Proceedings of ICAD2006
4th International Conference on Axiomatic Design
Firenze – June 13-16, 2006
ICAD-2006-02

AMENDING AXIOM II TO ACHIEVE A SIX SIGMA DESIGN

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ABSTRACT

A six sigma design delivers the design functionality on target with little variation around it. Consequently, it has a wide margin to accommodate errors in the manufacturing and usage of the design. Over the years, methods and metrics have been developed to achieve six sigma designs. They are collectively called Design for Six Sigma (DFSS).

One design principle behind DFSS is that it must focus on the target rather than the range of design functionality. Axiom I conforms to such principle since functional independence provides a metric for ease of tuning to the design target. By contrast, information content in Axiom II by definition is focused on the probability of exceeding the design range. Therefore Axiom II needs to be amended. In this paper, we proposed and developed the sensitivity of the design to deviate from its target as a metric to implement Axiom II. Together, the design matrix of Axiom I and the proposed sensitivity matrix of Axiom II, should bring Axiomatic Design to the forefront of DFSS.

An example is used to illustrate the proposed method and metric for implementing Axiom II to achieve a six sigma design.

Keywords: Design for Six Sigma, Axiomatic Design, Robust Design, independence axiom, information axiom, design matrix, sensitivity matrix, variation reduction

1 INTRODUCTION

Sigma is a process metric based on the statistical measure called the standard deviation denoted by the Greek symbol σ. For a 3σ specification limits, a 3σ process would have 99.73% of its performance within the specification limits. It has a 0.27 % chance that a defect will be produced. By contrast, a 6σ process performance fits well within the specification limits, 99.9999998% of it. So that not only is the chance of producing a defect negligibly small, but there is also an additional margin to absorb any shift in the process performance that may arise. Because testing, analyzing and fixing defects increase cycle time, systematic methods have been developed over the years to eradicate defects through process performance improvement and the achievement of 6σ. These systematic methods, e.g., DMAIC, are collectively called the Six Sigma process.

Experiences have shown that there is a diminishing return in Six Sigma process as we test, analyze and fix defects to approach a larger sigma. This is because an ill-conceived design cannot be improved by subsequent test, analyze and fix. It is from the realization that quality of a product can not be evolved but has to be designed that the Design For Six Sigma (DFSS) was born.

There are many versions of DFSS in the market place today. They generally consist of define design requirements, conceive design solutions, optimize the concept and validate the design. Of the four phases, methods for design definition and solution conception are the least developed; optimization using Robust Design being the most matured. Thus, Robust Design has been the centerpiece of many DFSS training curricula. However, the performance of a poorly defined and ill-conceived design can never be improved through subsequent optimization. It is therefore imperative that methods for design definition and solution conception be developed to further strengthen DFSS. Axiomatic Design with its two axioms can strengthen DFSS in these two phases. The independent axiom, Axiom I, is particularly suitable for defining the ease with which the performance of a design can be tuned to its target. However Axiom II, the information axiom, needs to be amended for implementation in DFSS. This paper shows the why and how of the amendment.

2 THE NEED FOR AN AMENDMENT

Information content I for a given functional requirement FR is defined as

$$I = \log_2 \frac{1}{P} = -\log_2 P$$

where P is the probability of satisfying FR given by

$$P(FR) = \int_{\text{design range}} f(FR)\,dFR$$

For a system with *n* FRs, the information content is

$$I(FR_1, FR_2, \cdots, FR_n) = -\log_2 P_{1,2,\ldots,n} \qquad (1)$$

where $P_{1,2,\ldots,n}$ is the joint probability satisfying all *n* FRs given by

$$P_{1,2,\ldots,n} = \int_{\text{design space}} f(FR_1, FR_2, \cdots, FR_n)\,dFR_1 dFR_2 \cdots dFR_n \qquad (2)$$

Axiom II, the information axiom, states that the design that has the smallest information content is the best design.

We now discuss the reasons why information content is not a logical metric for implementing DFSS. First of all, the computation of information content as defined by Equations (1) and (2) is complicated and tedious. To evaluate the probability $P_{1,2,\ldots,n}$ in Equation (2), we have to

1. identify the **DP** responsible for random deviation in **FR**;
2. relate **FR** to **DP** through the transfer functions $\phi(\mathbf{DP})$; and
3. characterize the randomness of **DP** in term of probability density functions $\mathbf{g}(\mathbf{DP})$.

