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THE MINIMUM CONSTRAINT DESIGN AND THE FIRST AXIOM

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ABSTRACT

Minimum Constraint Design (MinCD) is a design approach for mechanical systems yielding to the required performance with a minimum number of external constraints at each one of the system's components.

The method is claimed to be appropriate for the design of mechanical systems, allowing for assembling with zero looseness and binding, zero distortion and no residual stress. Such designs can be implemented with simple machining, loose manufacturing tolerances and semiskilled labour for both assembling and disassembling.

As a result, MinCD allows for cost decreasing in both design and manufacturing, and to a noteworthy increasing in reliability and maintainability.

According to the 1st Axiom of Suh's Axiomatic Design (AD), ideal designs are independent, that is, the mapping between design requirements and design parameters must be achieved in such a way that the variation of any specific design parameter affects one only design requirement.

This paper shows through selected examples that Minimum Constraint Design is a particular case of AD's 1st Axiom.

Keywords: MinCD - Minimum Constraint Design; Least Constraint; Kinematic Design; Axiomatic Design; First Axiom.

1 INTRODUCTION

The designing and construction of prototypes, or "one-off" products, always involves uncertainty of several types and different origins, which reduction must be achieved in the current competitive marketplace that requires high quality products with increased performance and reliability at a low cost and in a short time.

One way to make products unresponsive to geometrical uncertainties within certain limits is to apply the Minimum Constraint Design (MinCD) approach. MinCD recommends supporting and guiding each body only at points — and at as few points as possible — to attain the desired performance, at the same time conferring to the product the adequacy to the real world conditions.

A brief presentation of the Minimum Constraint Design (MinCD) approach and a short explanation of Axiomatic Design (AD) will be made in this paper, as a means to provide a basic understanding on the relationships between them.

In addition, two worked examples will be presented here with the purpose of showing that MinCD is in accordance with the 1st Axiom of Axiomatic Design.

2 THE MINIMUM CONSTRAINT DESIGN

The application of Minimum Constraint Design methodology (MinCD) [1], means to support and guide each body only at points, and at as few points as possible to attain the necessary performance.

This method, firstly named by Gauss in 1829 as the "Principle of Least Constraint", in a paper called "Über ein neues Grundgesetz der Mechanik" [2], leads to the minimum number of constraints that are needed to allow the required functionality with no extra degrees of freedom.

The physical components with mechanical links must be chosen in such a way that they do not bring in superfluous constraints [3]. A typical example is the use of self-aligning bearings allowing rotations in any direction, compensating for misalignments, instead of rigid ball bearings that just allow axial rotation and have no ability to adjust its shape to the imperfect real conditions.

Minimum Constraint Design provides zero binding and zero looseness of moving parts. In addition, it provides zero stresses and strains for assembling and installing stationary assemblies, without the need of rework.

This method increases both reliability and maintainability, even with loose manufacturing tolerances and semiskilled assembly labour, which provides major cost reductions in product design and manufacturing.

When a minimum number of constraints is used, all the forces acting in the parts are determined by equilibrium alone. When there are more than the minimum number of constraints, the forces in the parts depend critically on errors in manufacturing, temperature difference between one part and another, and so on [4].

For example, when someone sits on a three-legged stool, the load on each leg can ideally be calculated from equilibrium considerations, whereas this is not the case with a four-legged chair. If one slips a thin piece of wood under one leg of the chair, the load in the legs will change. If one does the same with a three-legged stool, the load will not change.

However, pure MinCD, also known as "Kinematic Design" [4], has limitations, due to high stress at the contact points and elasticity of the materials. An illustration of this would be a MinCD ball bearing with only three balls.

Usually, some degree of compromise with absolute purity of MinCD has negligible, unwanted effects and can be economically justified.

2.1 THE REDUNDANT CONSTRAINT DESIGN

A limitation of MinCD is related to the elasticity of materials, which effects are more important in the case of large parts subjected to distributed loads.

In that case, it is economically preferable to provide a redundant number of distributed fasteners — Redundant Constraint Design (RedCD) — instead of sizing the part to carry the load with MinCD fasteners.

There are several examples, such as heads of cylinders, flanges for pipes or thread screws, where MinCD is not preferred and another principle may be invoked, which is called "Elastic Design" [4].

The principle of elastic design might be stated: "if there is going to be a fight, let it be a very uneven one to ensure that the loser is not hurt" [4]. An example of elastic design can be observed in modern design of power line towers, which are allowed to bend until the loads in the cables are nearly equal. The cables can easily overload the tower, but the tower is flexible enough not to be harmed [4].

