F-GRANULAR DESIGN INFORMATION BASED INFORMATION AXIOM

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ABSTRACT

Design-relevant information is sometimes vague or linguistic (technically speaking, f-granular) (e.g., design a slim shaft with light weight where dimensions are critical than weight). This puts obstacle in practicing the information axiom (minimize the information content of the design) of axiomatic design theory, a theory that establishes a science base for design. In such a situation, how one should practice the axiom is an important issue for investigation. This paper proposes a version of information axiom, suitable for f-granular information, as follow: "maximize the coherency (overall definiteness) of design information." It is the consequence of two linear measures of definiteness of f-granular information. As axiomatic design farther moves into the age of design automation, various machine intelligence techniques capable of computing words rather than numbers will be a matter of investigation. The proposed version of information axiom might help achieve the goals of such investigation.

Keywords: Axiomatic Design, Information Axiom, f-granular Information, Definiteness (Fuzzy) Measures.

1 INTRODUCTION

Axiomatic design theory [Suh, 1990] establishes a science base for design. One of the key concepts of axiomatic design is to apply the *information axiom*, while mapping functional requirements (design goals) into design parameters (means to meet the goals). The axiom is as follows.

Minimize the information content or maximize the probability of success of the design.

In applying this axiom, the identification of information content from design-relevant information is very important. The exiting methods for practicing information axiom are mostly based on probability theory, which by nature assumes that information available should be either in numeric form (e.g., length is 250 mm) or in c-granular form (e.g., length is in between 240 mm to 280 mm). See, for example, the works of Frey, et al. [2000], Lim and Helander [2000]. When a design becomes decoupled or coupled, i.e., the relationships among functional requirements and design parameters becomes complex, resulting tedious calculation in finding out the exact amount of information content.

Sometimes, design-relevant information is rather vague and expressed linguistically (e.g., easy to use, length should be long, surface is smooth, color is bright or dark, diameter could be small, and so on). See, for example, the works of Feng, et al. [2001] and Shin, et al. [2001]. This kind of information is known as f-granular information [Zadeh, 2001]. As f-granularity of information put it beyond the reach of probability theory based predicate logic [Zadeh, 2001], there is a need for developing methods that help practice information axiom when design-relevant information is predominately f-granular.

Conceptually, a granule is a *perception* that naturally comes into being in our mind. Computationally, a granule is a clump of objects (crisp points). While clumping, the granule assigns a degree of belief to each crisp point showing how strongly the point belongs to the granule. In other words, a granule (g) corresponds to a set of ordered pairs $g = \{\ldots, (c, B(g, c)),\ldots\}$, which is known as fuzzy set [Zadeh, 1965]. The first coordinate of such an ordered pair contains a crisp point (c), which is a member of a classical set known as universe of discourse (U), $c \in U$, containing all the crisp points of interest. And, the second coordinate contains the amount of degree of belief, $B(c, g) \in [0, 1]$ showing how strongly g members c or dismembers c. For example, in a granule called long, the ordered pair (250 mm, 0.7) encodes the information that length = 250 mm belongs to the granule *long* with a degree of belief 0.7.

This paper therefore proposes a version of information axiom applicable when the design-relevant information is predominantly f-granular. The section called definiteness axioms deals with different facets of definiteness of information expressed by using granules. The next section presents the measures to quantify the definiteness. Then overall definiteness is discussed in the following section that suggests a version of information axiom suitable for f-granular information. The example section shows how to apply the axiom in order to continue the design in a definite way, even though there is an amount of imperfection in the information used.

2 DEFINITENESS AXIOMS

The following four definiteness axioms refer to some self-evident truths of information expressed by using granules. They also provide some notions helpful in identifying definiteness measures.

Axiom 1 (Local Definiteness)

Local definiteness axiom refers to the definiteness of information of "c" in term of "g," a granule. As B(c, g) = 0 means that g totally dismembers c and B(c, g) = 1 means that g fully members c, for B(c, g) = 1 and 0, the information is *local definite*. On the other hand, as B(c, g) = 0.5 means that g equally members and dismembers c, for B(c, g) = 0.5, the information is *local indefinite*. For B(c, g) ≠ 0, 0.5, 1, the information is in between local definite and local indefinite, i.e., *partial local definite*.

