the mean is on target. For uniformly distributed DPs, let us assume that the probability density is uniformly distributed throughout the entire specification range.

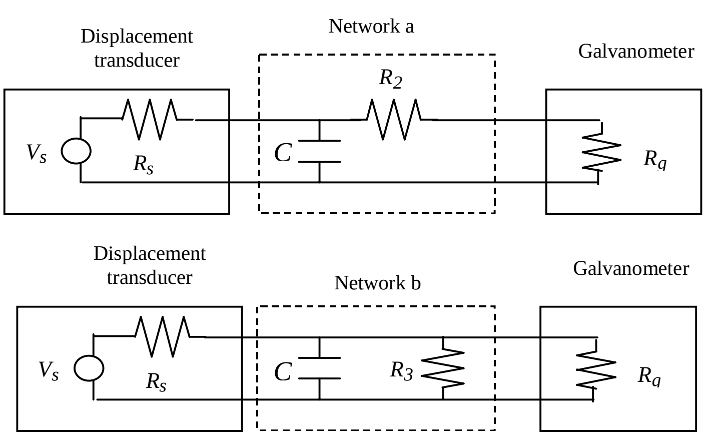

Figure 5. Two proposed network designs for passive filters (adapted from [1])

Table 1. Variable values for the displacement transducer and galvanometer.

Variable	Nominal Value
R_s	120 Ω
V_{in}	0.015 V
R_g	98 Ω
G_{sen}	657.58 μV/in.

Table 2. Equations for Networks A and B.

	Network A	**Network B**
ω_c (rad/sec)	$\dfrac{R_s + R_g + R_2}{CR_s(R_g + R_2)}$	$\dfrac{(R_g + R_3)R_s + R_3 R_g}{CR_3 R_g R_s}$
D (inches)	$\dfrac{R_g V_{in}}{G_{sen}(R_s + R_g + R_2)}$	$\dfrac{R_3 R_g V_{in}}{G_{sen}[(R_g + R_3)R_s + R_3 R_g]}$

Table 3. Design parameter values for networks A and B.

	Network A		**Network B**	
	Mean	Tolerance	Mean	Specification
Capacitor	C=231μF	±15μF	C=1474μF	±15μF
Resistor	R_2=527Ω	$\pm 10\% \cdot R_2$	R_3=22.3Ω	$\pm 10\% \cdot R_3$

Table 4. Design matrices for networks A and B.

Network A		**Network B**	
-1.86x10^5 $\left(\dfrac{\text{rad}}{\text{sec}\cdot\text{F}}\right)$	-1.10x10^{-2} $\left(\dfrac{\text{rad}}{\text{sec}\cdot\Omega}\right)$	-3.00x10^4 $\left(\dfrac{rad}{\text{sec}\cdot F}\right)$	-1.38 $\left(\dfrac{\text{rad}}{\text{sec}\cdot\Omega}\right)$
0 $\left(\dfrac{\text{in}}{\text{F}}\right)$	-4.03x10^{-3} $\left(\dfrac{\text{in}}{\Omega}\right)$	0 $\left(\dfrac{\text{in}}{\text{F}}\right)$	9.50x10^{-2} $\left(\dfrac{\text{in}}{\Omega}\right)$
Reangularity = 0.982		Reangularity = 0.707	

The result of applying the algorithms presented in this paper to the passive filter designs under the assumptions outlined above are summarized in Table 5. In all instances, the probability of success and information content of the designs computed by Equation 15 or 16 were confirmed by Monte Carlo simulation over the non-linear equations in Table 2. This lends evidence that the algorithms are free of error. It also shows that it was reasonable to assume linearity of the FRs within the range of the variability of the DPs.

Table 5. Information content and coupling for networks A and B.