2.2 ADHERENCE TO THE REAL WORLD

In pure MinCD, each support or guide acts only at points of contact. This can be disadvantageous because high stress concentration would appear at the point supports.

In real word materials, high contact stresses cause some combination of elastic, inelastic, and wear deformation, which will enlarge the point contact to an area contact, where the load will spread and the stresses will be reduced until the equilibrium is attained.

Design with such compromise is called semi-MinCD, since it uses finite contact areas instead of using point contacts, in an otherwise MinCD configuration.

There is a special class of semi-MinCD, named "matched sets", that uses elastic, inelastic, and wear deformation, to "match" both surfaces in contact, instead of requiring high manufacturing accuracy.

For the purpose of this paper, it will be considered the semi-MinCD approach.

3 AXIOMATIC DESIGN

Axiomatic Design (AD) [5] is the only design theory that mathematically describes the whole design process, according to the mode that the human mind operates during the development of designing tasks.

Therefore, AD provides a systematic approach to design, based on scientific thinking, by introducing axioms and theorems, as well as the concepts of domains, zigzag decomposition, and design matrices, for all the levels of the design process.

3.1 THE DESIGN ENVIRONMENT

In the AD terminology, the world of design is made up of four domains: the customer domain, the functional domain, the physical domain, and the process domain [6]. These domains are shown in figure 1, and its contents can be described as follows [7]:

Customer Domain: contains the Customer Needs (CN) or Attributes that the customer seeks in the product or in the system;

Functional Domain: contains the Functional Requirements (FR) of the design object. In a good design, they are the minimum set of independent requirements that completely describe the functional needs of the design solution;

Physical Domain: contains the Design Parameters (DP) of the design solution. They are the elements of a design solution that are chosen to satisfy the specified FRs;

Process Domain: contains the Process Variables (PV) that characterise the production process of the design solution, i.e. satisfies the specified DPs.

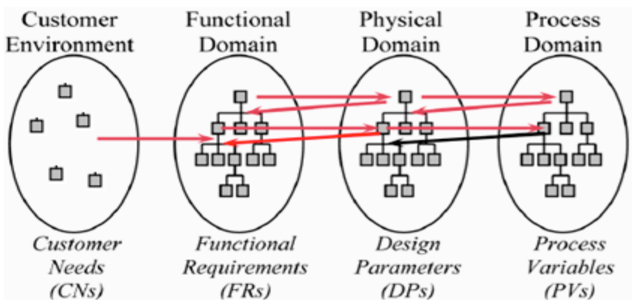

Figure 1. Axiomatic design domains, their contents and relationships.

In each pair of adjacent domains, the domain on the left, relative to the domain on the right, represents 'What is required to achieve — *i.e.* the goal'. The other domain represents 'How it is achieved — *i.e.* the way to achieve the goal'.

Design is attained by interactions between the goals of the design and the way that is used to achieve the goals. The goals of the design are specified in the functional domain (where only immaterial items exist), and the way of achieving them is proposed in the physical domain (where some possible real solutions able of performing the specified functionalities are represented).

The design process is the mapping of relationships between the domains, as represented by design matrices: for example, a product design matrix, which represents the relationships between FRs and DPs; and a process design matrix, which depicts the relationships between DPs and PVs.

Constraints are the bounds of acceptable solutions, which can be of two different kinds: input constraints and system constraints. Input constraints are imposed as part of the customer needs, and system constraints are imposed by the generated design solution.

The mapping between the FRs and DPs can be summarized in equation (1), where {FR} is the FR vector, {DP} is the DP vector, and [A] is the design matrix.

$$\{FR\} = [A]\{DP\} \tag{1}$$

Where
$$A_{ij} = \frac{\partial FR_i}{\partial DP_j} \tag{2}$$

If DP_j affects FR_i, then the corresponding element A_{ij} in the design matrix is non-zero. Otherwise it is zero.

3.2 Hierarchies and Zigzagging

Another important concept in AD is the hierarchical decomposition through zigzagging between domains, starting from the 'What' domain to the 'How' domain, in a top-bottom way, beginning at the system level and continuing through levels of more detail [5].

After solving the top-level, FRs and DPs are identified to provide enough design information, and they should be decomposed until the design reaches the final stage, the leaf level, creating a design that can be implemented. The hierarchies that were established between FRs and DPs represent the design structure, which is also known as the system architecture [7].

This means that the DPs at the leaf level should not need either redesigning or further decomposition.

An example of a possible zigzagging path between the functional and the physical domains relating to the design of an engine is shown in Figure 2.