In the above and hereafter, boldface quantities are vectors. By definition, design range of *n* $FR_i = \pm A_i$, a constant, are *n* orthogonal lines in the **FR** domain. Using the transfer function $\phi(\mathbf{DP})$ defined in step (2), we map these lines into curves that bound the region of integration in the **DP** domain identified in step (1). We then carry out the integration in the **DP** domain per Equation (3).

$$P_{1,2,\ldots,n} = \int_{\mathbf{FR}\text{ design range}} f(\mathbf{FR})\,d\mathbf{FR} = \int_{\mathbf{DP}\text{ design range}} \prod_{i=1}^{n} g_i\{\mathbf{DP}\}\,d\mathbf{DP} \qquad (3)$$

Thus to compute the information content per Equations (1) and (3), we need to know the probability density functions $\mathbf{g}(\mathbf{DP})$. We also need to carry out the integration in Equation (3) numerically if the transfer functions $\phi(\mathbf{DP})$ is not linear in **DP**.

The expression for the joint probability $P_{1,2,\ldots,n}$ in Equation (3) assumes the *n* FRs are independent. In a decoupled system in which the FRs are not independent, the computations are further complicated by the need to compute conditional probability:

$$P_{1,2,\ldots,n} = \int\int\cdots\int f(FR_1, FR_2, \ldots, FR_n)\,dFR_1 dFR_2 \ldots dFR_n$$

$$= \int\int\cdots\int f(FR_n | FR_1, FR_2, \ldots, FR_{n-1}) f(FR_{n-1} | FR_1, FR_2, \ldots, FR_{n-2}) \cdots$$

$$\cdots f(FR_2 | FR_1) f(FR_1)\ \ dFR_1 dFR_2 \ldots dFR_n$$

When mapped into the **DP** domain, the above integrals are nested, with limits of integration of the inner integrals being function of the outer integrands. In short, the information content as defined is complicated and tedious to compute. Moreover, the contribution to information content by the design is not explicitly delineated from that by the errors in **DP** affecting the design.

Perhaps the most serious shortcoming of using information content as a metric is that it will not lead us to a six sigma design. Consider for example two designs A and B as shown in Figure 1. System range of both designs is within the design range. Thus both designs have zero information content. Per Axiom II, they are at their best and therefore equally good. Yet intuition tells us that Design B is the better design because it has a larger margin to absorb any shift in process performance that may arise. By contrast slight shift in the process performance of Design A will result in failure.

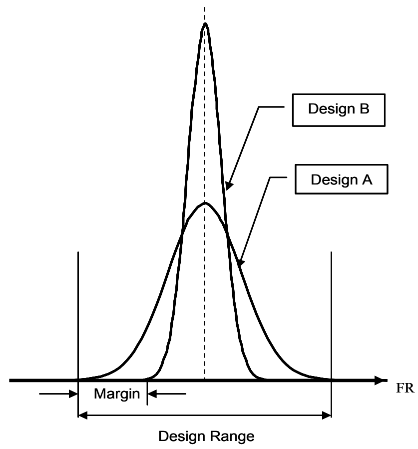

Figure 1

Consider another example, the practical example of the manufacturer specifying and the supplier satisfying component tolerance. The manufacturer can specify a tight tolerance, i.e., design range, in an attempt to obtain quality components. The supplier can satisfy the manufacturer by merely doing more inspection, rejecting components that are out of tolerance, submitting only those components that are within the tolerance. As a consequence, the manufacturer is receiving components with a truncated probability distribution function. The components are slightly better, but still sensitive to shift in process performance and at a higher cost.

The point behind these two examples is that to achieve DFSS, we must focus on the target rather than the range of design functionality. Information content by definition is focused on the probability of exceeding the design range. It therefore cannot lead us to six sigma design. It cannot be used as a metric to implement information axiom in DFSS. We need to amend Axiom II using a more relevant metric.