Axiom 2 (Global Definiteness)

Global definiteness axiom refers to the definiteness of information of c in term of all granules considered, g_i, (i = 1,…,ng), (ng is the number of granules considered). If only one of the granules fully members c and others fully dismembers it, (B(g_j, c) = 1, B(g_i, c) = 0, for all i, = 1,…, ng, $i \neq j$), then the information is *global definite*. Contrary to this situation, if all granules equally member and dismember c, (B(g_i, c) = 0.5 for all i = 1,…,ng), the information is *global indefinite*. This is a case which reflect the fact that the perception is not at all clear, the

Axiom 3 (Granular Definiteness)

Granular definiteness axiom refers to such definiteness of information which is affected by the number of granules (ng), i.e., number of granules considered in expressing a piece of information is an important issue. Sometimes the enhanced number of granules makes a piece of information more definite, sometimes not. In general, granular definiteness increases with increase in ng if there are possibilities of getting more local definite information. Otherwise, the information becomes less granular definite. For example, consider the cases in Table 1. For Case 1 and 2, the crisp value is constant. For Case 1, 3 granules has been used to express the information, for Case 2 which is 5. Case 2 is more definitive than Case 1 because for Case 2 three granules are local definite but for Case 1 only one granule is logical definite.

However, ng depends on how critical or important an issue is. The more critical we are, the more granules should come into consideration. For example, one may define f-granular information of length using three granules, namely, short, moderate, and long. Someone else may use one more, very short, giving more importance to length than the previous person does.

Table 1. F-granular information.

	Granule(g)	**B(g, c)**
Case 1	High	0.85
	Medium	0.25
	Low	0
Case 2	Very High	1
	High	0.85
	Medium	0.25
	Low	0
	Very Low	0

Axiom 4 (Desire Definiteness)

Desire definiteness axiom refers to the definiteness of information in terms of the granule desired (dg) and other important granules (e.g, the granule corresponding to the maximal degree of belief, i.e., the most significant granule (sg), the granule corresponding to the minimal degree of belief, i.e., the most insignificant granule (ig)). The information is *desire definite* if B(dg, c) ≥ B(sg, c). The information is *desire indefinite* if B(dg, c) ≤ B(ig, c). The information is in between desire definite and desire indefinite, i.e., *partial desire definite*, if B(ig, c) < B(dg, c) < B(sg, c).

For example, if dg : = High, Case 1 in Table 1 becomes desire definite because the granule desired overlaps the most significant granule, B(dg, c) = B(sg, c). But the other case is partial desire definite because B(ig, c) < B(dg, c) < B(sg, c).

3 DEFINITENESS MEASURES

There are measures to express the fuzziness (definiteness) of information expressed by using granules. See, for example, the works of Pal and Bezdek [1994], Pal [1999], Marichal and Roubens [2000]. The measures developed are based on Choquet integral, Sugeno's fuzzy integral, Shannon's (non-f-granular) information theoretic-measures (entropy), and Kaufmann's linear index.

However, the approach here is to derive measures that are very simple in nature and recognize the local, global, granular, and desire definiteness axioms as straightforwardly as possible. As such, the measures take the concepts of "B(g, c)," "ng," "dg," "sg," and "ig" because they are related to the definiteness axioms as mentioned in the above.

The start is defining a measure based on local definiteness axiom. The following linear function of B(g, c) called linear local function, denoted by L(g, c), can be used in order to find out the amount of local definiteness:

$$L(g, c) = \frac{B(g, c)}{0.5} \qquad \text{if } B(g, c) < 0.5$$
$$= \frac{1 - B(g, c)}{0.5} \qquad \text{Otherwise} \qquad . \qquad (1)$$

L(g, c) becomes zero when the information is local definite, becomes unit when the information is local indefinite, and becomes a value in between zero and unit when the information is partial local definite. See Figure 1 for the illustration.

