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THE THEORETICAL ASPECTS OF RELIABILITY DESIGN ANALYSED USING AXIOMATIC DESIGN

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ABSTRACT

This paper proposes the use of Axiomatic Design for dealing with issues related to Design for Reliability.
Starting from traditional Axiomatic Design theory, a theoretical approach is studied which enables designers to deal with reliability aspects in a systematic and friendly way.
The aim of this work is to study Reliability theory using and developing Axiomatic Design axioms and theorems. A *Reliability Matrix* is then used to analyze aspects related to the management of serial and parallel mechanical systems/components reliability (*Axiomatic Design for Reliability*).
This kind of approach shows how Axiomatic Design can be integrated with other design methodologies (e.g. reliability techniques), thereby improving management and making it easier to understand the project right from the beginning.

Keywords: Design Matrix, Reliability Matrix, Axiomatic Design for Reliability.

1 INTRODUCTION

In order to obtain a good design, which means, developing the definitive design immediately with no mistakes, experience alone cannot be relied upon but has to be supported by efficacious and efficient design theories and methodologies.
Axiomatic Design is a revolutionary design decision management tool that brings innovation, speed and control to the design process. It supports design development so that functional and reliability requirements and constraints can be satisfied, whilst also considering growing customer needs, especially those related to the fulfillment of *time to market requirements*.
Axiomatic Design "qualitatively" defines the project structure and finds physical solutions (DPs) which satisfy the functional requirements (FRs) with the axioms, allowing this phase to be performed efficaciously and efficiently. *Mapping* and *Zig-Zagging* are used to condense the design into two tree-structures, one for the FRs and one for the DPs; these are hierarchically ordered in levels of increasing detail, and correlated with the design matrices.
The aim of this approach is to optimize design by choosing the components and solutions that provide the best compromise between reliability characteristics and constraints (fixed and maintenance costs, weight, dimensions, etc.). This can be achieved by using a *Reliability Matrix*, [R] and an index, I_R, (*Relative Reliability Information Content*), to define the best "base components", and by then carrying out a "*Reverse Zig-Zagging*", moving from the elementary levels to those immediately above.
This approach has procedures in common with the reliability design of mechanical systems where the methodologies applied are integrated complementarily (i.e. FTA and FMEA/FMECA), so as to provide designers with a general overview, beginning with the idea that creates the design, up to the definition of solutions that characterizes it.

2 AXIOMATIC DESIGN FOR RELIABILITY

To develop a design that will satisfy customer needs, respect constraints, and define products/systems with high reliability, it is necessary to carry out:

1) a functional breakdown which enables the functions of the system and their hierarchy to be defined (system functions, subfunctions, …. elementary functions), and a physical breakdown which characterizes the system structure and identifies all its subsystems, including the elementary components; this breakdown into a hierarchical structure is obtained by zigzagging between the functional and the physical domain.
2) reliability evaluation and design solution management to optimize the system.

The first phase is part of classical Axiomatic Design theory: using a "Top-Down" approach, the system is defined by increasingly detailed levels; Design Matrices are then used to verify that the choice of the DPs "correctly" (that is, satisfying the first axiom) satisfies the FRs for each level.

The second phase known as *Axiomatic Design for Reliability*; which represents an integration to Axiomatic Design, can be divided into:

I) a reliability estimate of the "elementary components" and their subsystems[1];
II) the choice of the best design alternative.

In phases 1) and 2) a mainly qualitative "Top-Down" analysis is carried out whilst phase 3) provides a mainly quantitative optimal definition of the system, working up from the elementary to the system level, with a "Bottom-Up" approach.

I) Reliability Evaluation

The Design Matrices show whether ("X") or not ("0") there is a correlation between FRs and DPs. In Axiomatic Design, for a generic pair FRi-DPj, the question to be considered is: "Does DPj influence the value of FRi or not?". If the answer is "Yes", the element a_{ij} of the Design Matrix will be an "X", if it is "No", it will be a "0".

[1] The term "subsystem" or "subset" means the set of functions (FRs) and their related solutions (DPs) at a certain level. The term "elementary component" means a DP that cannot be broken down any further.