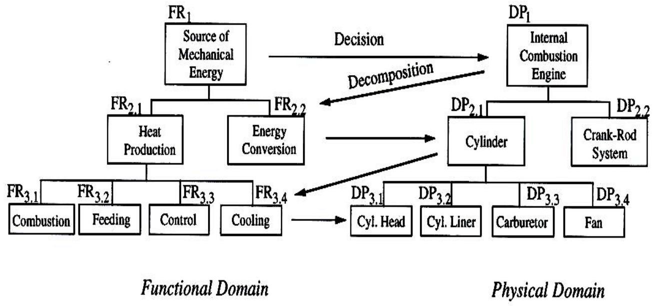

Figure 2. The zigzag path of the hierarchical decomposition (adapted from [8]).

3.3 The Principles of Design

The underlying hypothesis of AD is that there exist fundamental principles that govern good design practice.

There are two design principles, or axioms, that are used in AD, which provide a tool for analysis.

The two design axioms may be stated as follows [5], [6], [7]:

The Independence Axiom (first axiom):

Maintain the independence of functional requirements.

This means that each one of the FRs should be satisfied by adjusting one only DP without affecting the accomplishment of any other FR.

The Information Axiom (second axiom):

Minimize the information content of the design.

The purpose of this axiom is to help in finding out the alternative design solution with the highest probability of achieving the FRs.

During the decomposition, the independence axiom and constraints should be applied to the design matrix to ensure that an uncoupled or a decoupled design matrix is obtained at each level of the design process.

Since the design process does not lead to a unique solution, the information axiom should be used to compare the alternative solutions that were previously found.

3.4 The First Axiom

The first axiom states that in an ideal design, a strict one-to-one relationship between FRs and DPs should be observed. Thus, the number of FRs and DPs should be equal. Such design is known as uncoupled design and is characterized by a diagonal design matrix, which indicates a one-to-one relationship between FRs and DPs while ensuring that FRs can be fulfilled in an absolutely independent way.

However, complete uncoupling may not be easy to accomplish in the real world, where interactions between factors are usual.

Designs where the accomplishment of any FR depends on more than one DP are acceptable, as long as the design matrix [A] is triangular. This is called a decoupled design. A decoupled design also fulfils the independence axiom, since the DPs can be specified in a sequence such that each FR can ultimately be controlled by one only DP [5].

Any other shape of the design matrix that cannot be transformed into a triangular one by simple permutation of columns or rows represents a coupled design, i.e. contains interdependences between the FRs that cannot be avoided. Therefore coupled designs should be avoided according to Axiomatic Design.

The basic categories of design that are based on the shape of the design matrix are shown in Figure 3, where “X” represents non-zero elements.

$$\begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \qquad \begin{bmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{bmatrix} \qquad \begin{bmatrix} X & X & X \\ X & X & X \\ X & X & X \end{bmatrix}$$

Uncoupled Decoupled Coupled

Figure 3. Basic categories of design matrices.

3.5 THE SECOND AXIOM

The Information Axiom provides a means of evaluating the quality of designs, thus facilitating the selection among the available design alternatives.

This is accomplished by comparing the information content of the existing alternative solutions in terms of their probability of satisfying the FRs.

The minimum information criterion is a powerful tool in optimization and simulation of design/manufacturing processes when there are several variables with respect to which the solution must be optimized. [6] The information content evaluation can be used to select the best solution among those proposed, regardless of the number of variables involved. In terms of the information axiom, the best solution is the one that possesses the minimum information content and simultaneously satisfies Axiom 1.

The information content in a one FR and one DP design [5] is expressed as the logarithm of the inverse of the probability of success p:

$$I = \log_2 \frac{1}{p} \tag{3}$$

In the simple case of uniform probability distribution, the equation above can be written as:

$$I = \log_2 \left(\frac{area\ \ of\ \ the\ System\ Range}{area\ \ of\ \ the\ Common\ Range} \right) \tag{4}$$

where the area of the system range is computed from FR’s probability density function and the area of the common range is the fraction of the above mentioned area that is inside of the range limits, as shown in Figure 4.

In other words, the system range depicts the capability of the current system in terms of tolerances, the design range defines the acceptable range associated with the specified DP, and the common range refers to the amount of overlap between the design range and the system range.