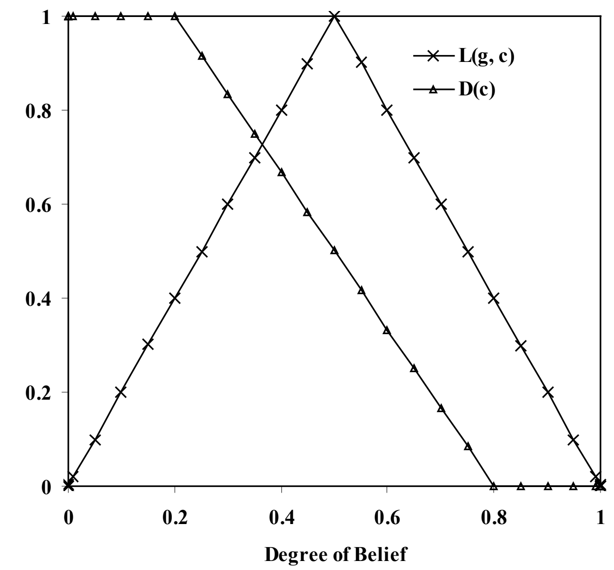


Figure 1. Illustration of L(g, c) and D(c).

In order to capture the aspects of global definiteness and granular definiteness along with local definiteness into a single function, L(g*i*, c) for all *i* = 1,…,ng should be added and, then, should be factored by "ng" so that for when the condition of global indefinite prevails, the function produces it maximal value equal to unit. Such a function is called local, global and granular definiteness function, denoted by G(c). As such, the expression of G(c) is as follows:

$$G(c) = \frac{1}{ng} \times \sum_{i=1}^{ng} L(gi, c) \, . \qquad (2)$$

G(c) becomes zero when the information is global definite, becomes unit when the information is global indefinite, which is most unlikely to occur. Otherwise, it is in between zero and unit and the value depends on the number of granules. For the Case 1 in Table 1, G(c) = (1/3)×(0.3 + 0.5 + 0) = 0.8/3 = 0.267. For the other case, G(c) = (1/5)× (0 + 0.3 + 0.5 + 0 + 0) = 0.8/5 = 0.16. This indicates that the information in Case 1 is less definite than that of Case 2.

On the other hand, in order to measure the aspect of desire definiteness axiom, degree of belief of dg, sg, and ig should be considered in a measure. One of the considerations is to compare the difference between B(dg, c) and B(dg, c) with that of B(sg, c) and B(ig, c). Such a measure is called desired measure, denoted by D(c). The expression is as follows:

$$D(c) = 1 \qquad \text{if } B(dg, c) < B(ig, c)$$

$$= 0 \qquad \text{if } B(dg, c) > B(sg, c) \, .$$

$$= \frac{|\, B(dg, c) - B(sg, c) \,|}{B(sg, c) - B(ig, c)} \qquad \text{Otherwise}$$

(3)

Figure 1 shows the nature of D(c) when B(ig, c) = 0.2, B(sg, c) = 0.8, and B(dg, c) varies from zero to unit.

D(c) becomes zero when the information is desire definite, becomes unit when the information is desire indefinite, and becomes a value in between zero and unit when the information is partial desire definite. For Case 1 in Table 1, if dg = High, then D(c) = 0, i.e., the information is desire definite. For the other case, if dg remains the same, then D(c) = 0.15/1 = 0.15, i.e., partial desire definite.