3 A PROPOSED AMENDMENT

Given a design that exhibits variability in its FR as described by a probability density function f(FR), we need a measure for "success" of the design. If we were contented with the design functionality falling within a range from –A to +A, then the metric would be the “goal post” loss function shown in Figure 2:

$$\int_{-A}^{+A} f(FR)dFR \, .$$

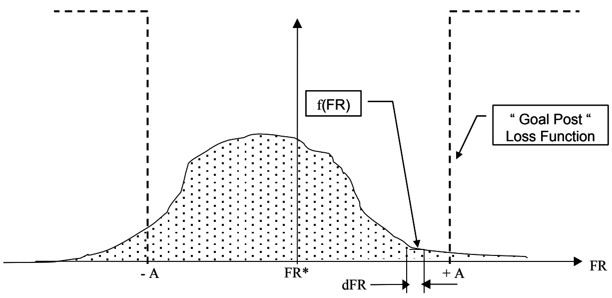

Figure 2

We recognize the above metric as the probability of success, the larger being the better. Axiom II uses this metric in the formulation of information content. To implement this metric, we need to know the probability density function f(FR) either by assumption or obtained experimentally. As discussed earlier, this metric can not be used to implement Axiom II in DFSS.

The aim of DFSS is to get the performance close to its target value FR*. Thus a relevant metric for “success” of the design is the quadratic loss function shown in Figure 3. In this case, the metric would be:

$$\int_{-\infty}^{+\infty} \left(FR - FR^*\right)^2 f(FR)dFR \qquad (4)$$

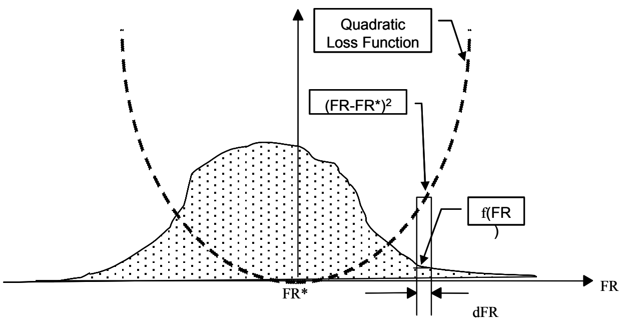

Figure 3

We recognize the above metric as the mean squared deviation (MSD) in FR, the smaller being the better. Note that per the “goal post” loss function, the smaller the MSD, the higher is the probability of success; thus the smaller is the associated information content. In other words, smaller information content is a by-product if we use MSD as the metric. Also for small deviation in FR, we will show later that its MSD can be expressed in terms of the MSD in **DP**, independent of the probability density function of **DP**. Namely, MSD is distribution free.

To extend Equation (4) to design involving multiple FR, we carry out the integration over the joint probability density function of the vector **FR**. Namely, $E\{(\mathbf{FR} - FR^*)^2\}$, where the notation $E\{\bullet\}$ denotes the expected value. Since MSD is related to bias and variance through the identity:

$$E\left\{\left(\mathbf{FR} - \mathbf{FR}^*\right)^2\right\} \equiv \left(E\{\mathbf{FR}\} - \mathbf{FR}^*\right)^2 + E\left\{\left(\mathbf{FR} - E\{\mathbf{FR}\}\right)^2\right\}; \qquad (5)$$

we can reduce MSD by (1) reducing the bias, the first term on the RHS of Equation (5), to zero; and (2) minimizing the variance, the second term on the RHS of the equation.

To achieve zero bias, methods for finding the **DP** that adjust the mean value $E\{\mathbf{FR}\}$ to the target value $\mathbf{FR}^*$ are well documented for design involving single FR, see Reference [1]. The strategy is to find the **DP** that significantly affects the mean but not the variance of FR. These same methods and strategy carry directly over to design with multiple FRs if the design matrix [A] is diagonally dominant, i.e., uncoupled:

$$\mathbf{FR} = [A]\,\mathbf{DP}$$

If [A] is lower triangular, i.e., decoupled, the same methods and strategy still apply to each component of **FR**; with the added provision that the application follows the sequence dictated by the architecture of the design matrix [A]. When [A] is coupled, reducing bias to zero may not be possible. In short, the methods and strategy for bias reduction are built on the independence axiom; with design matrix [A] serving as the metric for characterizing explicitly the ease of bias reduction.

To minimize variance, the methods and strategy should be based on the information axiom. The focus should not be on how much the variance, i.e., system range has extended outside the design range. Rather, they should be on how little the variation is scattered around the target value **FR***. By focusing on achieving the latter, the former becomes its by-product. With this philosophy in mind, we develop the metrics that characterize variation reduction as follow.