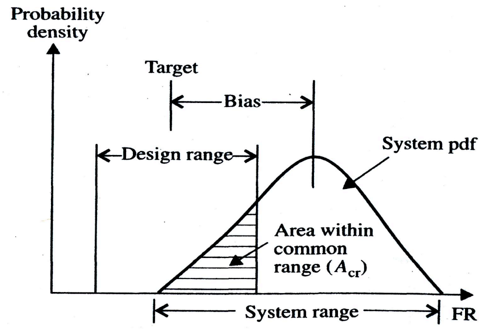

Figure 4. Probability calculation of success in a one FR and one DP design [6].

For an uncoupled design with n FRs, the total information content, I, can be computed through

$$I = \sum_{i=1}^{n} -\log_2 p_i = \sum_{i=1}^{n} I_i \tag{5}$$

where p_i is the probability of FR_i being satisfied by DP_i [5]. When all the probabilities, p_i, are equal to one, then the information content is zero. Conversely, the information content is infinite when one or more of the p_i are equal to zero.

Complex systems require a substantial amount of information to perform as expected, and large systems that are made with many components are not necessarily complex. On the other hand, systems with a small number of components are complex if their probability of accomplishment is low.

4 EXAMPLES

A method to develop the design process of supporting and guiding machine parts, in order to make the product indifferent to the geometrical uncertainties is analysed in the following examples.

The application of Minimum Constraint Design (MinCD), recommends supporting and guiding each body at as few points as possible, to attain the desired performance. This makes the effects at each support to be independent of the effects at the other supports, so that the design solution is in accordance with the 1st Axiom.

In our examples, the functional requirements {FR} are the positions of the supporting points of the machines’ parts, which goal is to maintain them fixed or guided through a defined trajectory; the design parameters {DP} are the loads that are imposed to the parts of the machines by the supporting components, in order to maintain the support points in the required positions.

Therefore, Eq. (1) can be re-written as follows:

$$\{\text{Positions}_i\} = [\text{A}]\{\text{Supports}_j\} \tag{6}$$

$i = 1, \dots n; \qquad j = 1, \dots n;$

where "Position$_i$" comprises the localization and orientation; and "Support$_j$" ensures the position of the point by applying loads that impose the static equilibrium of the body.

4.1 SUPPORTING A SCREW CONVEYOR

In the real World, there is always uncertainty of several types and several origins, which elimination is not possible without paying a cost.

Considering the geometrical uncertainties, it is not possible to warrant that the screw shaft follows a strait line, or the base plates where the supports rest are coplanar, or the axes of the supports and the screw shaft are co-linear, mostly due to weight and elasticity of materials, as well as due to the manufacturing processes.

In this case, we intend to support the screw conveyor by its ends allowing axial rotation, which means that we have to constrain five from the possible six degrees of freedom of the screw, as shown in Figure 5.

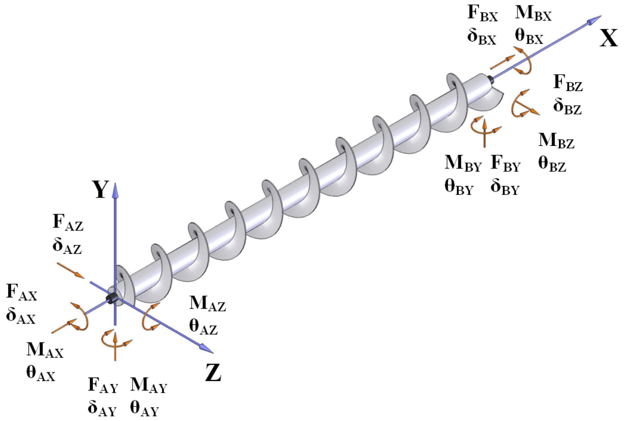

Figure 5. Screw conveyor and its supports.

Supposing that the shaft would be supported by rigid ball bearings that do not allow either linear or angular displacement in any direction, Eq. (6) would be written as follows:

$$\begin{Bmatrix} \text{Position of A} \\ \text{Position of B} \end{Bmatrix} = \begin{bmatrix} x & x \\ x & x \end{bmatrix} . \begin{Bmatrix} \text{Support at A} \\ \text{Support at B} \end{Bmatrix} \qquad (7)$$

Substituting the position of each point by its linear and angular displacements, and substituting the support by its reactions (forces and torques), Eq. (7) can be re-written as Eq. (8), where: δ are the linear displacements of each point in the directions of the co-ordinate axes; θ are the angular displacements of each point around the same directions; F are the forces that are induced by each support in the directions of the co-ordinate axes and M are the torques that are induced by each support at the same directions.