		Network A		**Network B**	
		I (bits)	p_s	I (bits)	p_s
Information content for normally distributed DPs	Integration of pdf (Equation 15)	0.084	94.4%	0.059	96.0%
	Monte Carlo	0.095	93.6%	0.063	95.7%
	Summing information of each FR	0.084	94.4%	0.107	92.9%
Information content for uniformly distributed DPs	Ratio of volumes (Equation 17-20)	0.880	54.4%	0.576	67.1%
	Monte Carlo	0.844	55.7%	0.593	66.3%
	Summing information of each FR	0.887	54.1%	1.038	48.7%

The algorithms of this paper reveal that network B has lower information content than network A and therefore is to be preferred according to the Information Axiom. B was preferred to A regardless of the distribution shape although the uniform distribution led to higher information content in all cases since it is a more pessimistic assumption. This is notable since network B is more coupled than network A. Network B is decoupled and therefore is still an acceptable design according to the Independence Axiom. This suggests that when both uncoupled and decoupled alternatives exist, it is important to evaluate the information content of all the designs before discarding any alternatives. The higher probability of success of a decoupled design may more than compensate for the requirement of selecting the DPs in proper order.

The process of summing information content provided excellent estimates of the information content for network A. The degree of coupling of network A is low enough, as indicated by its high Reangularity, that the FRs can be considered probabilistically independent. When all the design alternatives being considered are fully uncoupled and the DPs are probabilistically independent, the procedure of summing information is reliable.

On the other hand, the process of summing information content provided poor estimates of the information content for network B due to its higher degree of coupling. More importantly, these poor estimates would lead to the wrong choice of design. *In both the normally distributed and uniformly distributed cases, the sum of information content incorrectly indicates that network A is superior to network B.* It is essential to design decision making that the information content of decoupled designs is computed correctly and not simply summed.

7 CONCLUSIONS

This paper has proven mathematically that the information content of a decoupled design is never exactly the sum of the information of the functional requirements (FRs). Decoupled designs always induce some correlation among the functional requirements even when the design parameters (DPs) are independent.

We presented two algorithms for properly computing the information content of decoupled designs. One is a general procedure applicable to any form of distribution of the DPs. The key innovation of the algorithm is the proper computation of the limits of integration based on the tolerances on the FRs, specifications on the DPs and the design matrix. The other algorithm is applicable only if one assumes that all the DPs are uniformly distributed. The technique is based on calculation of the volumes of convex polytopes.

The example application demonstrated that, in some cases, the Information Axiom requires that decoupled designs are to be preferred to uncoupled designs. Decoupled designs induce correlation in the FRs which, all other things being equal, tends to reduce information content of the design. However, the only reliable means for making decisions consistent with the Information Axiom is to accurately estimate the information content of all the feasible design alternatives.

Most importantly, the example application shows that the information content of decoupled designs can sometimes be significantly different from the sum of information of the FRs and that these differences can sometimes critically affect engineering decision making. To ensure that one's design process is consistent with the Information Axiom one absolutely requires algorithms such as those presented in this paper which properly account for correlation among FRs.

8 ACKNOWLEDGEMENTS

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APPENDIX -- IMPLEMENTATION DETAILS OF THE ALGORITHM

Equation 19 is a recursive algorithm which Lasserre used to find symbolic expressions for the volume of a polytope. The algorithm was adapted to create a numerical solution technique by recognizing that the reduced system $\tilde{\mathbf{A}}\mathbf{x} \le \tilde{\mathbf{b}}$ can by an equality constraint and a set of inequality constraints

$$\mathbf{A}_p\mathbf{x} = \mathbf{b}_p \quad \text{and} \quad \tilde{\mathbf{A}}\begin{Bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{q-1} \\ \mathbf{x}_{q+1} \\ \vdots \\ \mathbf{x}_n \end{Bmatrix} \le \tilde{\mathbf{b}} \tag{21}$$

where

$$\tilde{\mathbf{A}}_{i,j} = \mathbf{A}_{\begin{bmatrix} i \text{ if } i<p \\ i+1 \text{ otherwise} \end{bmatrix},\begin{bmatrix} j \text{ if } j<q \\ j+1 \text{ otherwise} \end{bmatrix}} - \mathbf{A}_{\begin{bmatrix} i \text{ if } i<p \\ i+1 \text{ otherwise} \end{bmatrix},q} \frac{\mathbf{A}_{p,\begin{bmatrix} j \text{ if } j<q \\ j+1 \text{ otherwise} \end{bmatrix}}}{\mathbf{A}_{p,q}} \tag{22}$$