If **DP*** is the value of **DP** that brings **FR** to its target value **FR***, any random deviation in **DP** from **DP*** yields a random deviation (**FR** - **FR***) that may be approximated by a Taylor series expansion of **FR** around **FR***:

$$\begin{aligned}\mathbf{FR\text{ - }FR}^* &= \sum_j \left.\frac{\partial \mathbf{FR}}{\partial DP_j}\right|_{\mathbf{DP}^*} \left(DP_j - DDP_j^*\right) \\ &= [B]\left(\mathbf{DP\text{ - }DP}^*\right)\end{aligned} \qquad (6)$$

The [B] matrix is the sensitivity matrix characterizing the sensitivity of **FR** to the deviation in **DP**. It is evaluated at **DP*** as follows.

$$[B] = \left.\frac{\partial \mathbf{FR}}{\partial DP_j}\right|_{\mathbf{DP^*}} = \left.\begin{bmatrix} \frac{\partial FR_1}{\partial DP_1} & \frac{\partial FR_1}{\partial DP_2} & \cdots & \cdots & \frac{\partial FR_1}{\partial DP_m} \\ \frac{\partial FR_2}{\partial DP_1} & \frac{\partial FR_2}{\partial DP_2} & \cdots & \cdots & \frac{\partial FR_2}{\partial DP_m} \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ \frac{\partial FR_n}{\partial DP_1} & \frac{\partial FR_n}{\partial DP_2} & \cdots & \cdots & \frac{\partial FR_n}{\partial DP_m} \end{bmatrix}\right|_{\mathbf{DP^*}} \qquad m \ge n$$

Taking the expected values of both sides of Equation (6), we have,

$$E\{\mathbf{FR}\} - \mathbf{FR}^* = [B]\left(E\{\mathbf{DP}\} - \mathbf{DP}^*\right) \qquad (7)$$

Subtracting Equation (7) from Equation (6), we obtain deviation from the mean as:

$$\mathbf{FR} - E\{\mathbf{FR}\} = [B]\left(\mathbf{DP} - E\{\mathbf{DP}\}\right).$$

Or,

$$\delta\mathbf{FR} = [B]\delta\mathbf{DP};$$

where the symbol δ denotes deviation from the mean.

The total squared deviation from the mean is the inner product of δ **FR**:

$$\begin{aligned}\delta\mathbf{FR}^T\delta\mathbf{FR} &= ([B]\delta\mathbf{DP})^T([B]\delta\mathbf{DP}) \\ &= \delta\mathbf{DP}^T[B]^T[B]\delta\mathbf{DP}\end{aligned}. \qquad (8)$$

The above equation shows that there are two groups of factors affecting the total squared deviation in **FR**: the δ**DP** coming from errors in manufacturing and usage; and the inner product of [B] matrix, which is solely a characteristic of the design. Furthermore, on invoking the Schwartz's inequality, it can be shown that

$$\begin{aligned}\delta\mathbf{FR}^T\delta\mathbf{FR} &= \delta\mathbf{DP}^T[B]^T[B]\delta\mathbf{DP} \\ &\le \mathrm{Trace}\left\|[B]^T[B]\right\| \bullet \left(\delta\mathbf{DP}^T\delta\mathbf{DP}\right)\end{aligned}. \qquad (9)$$

So that the total variance of **FR** is directly proportional to the total variance of the **DP**:

$$E\left(\delta\mathbf{FR}^T\delta\mathbf{FR}\right) \le \mathrm{Trace}\left\|[B]^T[B]\right\| E\left(\delta\mathbf{DP}^T\delta\mathbf{DP}\right).$$

Note in the inequality above that the contribution of $[B]^T[B]$, a design characteristic, is explicitly delineated from that of δ**DP**, the errors affecting the design. Its effect is to amplify the total variance of **DP** into the total variance of **FR**. Note further that relationship between total variance of **FR** and total variance of **DP** is established independent of the probability density function of **DP.** Just as the design matrix [A] is the metric for implementing the independence axiom for reducing bias, so is the inner product $[B]^T[B]$ a metric for implementing the information axiom for reducing variance. Thus we propose the use of the two metrics [A] and $[B]^T[B]$ together for implementing Axiomatic Design to achieve 6σ design in DFSS.