In order to study system reliability, a *Reliability Matrix* [R] is introduced, which shows the relations, in terms of reliability, between achieving functions (FRs), and the components/subsystems (DPs) that satisfy them. To find out whether a_{ij} will be an "R" or an "0", this time the question to be posed is: "Does the probability of satisfying the function *i* (FRi) depend on the reliability of the component *j* (DPj)?". R_{ij} shows the reliability value of the component *j* in relation to the function *i*. So the passage from Design Matrix to Reliability Matrix usually implies more than a simple substitution of "Xs" with "Rs".

It should be noted that the elements of [R] can also be determined by more components in series or parallel; for these[2] we have:

$$R_{Series} = \prod_{i=1}^{n} R_i \tag{1}$$

$$R_{Parallel} = 1 - \prod_{i=1}^{n} \left(1 - R_i\right) \tag{2}$$

The values for R_i can be found in manuals [RAC, 1995] or in reliability databases.

The contribution of each component to the reliability function can be assessed for each row [R]: the product of the Rij in a row (neglecting the "0") gives the function reliability for the corresponding function *"i"* (as it corresponds to the series of these elements).

"Measuring" the number of functions that risk being damaged if one component fails provides an idea of the "criticality" of the components used in that system/subsystem. Analysis of the [R] columns enables the functions to be optimized using the Relative Reliability Information Content, which maximizes the reliability value of each component, whilst respecting the constraints imposed by design decisions already taken (DPs).

Generally, as a design solution, the diagonal matrix is to be preferred; however, a triangular matrix may be used in cases where the use of high reliability components, although in series, might give a higher reliability value for the function than would be obtained with one single component with a lower

[2] The following hypotheses must be satisfied:
1) a constant failure rate for all the components;
2) the components must be either in perfect working order or broken (intermediate working conditions are excluded);
3) component failures must be independent of one another (i.e. there must be no connection between the failure of one component and the failure of others).

reliability value (for instance, if we replace one element with R = 0.75 with two elements in series, both with R = 0.9, this series will give $R_{Function}$ = 0.81>0.75).

II) Choice of the best design alternative through the information content

As said above, the reliability of each function is obtained from the product of the reliability values in each row; if this value is below the minimum consented by the project specifications, it has to be increased. There are different ways of doing this; for instance:

a) using components with higher reliability;

b) putting two (or more) elements in parallel.

Solution b) may result in a complex design because if the number of physical components increases it is more likely that one of them will not meet specification requirements.

Generally these solutions result in an increase in costs and/or weight and dimensions; it is, therefore, advisable to under-take a "cost/benefit" analysis before deciding whether or not to make these changes. If this is not sufficient it is necessary to return to the upper levels and reexamine the design decisions previously taken, i.e., to choose different DPs; for instance, a solution that uses fewer elements, on the principle that "it can't break, if it is not there".

Similarly to the second axiom, it is possible to define the index I_R, (*Relative Reliability Information Content*). I_R ($=I_1/I_2$) makes it possible to choose the solution that optimizes the system, i.e. the one that represents the best compromise between reliability and constraints. The number and type of factors to be considered may differ from one project to another, depending on the aims to be achieved: in this paper, initial costs, maintenance costs, weight and dimensions are used.

So I_R is:

$$I_R = \frac{R_2}{R_1} * K_A * K_B * K_C * K_D \qquad (3)$$

where:

R_1, R_2 = Reliability for components 1 and 2;

K_A = (Initial costs component 1) / (Initial costs component 2);

K_B = (Maintenance costs component 1) / (Maintenance costs component 2);

K_C = (Weight component 1) / (Weight component 2);

K_D = (Dimensions[3] component 1) / (Dimensions component 2).

If $I_R<1$ (i.e. $I_1<I_2$) solution "1" is more convenient, vice versa if $I_R>1$ solution "2" is to be preferred. If one or more of the above factors (initial costs, maintenance costs, weight and dimensions) are not important for the comparison between solution "1" and "2", the corresponding factor is assigned a fixed value of one (e.g., if the dimensions are not important for the choice, then $K_D = 1$).