and

$$\tilde{\mathbf{b}}_{\begin{bmatrix} i \text{ if } i<p \\ i+1 \text{ otherwise} \end{bmatrix}} = \mathbf{b}_{\begin{bmatrix} i \text{ if } i<p \\ i+1 \text{ otherwise} \end{bmatrix}} - \mathbf{A}_{\begin{bmatrix} i \text{ if } i<p \\ i+1 \text{ otherwise} \end{bmatrix},q} \frac{\mathbf{b}_p}{\mathbf{A}_{p,q}} \tag{23}$$

where $i \in 1..2(m-1)$, $\mathrm{j} \in 1..2(\mathrm{n}-1)$.

The implementation of the recursive algorithm defined by Equation 19 requires some care in implementation. On each evaluation of $V(n,\mathbf{A},\mathbf{b})$, any duplicate inequality constraints must be removed. That is, if two rows of **A** are identical and the corresponding elements of **b** are also the same, then one of the rows must be removed from the system. If not, the algorithm will sum the volume of a single face more than once. It also requires care in selecting which $\mathbf{x}_q$ to remove for any given i since if $\mathbf{A}_{i,q}$ is zero (or very small) an overflow will result.

A Mathcad implementation of the recursive algorithm appears in Figure 6. The first two arguments of the function V(m,n,C) are the number of rows and columns respectively in the matrix **A**. The third argument is the matrix **A** and vector **b** augmented into a single matrix C=$[\mathbf{A}\ \mathbf{b}]$. Note that the function calls itself making the algorithm recursive.

Note that if the polyhedron is unbounded, then the function will return a very large number. This cannot occur in computing the volume of the common range as defined in Equation 20. Also note that if the volume of the i^{th} face as computed by the algorithm below is zero, then the i^{th} constraint is redundant.

The Mathcad implementation in Figure 5 was used for computing values for the example application. It has been validated by reproducing results from the open literature and by comparison to results from Monte Carlo simulations.


```
V(m,n,C) := | if n>1
            |   for i ∈ 2..m
            |     same ← not[ ∏_{i2=1}^{i-1} not[[[(C^T)^<i>]·|(C^T)^<i2>| = [(C^T)^<i2>]·|(C^T)^<i>|]·|(C^T)^<i2>|·|(C^T)^<i>|] ]
            |     ( for j ∈ 1..n+1 )  if same
            |     (   C_{i,j} ← 0  )
            |   [ (1/n)·[ Σ_{i=1}^{m} [ | for j_elim ∈ 1..n
            |                           |    break if C_{i,j_elim} ≠ 0
            |                           | 0 if C_{i,j_elim} = 0
            |                           | otherwise
            |                           |   | for i2 ∈ 1..m
            |                           |   |   | Cp_{i2,n} ← C_{i2,n+1} − (C_{i2,j_elim} · C_{i,n+1} / C_{i,j_elim})
            |                           |   |   | for j ∈ 1..n−1
            |                           |   |   |   Cp_{i2,j} ← [ C_{i2,[| j if j<j_elim ; (j+1) otherwise]} − C_{i2,j_elim} · C_{i,[| j if j<j_elim ; (j+1) otherwise]} / C_{i,j_elim} ]
            |                           |   | ( C_{i,n+1} / |C_{i,j_elim}| · V(m,n−1,Cp) )  if C_{i,j_elim} ≠ 0
            |                           |   | 0 otherwise
            |                         ] ] ]
            | otherwise
            |   | EMPTY ← 0
            |   | for L ∈ 1..m
            |   |   | (EMPTY ← 1) if (C_{L,1} = 0)·(C_{L,2} < 0)
            |   |   | pos_L ← [ | C_{L,2}/C_{L,1}  if C_{L,1} > 0
            |   |   |           | 10^10  otherwise ]
            |   |   | neg_L ← [ | C_{L,2}/C_{L,1}  if C_{L,1} < 0
            |   |   |           | -10^10  otherwise ]
            |   | max([ 0 ; min(pos) − max(neg) ])·not(EMPTY)
```

Figure 6. Mathcad code for computing the volume of a convex polyhedron.