4 OVERALL DEFINITENESS

The notion of overall definiteness refers to the combined definiteness of f-granular information measured by local, global and granular definiteness function, G(c), and by desire definiteness function, D(c). As such, the ordered pair (G(c), D(c)) expresses the overall definiteness of a piece of f-granular information. Therefore, overall definiteness can be visualized by the graph where D(c) is on the vertical axis and G(c) is on the horizontal axis. A point on such a graph is called *definiteness position.*

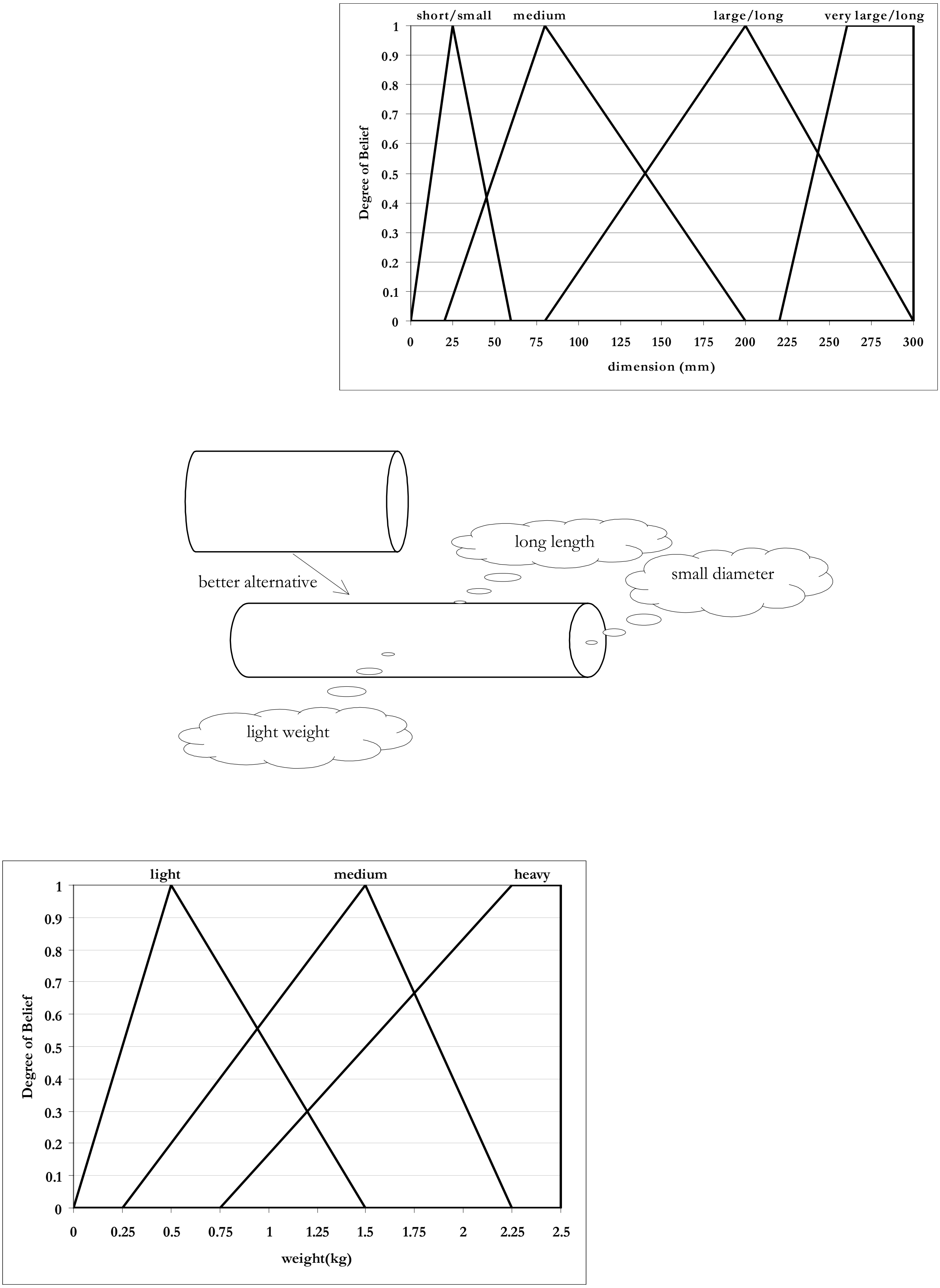


Figure 2. Design perception with f-granular information.