To optimize the Reliability Matrices the "best component" for each function must be chosen; it is then necessary to check whether it convenient to use two (or more) identical elements in parallel (i.e., two identical elements in parallel with R = 0.6 each, are equivalent to a single element with R = 0.84; this increase in reliability may, however, result in an increase of costs, weight and dimensions, that exceeds the limits set by project specifications). As in the previous situation, the ratio between the reliability information content of the solution with one component ("1") and the solution with two (or more) components in parallel ("2") is considered.

On the basis of what has been said so far, the following *Reliability Theorem* can be enunciated*:*

"The better of two given systems is the one that represents the best compromise between reliability and constraints , i.e the one with the Lower Reliability Information Content ($I_{min} = I_i$ *when* $I_i<I_j$)*".*

[3] For the dimensions, we can consider the volume, the area (if one of the dimensions is much smaller than the others) or the length, width, or height (if one of the dimensions is much larger than the others).

2.1 CASE STUDY

In order to analyze and apply *Axiomatic Design for Reliability*, an example is taken from a hydraulic circuit[4] (figure 1):

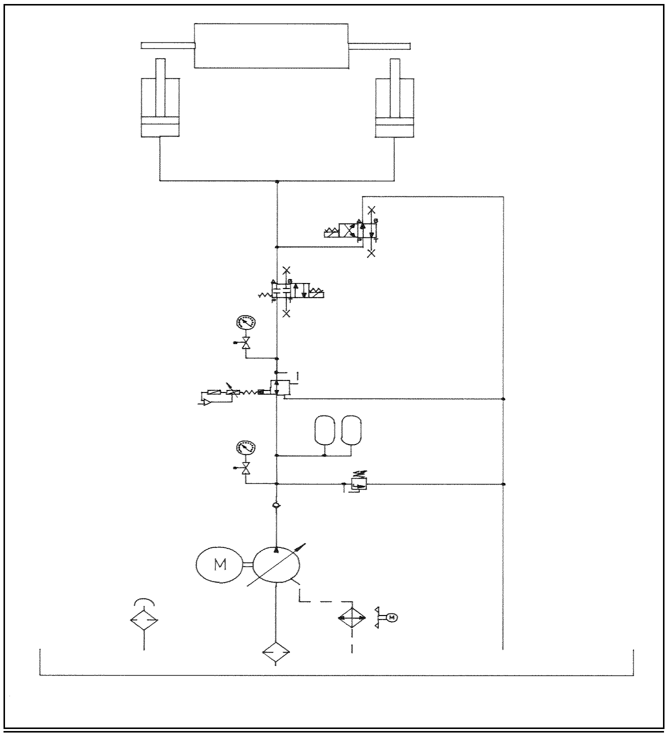

Figure 1 The hydraulic circuit for a rolling machine

Table 1. Functional Breakdown

Functions (FRs)
FR1 = provide oil thrust
FR2 = activate roller
FR3 = push roller
FR11 = filter oil
FR12 = rotate pump
FR13 = intake oil from the tank
FR14 = limit pressure in the system
FR15 = prevent oil from going back
FR21 = give instantaneous upper capacity
FR22 = measure pressure in the control group
FR23 = discharge cylinders
FR24 = separate cylinders from system pressure
FR25 = adjust system pressure
FR31 = measure leather width
FR32 = measure leather thickness
FR33 = transmit oil
FR34 = move roller

Table 2. Physical Breakdown

Components (DPs)
DP1 = power group
DP2 = control group
DP3 = user group
DP11 = filter
DP12 = electric motor
DP13 = pump
DP14 = relief valve
DP15 = check valve
DP21 = accumulators
DP22 = manometer
DP23 = directional valve normally open
DP24 = directional valve normally closed
DP25 = electrohydraulic proportional valve
DP31 = photoelectric barrier
DP32 = thickness measuring set
DP33 = piping
DP34 = cylinders

In the case being examined the Design parameters (DPs) represent physical components.