Since the screw is required to rotate around its axis, then it must have one only degree of freedom. Thus, according to MinCD, five constraints must be imposed in order to satisfy the above-mentioned requirement.

$$\begin{Bmatrix} \delta_{AX} \\ \delta_{AY} \\ \delta_{AZ} \\ \theta_{AX} \\ \theta_{AY} \\ \theta_{AZ} \\ \delta_{BX} \\ \delta_{BY} \\ \delta_{BZ} \\ \theta_{BX} \\ \theta_{BY} \\ \theta_{BZ} \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & X \\ 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X \\ 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 \\ 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & X & 0 \\ 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & X \\ X & 0 & 0 & 0 & X & X & X & 0 & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 \\ 0 & 0 & X & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 \\ 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X \end{bmatrix} \begin{Bmatrix} F_{AX} \\ F_{AY} \\ F_{AZ} \\ M_{AX} \\ M_{AY} \\ M_{AZ} \\ F_{BX} \\ F_{BY} \\ F_{BZ} \\ M_{BX} \\ M_{BY} \\ M_{BZ} \end{Bmatrix} \qquad (8)$$

Therefore, we should remove the unnecessary reactions letting the shaft be supported by two simple supports, one of them fixed and the other one axially movable.

In physical terms, this can be achieved by using self-aligning bearings, for they allow rotation in any direction, thus compensating both the shaft deflection and the possible mis-alignment of the end supports.

Additionally, one of the bearing housings must have an axial gap, to allow the axial displacement of one of the shaft ends, in order to allow for the compensation of thermally induced variations of the shaft length.

4.2 EFFECTS OF REAL VERSUS IDEAL SITUATION

To evaluate the results of the MinCD approach let us consider now that one of the supports, for example the support at point B, allows rotation in any direction and axial displacement, just restricting the displacements orthogonally to the axis of the screw. In addition, let us suppose that the other support, at point A, restricts five of the six degrees of freedom at that point of the screw's shaft, allowing only its axial rotation.

In this way, the screw would be supported by superfluous external constraints, which would lead to the lack of independence of each constraint, leading in turn to the lack of the screw's capability to adjust its shape to the real, imperfect, external conditions.

In a real situation, this condition could be achieved by utilizing a rigid ball bearing to support the screw at the point A, which would allow just axial rotation, and a self-aligning bearing, at support B, which would allow rotation in any direction.

Because support A is rigid and support B only restricts the displacements orthogonally to the axis of the screw, hence the elements related to the reactions F_{BX}, M_{BX}, M_{BY} and M_{BZ} at support B in Eq. (8) would be zeroes.

By eliminating the rows and the columns that are marked with a line in Eq. (9), where every element is zero, the rank of the design matrix becomes 8 x 8, and the relationships between the displacements and the reactions at the screw shaft end supports can be written as shown in Eq. (10), which represents a decoupled design matrix.

$$
\begin{Bmatrix} \delta_{AX} \\ \delta_{AY} \\ \delta_{AZ} \\ \theta_{AX} \\ \theta_{AY} \\ \theta_{AZ} \\ \delta_{BX} \\ \delta_{BY} \\ \delta_{BZ} \\ \theta_{BX} \\ \theta_{BY} \\ \theta_{BZ} \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & X \\ 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X \\ 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 \\ 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & X & 0 \\ 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & X \\ 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X \end{bmatrix} \begin{Bmatrix} F_{AX} \\ F_{AY} \\ F_{AZ} \\ M_{AX} \\ M_{AY} \\ M_{AZ} \\ F_{BX} \\ F_{BY} \\ F_{BZ} \\ M_{BX} \\ M_{BY} \\ M_{BZ} \end{Bmatrix} \qquad (9)
$$

Therefore, Eq. (9) can be rewritten as follows:

$$
\begin{Bmatrix} \text{Position of A} \\ \text{Position of B} \end{Bmatrix} = \begin{bmatrix} x & x \\ 0 & x \end{bmatrix} \cdot \begin{Bmatrix} \text{Support at A} \\ \text{Support at B} \end{Bmatrix} \qquad (10)
$$

In the real world it is impossible to ensure the theoretical, perfect conditions due to several factors, such as self-weight, loading, elasticity of the materials, imperfect shape, deformation originated by the fabrication processes, errors in the final or the intermediate assembly, etc.

This means that the axis of the screw is not a straight line; the surfaces of both bases where the supports of the shaft ends rests are not in the same plane; the axis of the screw is not parallel to the plane where the support's bases lay, etc.

For a matter of simplicity, one can consider that just one imperfection exists. For example, let us suppose that the base of the support at A is inclined by an angle γ_{AZ} around the axis Z, as shown in Figure 6.