If the definiteness position of a piece of f-granular information is (0, 0), then it is *perfect overall definite*, which is possible when global definite and desire definite exist together. Otherwise, the information is *partial overall definite*.

Two or more pieces of f-granular information (not necessarily belong to the same universe of discourse) are said to be *coherent* if their definiteness positions are reasonably close to each other.

While translating functional requirements into design parameters, the goal should be to keep the definiteness positions of each piece of information as near to the origin as possible keeping, at the same time, the coherency as strong as possible. Otherwise, a design contains design parameters that meet different functional requirements with different level of definiteness.

In synopsis,

"*maximize the coherency (overall definiteness) of the design information*"

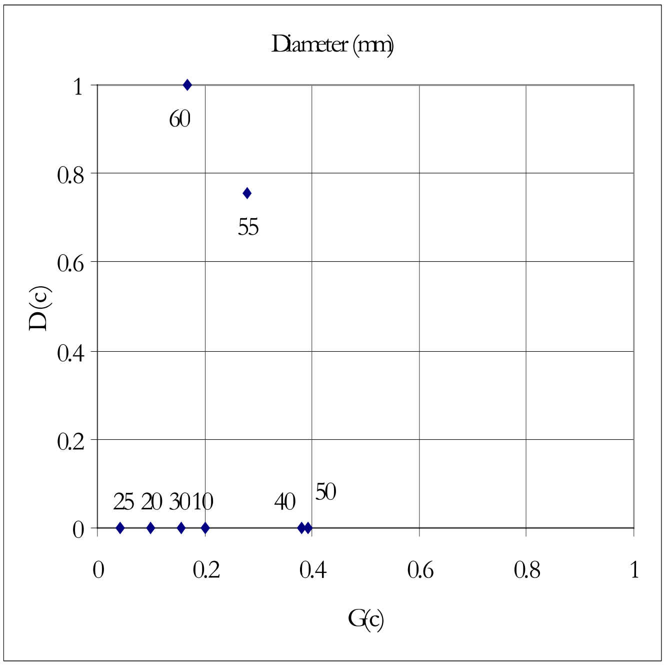
is perhaps another version of information axiom of the axiomatic design applicable if the design-relevant information is predominantly linguistic or perception based.

5 EXAMPLE

Consider that a designer is in the phase of *zigzag*. Due to the complexity of the design, the designer could (at best) express his/her perception on a design object (say, shaft) by the following expression.

"*Design a slim shaft with light weight where dimensions are critical than weight.*"

The expression can be visualized by the illustration in Figure 2. As seen from Figure 2, the designer desired a granule *long* for length, *small* for diameter, and *light* for weight. The dimension universe of discourse contains four granules labeled short/small, medium, large/long, and very large/long to divide the dimension points from 0 mm to 300 mm. The weight universe of discourse contains one granule less (labeled light, medium, and heavy) to divide the weight points from 0 kg to 2.5 kg. The weight universe of discourse contains less number of granule compare to that of the dimension universe of discourse due to the fact that "*dimensions are critical than weight.*"



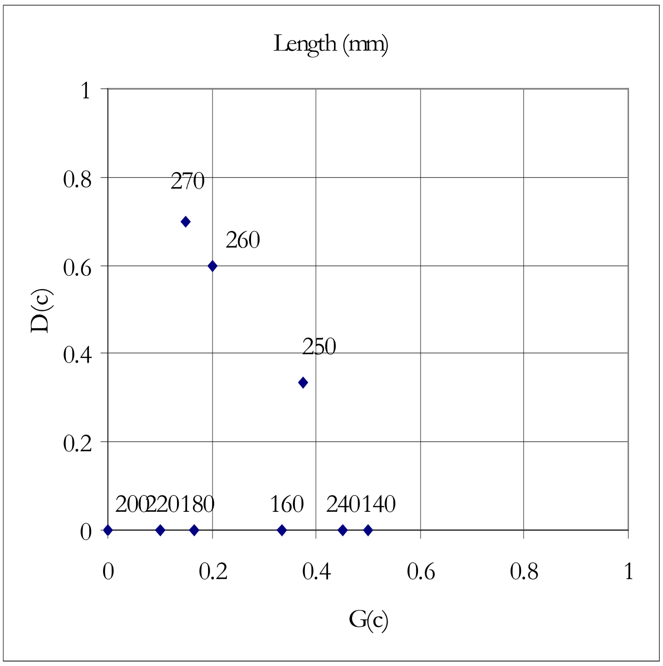


Figure 3. Definiteness positions of dimensions.