The Design Matrix at the first level is:

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix}$$

for the Power Group (henceforth called "P.G."), DP1, we have:

$$\text{P.G.} \quad \begin{Bmatrix} FR11 \\ FR12 \\ FR13 \\ FR14 \\ FR15 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 \\ 0 & X & X & 0 & 0 \\ 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & X \end{bmatrix} \begin{Bmatrix} DP11 \\ DP12 \\ DP13 \\ DP14 \\ DP15 \end{Bmatrix}$$

for the Control Group (called "C.G."), DP2, we have:

[4] The hydraulic circuit (here a simplified version is examined) for a rolling machine for sole leather, [Arcidiacono, 1997].

$$\text{C.G.} \quad \begin{Bmatrix} FR21 \\ FR22 \\ FR23 \\ FR24 \\ FR25 \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & X \end{bmatrix} \begin{Bmatrix} DP21 \\ DP22 \\ DP23 \\ DP24 \\ DP25 \end{Bmatrix}$$

The Reliability Matrices for DP1 and DP2 are:

$$\text{P.G.} \quad \begin{Bmatrix} FR11 \\ FR12 \\ FR13 \\ FR14 \\ FR15 \end{Bmatrix} = \begin{bmatrix} R11 & 0 & 0 & 0 & 0 \\ 0 & R12 & 0 & 0 & 0 \\ R11 & R12 & R13 & 0 & 0 \\ 0 & 0 & 0 & R14 & 0 \\ 0 & 0 & 0 & 0 & R15 \end{bmatrix} \begin{Bmatrix} DP11 \\ DP12 \\ DP13 \\ DP14 \\ DP15 \end{Bmatrix}$$

$$\text{C.G.} \quad \begin{Bmatrix} FR21 \\ FR22 \\ FR23 \\ FR24 \\ FR25 \end{Bmatrix} = \begin{bmatrix} R21 & 0 & 0 & 0 & 0 \\ 0 & R22 & 0 & 0 & 0 \\ 0 & 0 & R23 & 0 & 0 \\ 0 & 0 & 0 & R24 & 0 \\ 0 & 0 & 0 & 0 & R25 \end{bmatrix} \begin{Bmatrix} DP21 \\ DP22 \\ DP23 \\ DP24 \\ DP25 \end{Bmatrix}$$

It is obvious that if there is a correlation between the DPs and the FRs in the design matrix this correlation will also be found in the reliability matrix.

The reliability of components which although not directly satisfying a given FR can reduce the probability of meeting that functional requirement must also be considered.

This is because the failure of a component, even when it is not directly involved in performing a function, may prevent that function from being achieved (e.g., if the filter is blocked, then the pump can no longer "intake oil from the tank"). For instance, for the matrix P.G. it should be noted that the "most critical" elements, or rather, the elements which require greater attention during the design phase are the filter (DP11) and the electric motor (DP12), as their failure would damage other functions (respectively, FR11 and FR13 for the filter, and FR12 and FR13 for the motor).

The Reliability Matrix C.G. is diagonal and corresponds to the situation where the reliability of each function depends on the reliability of one component, while in the triangular matrices (i.e. P.G.) the reliability of one or more function depends on the reliability of more components. For instance, the probability of satisfying from the reliability point of view, FR13, "intake oil from the tank", depends on the reliability of the filter, the motor and the pump (in fact, if one of these components fails, it is no longer possible to intake oil); so for FR13 we obtain: P_{FR13} = R11*R12*R13. (It should be noted that in this preliminary phase of reliability analysis it was not considered necessary to study the presence of possible causes of breakdown.[7]

" The probability of satisfying the highest-level FRs is related to the probability of satisfying the lowest- level FRs. Therefore, the probability of satisfying the highest-level FRs is given by the product of all the probabilities associated with all the lowest-level FRs in the system architecture.(Bottom up)" [3]

The values for R_{ij} also depend on the kind of components to be used and on the operating conditions. For instance, suppose that for the filter (DP11) there is a choice between two different elements, filter "A" and "B", with the following characteristics[5]:

Table 3. Data for choosing the filter

	Filter A	**Filter B**
R	0.75	0.75
Initial cost	20 $	17 $
Maintenance costs	4 $/year	4.5 $/year
Weight	150 gr.	130 gr.
Dimensions[6]	-	-

When these values are applied to equation (3) the following value is obtained:

$$I_R = \left(\frac{I_A}{I_B} \right) = \frac{0.75}{0.75} * \frac{20}{17} * \frac{4}{4.5} * \frac{150}{130} \cong 1.20$$

As I_R>1 filter B represents the best compromise between reliability and constraints, as its Reliability Information Content is smaller than that of filter A.