4 AN ILLUSTRATIVE EXAMPLE

This example appeared in Reference [2]. A passive filter network is to be used in instrumentation for recording displacement of a mechanical system. The displacement signal is picked up by a transducer; amplitude modulated and passed on to a galvanometer. Figure 4 shows the schematic of the system where the transducer

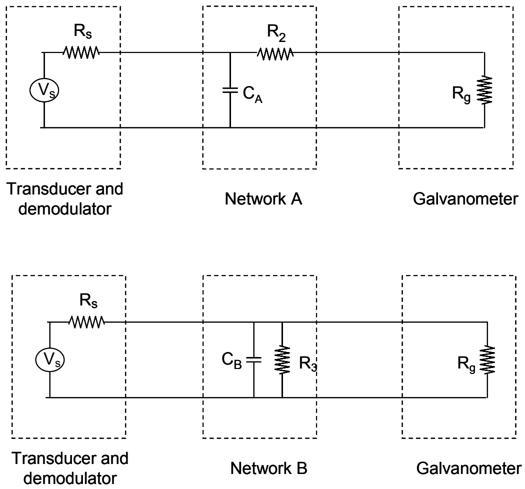

Figure 4

is represented by output impedance R_s and a demodulated signal excitation V_s, and the galvanometer by impedance R_g. The functions of the network are to filter out the carrier signal from the transducer and to attenuate the filtered signal to match the scale of the galvanometer. The engineering representations for the two functions are: $FR_1 = \omega_c$, the filter cutoff frequency above which the carrier signal is filtered out; and FR_2 = D, the full scale displacement of the galvanometer to which the filtered signal will match. Two candidate networks are considered. These are shown in Figure 4. The expressions for ω_c and D were derived in Reference [2] and reproduced here.

For Network A,

$$\omega_c = \left(\frac{2\pi}{C_A R_g}\right)\frac{R_g\left(R_2 + R_g + R_s\right)}{R_s\left(R_2 + R_g\right)} \qquad (10)$$

$$D = \left(\frac{|V_s|}{G_{sen}}\right)\frac{R_g}{R_2 + R_g + R_s} \qquad (11)$$

For Network B,

$$\omega_c = \left(\frac{2\pi}{C_B R_g}\right)\frac{R_g R_s + R_g R_3 + R_s R_3}{R_3 R_s} \tag{12}$$

$$D = \left(\frac{|V_s|}{G_{sen}}\right)\frac{R_g R_3}{R_g R_s + R_g R_3 + R_s R_3} \tag{13}$$

where R, subscripted 2, 3, g, and s, are the impedances; C subscripted A and B, are the capacitors, $|V_s|$ is the full scale transducer output, and G_{sen} is the gain of the galvanometer. This example was used in Reference [2] to illustrate the implementation of Axiom I in determining which network better meets functional independence. It was also analyzed in Reference [2] using the Taguchi method. The focus of Taguchi method is on variation reduction through optimizing the design robustness. The method is employed predominantly in Robust Design. In here, we will use the same example to illustrate the proposed method for implementing Axiom II to determine which network better meets the information axiom. The proposed method would serve as the link between Axiomatic Design and Robust Design.

For a common basis, we compare the two networks at design points that produce the same responses, ω_c and D. Thus for the same response in D, we equate Equation (11) to Equation (13):

$$\frac{R_g}{R_2 + R_g + R_s} = \frac{R_g R_3}{R_g R_s + R_g R_3 + R_s R_3}$$

Simplifying,

$$R_2 R_3 = R_g R_s \tag{14}$$

Similarly, we equate Equation (10) to Equation (12) for the same response value in ω_c:

$$\left(\frac{2\pi}{C_A R_g}\right)\frac{R_g(R_2 + R_g + R_s)}{R_s(R_2 + R_g)} = \left(\frac{2\pi}{C_B R_g}\right)\frac{R_g R_s + R_g R_3 + R_s R_3}{R_3 R_s}.$$

We make use of Equation (14) to simplify further:

$$C_B = C_A\left(1 + \frac{R_2}{R_g}\right) \tag{15}$$

As long as (R_2, C_A) in Network A and (R_3, C_B) in Network B are specified in accordance with Equations (14) and (15), both networks will output the same ω_c and D. With the two networks delivering the same response value, we implement the proposed amendment of Axiom II to determine which network is more consistent in their response in the presence of errors. Specifically, we develop and apply the sensitivity matrices [B] discussed in Section 3.