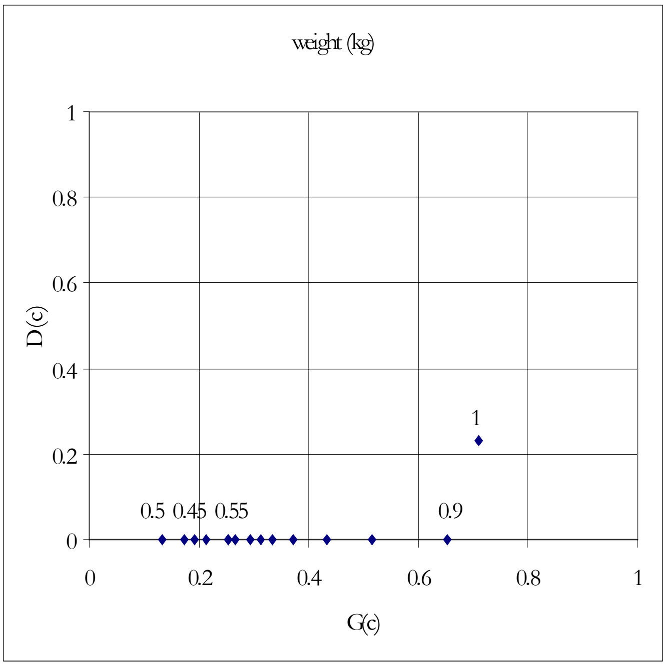


Figure 4. Definiteness positions of weight.

Figure 3 shows the definiteness positions of some length, diameter, and Figure 4 shows definiteness positions of some weight points. From these figures, it is clear that definiteness positions of some dimension and weight points are very close to each other, some are not.

According to the proposed version of information axiom the dimensions and weight points one the line D(c) = 0 with $0 \leq G(c) \leq 0.2$ are the most appropriate dimensions and weight for the shaft. As such, the shaft diameter = 25 mm, length = 200 mm, and weight = 0.5 kg are the best diameter, length, and weight as far as the definiteness of information is concerned.

However, weight point 0.5 kg corresponds to G(0.5 kg) = 0.133. The coherent diameters are the diameters with G(dimension points) $\approx$ 0.133, which are 18.5 mm and 29 mm. Similarly, the coherent length points are 184 mm and 221.5 mm.

Therefore, if we emphasize the phrase "design a slim shaft," we then need to sets shaft design parameters as follows: Length = 221.5 mm, Diameter = 18.5 mm, Weight = 0.5 kg.

If in the design perception contains a constraint, slenderness should be near to 5, then the design parameters should be set to: Length = 184 mm, Diameter = 29 mm, Weight = 0.5 kg.

Further investigation is however needed in order to find a suitable computational framework and underlying algorithms that enable efficient use of "maximize the coherency among the pieces of f-granular-design-relevant information."

6 CONCLUSIONS

This paper proposed a version of information axiom applicable when the design-relevant information is predominantly f-granular (linguistic or vague), as follows, "maximize the coherency (overall definiteness) of design information." It was the consequence of two linear measures of definiteness, incorporating the concepts of degree of belief, number of granules, most significant granule, most insignificant granule, and granule desired. . The example demonstrated that even though the design information is vague, the axiom was helpful in continuing the design in a definite way. As axiomatic design farther moves into the age of design automation, various machine intelligence techniques capable of computing words rather than numbers will be a matter of investigation. The proposed version of information axiom might help achieve the goals of such investigation.

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