We have to consider that a "cost/benefit" evaluation, such as the one carried out using the index I_R, provides the best alternative among those that satisfy the design requirements. Therefore, if the reliability

[5] Data relative to filters made by "SOFIMA HYDRAULICS"; the values are calculated from the failure rates, assuming an exponential distribution and an operating time of 5000 hours.

[6] The dimensions are not important, that is, K_D = 1.

value for a function is smaller than the minimum consented by project specifications, that design solution can no longer be considered. For instance, if the minimum value for R_{FR11} is 0.7, then the best design solution would be the use of a single filter "B".

However the reliability of filter "B" is quite low. If R_{FR11} is assumed to have the minimum value 0.9, another filter has to be chosen. To increase the filter's reliability, two "B" filters could be used in parallel (R=0.938). Another possibility would be to invest more money in filter maintenance (increased maintenance increases reliability); for instance, with $14 maintenance filter reliability increases to R=0.95 Once again, with the *Reliability Theorem*, we can choose between the use of two filters "B" in parallel (case 1) or a single filter "B" with increased maintenance costs (case 2);

Table 4. Data for choosing the filter

	Case 1	Case 2
R	0.938	0.95
Initial cost	34 $	17 $
Maintenance costs	9 $/year	14 $7year
Weight	260 gr.	130 gr.
Dimensions	-	-

Using equation (3):

$$I_R = \left(\frac{I_A}{I_B}\right) = \frac{0.95}{0.938} * \frac{34}{17} * \frac{9}{14} * \frac{260}{130} \cong 2.6$$

As $I_R>1$, increased maintenance costs represent a better way of increasing filter reliability than using two filters in parallel (case 2), as this solution has the lower Reliability Information Content. It should be noted (although this is not the case in the example examined here) that a reduction in component numbers does not automatically reduce the value of I because a design is defined as "complex" in two different situations: where the probability of meeting certain requisites is low and where the information content is high.

In the same way we can choose the other components for the Power Group. First of all, the *Reliability Theorem* is used to find the best element among those available (for the electric motor the elements available are the pump, the relief valve and the check valve). The reliability value of each element is examined; if it is smaller than the minimum consented in the project specifications, it is increased; the solution which represents the best compromise between reliability and design constraints is found using the index I_R.
For the Power Group of the hydraulic circuit the "optimized" values are: R_{11}=0.95; R_{12}=0.98; R_{13}=0.94; R_{14}=0.99; R_{15}=0.98. As the function reliability for function *"i"* is determined by the product of the R_{ij} on row *"i"*, the values obtained are: R_{FR11}=0.95; R_{FR12}= 0.98; R_{FR13}= 0.88; R_{FR14}=0.99; R_{FR15}=0.98. If one or more of these values is lower than those consented in the project specifications, it is once again necessary to look for solutions that increase that value. For instance, the minimum reliability value for R_{FR13} in this case is fixed at 0.85; if it were fixed at 0.9, it would be necessary to choose another filter, motor and/or pump, till the reliability value for the function became admissible.

3. CONCLUSIONS

This paper has attempted to define how Axiomatic Design can be integrated with reliability analysis to manage and optimize systems, in the field of *Design for Reliability*.
In particular, it has been shown how the introduction of reliability values ("R") for the components (DPs) chosen to satisfy the FRs, leads to the definition of a Reliability Matrix, [R]. This matrix presents two different "reading keys": a row by row analysis which provides the reliability of the system functions, and a column by column examination that provides information on the number of functions that a single component could prevent if it were to fail; in this way it is possible to ascertain which are the "most critical" components, that require more attention during the design phase.
Moreover, the *Relative Reliability Information Content,* I_R was defined, and a *Reliability Theorem* was introduced to identify the components which represent the best compromise between reliability and design constraints.

This allows a more complete design approach which is capable of taking the system's reliability specifications into consideration from the start. (*Axiomatic Design for Reliability*). Respecting the increased need for "Integrated Design", this research shows how Axiomatic Design can be integrated with other design methodologies (in this case, with "Top-Down" and "Bottom-Up" techniques), thereby providing and guaranteeing the choice of the best possible design solution, in terms of reduced developing times, increased quality and higher reliability of the designed system.

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