We first identify the sources of errors. Generally, there are two: manufacturing and usage. The actual values of C and R used in the manufacturing will typically deviate from their nominal values. Thus capacitor C and resistor R are sources of manufacturing errors. The filter network will be used with different transducers and galvanometers. Thus, R_s, R_g, $|V_s|$, and G_{sen} are sources of usage errors. The sensitivity of the network to these sources of errors is characterized by the matrix [B]. They are derived below for the two networks subscripted A, B respectively.

$$[B]_A = \begin{bmatrix} \left(\frac{V_g}{\omega_c}\right)\frac{\partial\omega_c}{\partial V_g} & \left(\frac{R_g}{\omega_c}\right)\frac{\partial\omega_c}{\partial R_g} & \left(\frac{R_s}{\omega_c}\right)\frac{\partial\omega_c}{\partial R_s} & \left(\frac{C_A}{\omega_c}\right)\frac{\partial\omega_c}{\partial C_A} & \left(\frac{R_2}{\omega_c}\right)\frac{\partial\omega_c}{\partial R_2} \\ \left(\frac{V_g}{D}\right)\frac{\partial D}{\partial V_g} & \left(\frac{R_g}{D}\right)\frac{\partial D}{\partial R_g} & \left(\frac{R_s}{D}\right)\frac{\partial D}{\partial R_s} & \left(\frac{C_A}{D}\right)\frac{\partial D}{\partial C_A} & \left(\frac{R_2}{D}\right)\frac{\partial D}{\partial R_2} \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & \frac{(R_g + R_2)}{(R_g + R_2 + R_s)} & 1 & \frac{R_2 R_s}{(R_g + R_2)(R_g + R_2 + R_s)} \\ -1 & 0 & \frac{R_s}{(R_g + R_2 + R_s)} & 0 & \frac{R_2}{(R_g + R_2 + R_s)} \end{bmatrix}$$

$$[B]_B = \begin{bmatrix} \left(\frac{V_g}{\omega_c}\right)\frac{\partial\omega_c}{\partial V_g} & \left(\frac{R_g}{\omega_c}\right)\frac{\partial\omega_c}{\partial R_g} & \left(\frac{R_s}{\omega_c}\right)\frac{\partial\omega_c}{\partial R_s} & \left(\frac{C_B}{\omega_c}\right)\frac{\partial\omega_c}{\partial C_B} & \left(\frac{R_3}{\omega_c}\right)\frac{\partial\omega_c}{\partial R_3} \\ \left(\frac{V_g}{D}\right)\frac{\partial D}{\partial V_g} & \left(\frac{R_g}{D}\right)\frac{\partial D}{\partial R_g} & \left(\frac{R_s}{D}\right)\frac{\partial D}{\partial R_s} & \left(\frac{C_B}{D}\right)\frac{\partial D}{\partial C_B} & \left(\frac{R_3}{D}\right)\frac{\partial D}{\partial R_3} \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 1 & \frac{R_g R_3}{(R_3 R_g + R_s R_g + R_3 R_s)} & 1 & \frac{R_g R_s}{(R_3 R_g + R_s R_g + R_3 R_s)} \\ -1 & 0 & \frac{R_s(R_g + R_3)}{(R_3 R_g + R_s R_g + R_3 R_s)} & 0 & -\frac{R_g R_s}{(R_3 R_g + R_s R_g + R_3 R_s)} \end{bmatrix}$$

In the above, the matrices are normalized with respect to nominal values of the factors and the responses. Also, we have combined $|V_s|$ and G_{sen} into one factor $V_g = (|V_s| / G_{sen})$.

We compute the trace and the total squared deviation from the mean of the responses per Equations (8) and (9) at two design points. At Design Point I, we use for the variables, the values that were in Reference [2] and shown in Table 1 below.