The support located at point A is embodied by a rigid ball bearing. The axis of the screw is inclined in the XY plane, and the screw end B' would not coincide with the position of support B until it would be forced to its final position.

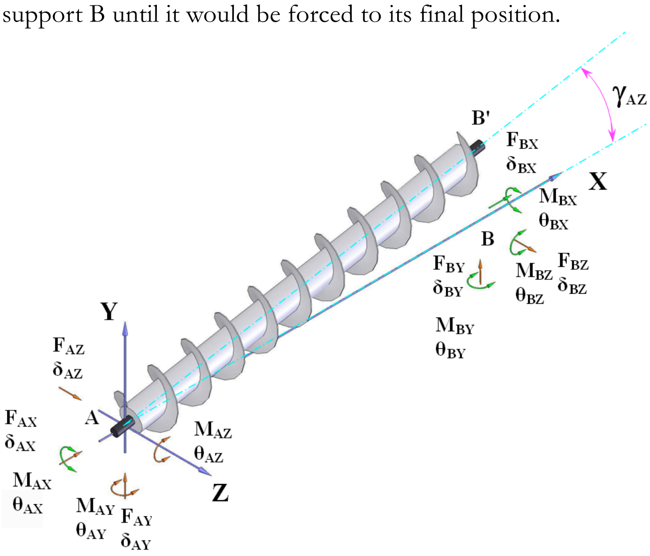

Figure 6. Screw conveyor supported by a rigid ball bearing at A and free at B.

To make the screw end B' to be coincident with the support B, it would be necessary to apply an external force that would cause deformation on the screw. This would bend its axis, as shown in Figure 7.

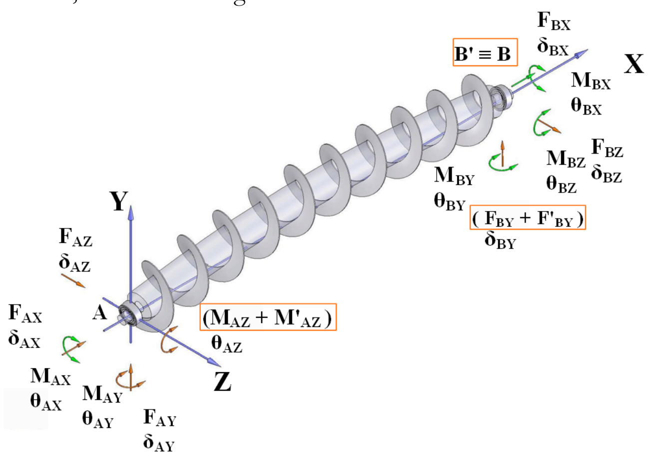

Figure 7. Screw conveyor forced to fit to support B

The vertical reaction at support B, $\mathbf{F_{BY}}$, must be increased by $\mathbf{F'_{BY}}$ to make the screw end B' coincident with point B. In order to preserve the static equilibrium, the torque $\mathbf{M_{AZ}}$ must increase by $\mathbf{M'_{AZ} = L_{AB} \times F'_{BY}}$ as well.

The increments of the aforesaid reactions are due to the fact that the ball bearing located at point A cannot adjust its position in order to absorb the imperfections of the support base. Therefore, one can say that the supporting condition at point A affects the reactions at point B. This induces the increasing of the shear force and the bending moment along the length of the screw, which in turn increases the contact pressure at the end supports.

Because the constraints are more than the minimum, the external forces applied to the screw by the supports depend critically on the real world imperfect conditions.

In this case, the worst effect would be caused by the large distance between the ends A and B, combined with the small width of the bearing inner bush, which would originate very high contact stresses, as shown in Figure 8.

This condition could cause premature failure of the shaft due to the high contact pressure, failure of the ball bearings due to high torque, and fatigue failure of the axis due to high stresses originated by rotating bending.

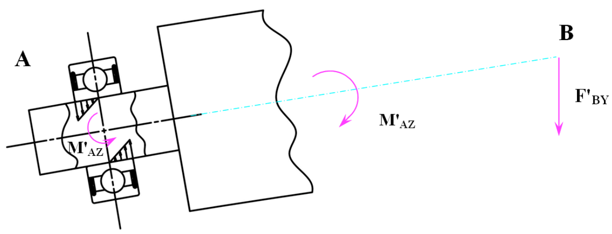

Figure 8. Contact pressure applied by a rigid ball bearing at Support A of the screw's shaft.

4.3 Living With Uncertainties

We will apply now the Minimum Constraint Design approach to the above mentioned screw conveyor, in order to make it unresponsive to geometrical uncertainties. Let us assume that both supports, A and B, allow rotation in any direction and restrict linear displacements in any direction. Additionally, let us suppose that support B allows axial displacement.

The conditions are depicted in the Figure 9 and in a real situation they can be achieved by utilizing self-aligning bearings at both supports, A and B, with an axial gap in one of the bearing housings to allow axial displacement, which will be the support B in the current example.

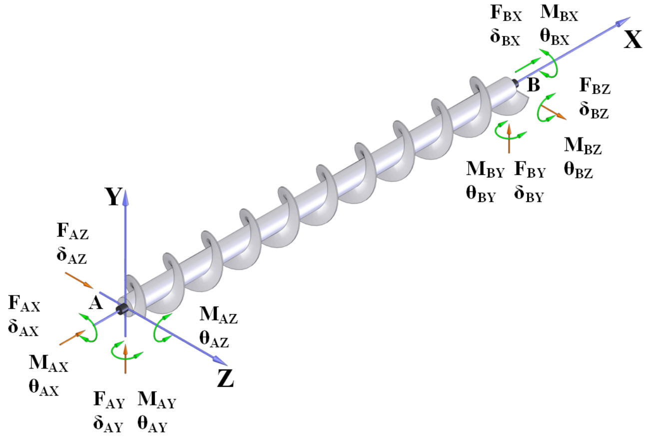

Figure 9. Screw conveyor and its MinCD supports.

The reactions marked in green colour in Figure 9, M_{AX}, M_{AY} and M_{AZ} at support A, and F_{BX}, M_{BX}, M_{BY} and M_{BZ} at support B, are nil.

In this manner, the screw is supported by the minimum number of external forces (marked in red colour in Figure 9). These external forces restrict five of the six degrees of freedom of the screw conveyor, allowing the screw's capability to adjust the shape of its supports to the real, imperfect, external conditions.

Zeroing the corresponding elements in Eq. (8), we have:

$$\begin{Bmatrix} \delta_{AX} \\ \delta_{AY} \\ \delta_{AZ} \\ \theta_{AX} \\ \theta_{AY} \\ \theta_{AZ} \\ \delta_{BX} \\ \delta_{BY} \\ \delta_{BZ} \\ \theta_{BX} \\ \theta_{BY} \\ \theta_{BZ} \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{AX} \\ F_{AY} \\ F_{AZ} \\ M_{AX} \\ M_{AY} \\ M_{AZ} \\ F_{BX} \\ F_{BY} \\ F_{BZ} \\ M_{BX} \\ M_{BY} \\ M_{BZ} \end{Bmatrix} \quad (11)$$

Eliminating the rows and the columns that are marked with a line in Eq. (11), where every element is zero, the rank of the design matrix becomes 5 x 5, which can be expressed as follows:

$$\begin{Bmatrix} \delta_{AX} \\ \delta_{AY} \\ \delta_{AZ} \\ \delta_{BX} \\ \delta_{BY} \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & X \end{bmatrix} . \begin{Bmatrix} F_{AX} \\ F_{AY} \\ F_{AZ} \\ F_{BX} \\ F_{BY} \end{Bmatrix} \quad (12)$$

Therefore, Eq. (12) can be re-written as follows:

$$\begin{Bmatrix} \text{Position of A} \\ \text{Position of B} \end{Bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} . \begin{Bmatrix} \text{Support at A} \\ \text{Support at B} \end{Bmatrix} \quad (13)$$

As one can see, now one has a diagonal design matrix, which means that we achieved an uncoupled solution and the external forces applied in the screw by its supports are all determined by equilibrium alone, independently of the real world imperfect conditions.

4.4 Pivoting Belt Conveyor

As it was found in the previous 2-D Example, also in the following case, a 3-D Example, there are always uncertainties of several types and several origins, which elimination usually is not possible.

Let us consider now a pivoting belt conveyor intended to have a pivoting reception section, so that the discharge section can move in order to distribute de conveyed load over a heap storage area, as shown in Figure 10.