	R_g *Ω*	*R_s* *Ω*	*V_g* *in.*	*R_2* *Ω*	*R_3* *Ω*	*C* *μF*	*Trace*	*Var*
Network A	120	98	23	694	∞	1671	4.396	0.022
Network B	120	98	23	0	17	11343	4.930	0.023

Table 1 Design Point I

Substituting above values into Equations (10) thru (13), both networks have same responses: ω_c = 6.84 Hertz and D = ± 3.00 inches. The sensitivity matrix for Network A is as follow.

$$[\mathrm{B}]_A = -\begin{bmatrix} 0 & 1.000 & 0.8926 & 1.000 & 0.0916 \\ -1.000 & 0 & 0.1074 & 0 & 0.7611 \end{bmatrix}$$

The trace is

$$[\mathrm{B}]_A [\mathrm{B}]_A^T = \begin{bmatrix} 2.8051 & 0.1656 \\ 0.1656 & 1.5908 \end{bmatrix}; \quad \mathrm{Trace} = 2.8051 + 1.5908 = 4.3959$$

Assume a deviation from the mean of 5% for all δDP. Namely,

$$\frac{\delta\mathbf{DP}}{\mathrm{E}\{\mathbf{DP}\}} = [0.05, 0.05, 0.05, 0.05, 0.05,]^T$$

Then the total variance in **FR** per Equation (8) is,

$$\mathrm{Var} = \left(\frac{\delta\mathbf{DP}}{\mathrm{E}\{\mathbf{DP}\}}\right)^T [\mathrm{B}]_A^T [\mathrm{B}_A] \left(\frac{\delta\mathbf{DP}}{\mathrm{E}\{\mathbf{DP}\}}\right) = 0.0223$$

Total deviation from the mean in **DP** is,

$$\left(\frac{\delta\mathbf{DP}}{\mathrm{E}\{\mathbf{DP}\}}\right)^T \left(\frac{\delta\mathbf{DP}}{\mathrm{E}\{\mathbf{DP}\}}\right) = 0.0125$$

The above multiplied by the trace is larger than the total variance in **FR**:

0.0125 x 4.3959 > 0.0223.

This verifies the validity of Equation (9).

The sensitivity matrix for Network B is calculated and shown below.

$$[\mathrm{B}]_B = -\begin{bmatrix} 0 & 1.000 & 0.1315 & 1.000 & 0.7611 \\ -1.000 & 0 & 0.8685 & 0 & -0.7611 \end{bmatrix}$$

The trace and total variance in **FR** are similarly calculated using $[\mathrm{B}]_B$ for Network B. The results are shown in Table 1. At the Design Point I, the two networks appear to have the same total variance in FR and therefore the same consistency in response.

Similar calculations were made for another design point that delivers the same responses ω_c = 6.84 Hertz and D = ± 3.00 inch. The results are shown in Table 2. At this design point, the total variance in **FR** of Network B is only 65% that of Network A. From a DFSS point of view, Network B is the better of the two designs at this design point.

	R_g Ω	R_s	V_g	R_2	R_3	C	*Trace*	*Var*
		Ω	*in*	Ω	Ω	µF		
Network A	38	456	46	90	∞	1457	3.980	0.019
Network B	38	456	46	0	194	4875	3.924	0.012

Table 2 Design Point II

5 CONCLUDING REMARKS

A six sigma design delivers the design functionality on target with little variation around it. Consequently, it has a wide margin to accommodate errors in the manufacturing and usage of the design. Over the years, methods and metrics have been developed to achieve six sigma designs. These methods and metrics are collectively called Design for Six Sigma (DFSS).

One of the design principles behind DFSS is that it must focus on the target rather than the range of design functionality. Axiom I conforms to such principle since functional independence provides a metric for ease of tuning the design to its target. By contrast, information content in Axiom II is focused on the probability of exceeding the design range. Axiom II therefore needs to be amended.

In this paper, we proposed and developed the [B] matrix as a metric to implement Axiom II. Unlike information content which requires knowledge of the probability density function of the DP, the [B] matrix is distribution free. Furthermore, the proposed metric explicitly delineates contribution of design characteristics separate from that of errors affecting the design. Perhaps the most compelling reason to use this metric is that it is focused on the design target. The metric characterizes the sensitivity of the design to deviate from its target in the presence of errors.

Together, the design matrix of Axiom I and the proposed sensitivity matrix of Axiom II, should bring Axiomatic Design to the forefront of DFSS.

6 REFERENCES

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