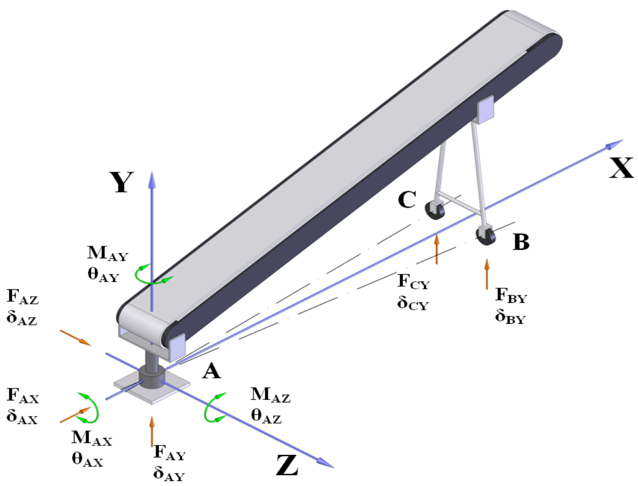

Figure 10. Pivoting Belt Conveyor and its Supports.

Since there are geometrical uncertainties mostly due to the manufacturing processes, it is not possible to ensure that, for example, both wheels would rest in a perfect plane surface, or that the pivoting axis would be precisely perpendicular to the plane that supports the wheels.

Let us suppose that the conveyor belt is supported by rigid components, which do not allow linear and angular displacement in any direction. In this case, Eq. (6) could be written as follows:

$$\begin{Bmatrix} \text{Position of A} \\ \text{Position of B} \\ \text{Position of C} \end{Bmatrix} = \begin{bmatrix} X & X & X \\ X & X & X \\ X & X & X \end{bmatrix} \begin{Bmatrix} \text{Support at A} \\ \text{Support at B} \\ \text{Support at C} \end{Bmatrix} \quad (14)$$

Again, one can remove the unnecessary reactions by installing a fixed, self-aligning bearing at support A, and wheels at supports B and C.

According to MinCD, the supports B and C should use spherical bearings running over a rigid flat surface, but wheels are used instead, given the impossibility of having those perfect conditions. However, the axes of the wheels must converge to the rotation centre A, in order to allow rotation of the conveyor around a vertical axis passing through point A, without slipping of the wheels.

Under a Minimum Constraint Design condition, the bearing and housing of the pivot do not allow the linear displacement of the shaft in any direction, but allow rotation around Y axis in any direction; the front wheels just react with vertical forces, and Eq. (14) can be written as follows:

$$\begin{Bmatrix} \delta_{AX} \\ \delta_{AY} \\ \delta_{AZ} \\ \theta_{AX} \\ \theta_{AY} \\ \theta_{AZ} \\ \delta_{BX} \\ \delta_{BY} \\ \delta_{BZ} \\ \theta_{BX} \\ \theta_{BY} \\ \theta_{BZ} \\ \delta_{CX} \\ \delta_{CY} \\ \delta_{CZ} \\ \theta_{CX} \\ \theta_{CY} \\ \theta_{CZ} \end{Bmatrix} = \begin{bmatrix}
X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 \\
0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & X & 0 & X \\
0 & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 \\
0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X & 0 & X & 0 & X & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 \\
0 & 0 & 0 & X & 0 & X & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 \\
0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 \\
0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 \\
X & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & 0 & 0 \\
0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & X & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 \\
0 & X & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 & 0 \\
X & 0 & X & 0 & 0 & 0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 \\
0 & X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X
\end{bmatrix} \begin{Bmatrix} F_{AX} \\ F_{AY} \\ F_{AZ} \\ M_{AX} \\ M_{AY} \\ M_{AZ} \\ F_{BX} \\ F_{BY} \\ F_{BZ} \\ M_{BX} \\ M_{BY} \\ M_{BZ} \\ F_{CX} \\ F_{CY} \\ F_{CZ} \\ M_{CX} \\ M_{CY} \\ M_{CZ} \end{Bmatrix} \quad (15)$$

where the position of each point is represented by its linear and angular displacements, δ_i and θ_i, respectively, and each support is represented by its reactions (the forces F_j and the torques M_j). In a compact mode,

$$\begin{Bmatrix} \text{Position of A} \\ \text{Position of B} \\ \text{Position of C} \end{Bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \begin{Bmatrix} \text{Support at A} \\ \text{Support at B} \\ \text{Support at C} \end{Bmatrix} \quad (16)$$

Thus, the Design Matrix is diagonal, corresponding to an uncoupled design.

5 CONCLUSION

The worked examples presented in the paper show that MinCD is a design approach that allows making the response of each support independent of the loads applied at the other supports.

In fact, supporting or guiding a component using a minimum number of constraints makes that component externally isostatic and all the external connection forces are determined by equilibrium alone.

In other words, MinCD allows transforming coupled and decoupled designs into uncoupled designs, which are the preferable design solutions according to the 1st axiom of Axiomatic Design. This means that Minimum Constraint Design it is a good practice.

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